Optica Applicata, Vol. XXIV, No. 4, 1994

Influence of nonlinear effect on modulation bandwidth in DFB lasers

A. MOSSAKOWSKA-WYSZYŃSKA

Institute of Microelectronics and Optoelectronics, Warsaw University of Technology, ul. Koszykowa 75, 00–662 Warszawa, Poland.

An influence of the gain saturation effect and spatial hole burning effect on the modulation bandwidth in distributed feedback (DFB) lasers is analyzed. We consider two cases: the dynamic model of DFB laser with pure index modulation and with the complex coupling coefficient. The strong dependence of the nonlinear effects on the modulation bandwidth is shown for two different dielectric laser structures.

The high-speed performance of high-data-rate lasers is often characterized by 3dB modulation bandwidth f_{3dB}, defined as the modulation frequency at which the small-signal response of the laser reduces by a factor of 2 relative to the zero frequency or dc response [1]. The 3dB modulation bandwidth can be expressed by the frequency Ω and the damping rate λ of the relaxation oscillation [2], which are a valuable tool in analyzing various laser parameters as spontaneous lifetime and cavity losses.

We present the influence of nonlinear effects (such as gain saturation and spatial hole burning) on the 3dB modulation bandwidth in planar and fibre dielectric waveguide distributed feedback lasers with pure modulation of the index coefficient (resulting in pure real coupling coefficient [3]) and with modulation of the complex coupling coefficient (for two cases of coupling: gain and loss). We based our method of analysis on the approach developed earlier for distributed feedback injection lasers [1], which was modified by taking into account spatial hole burning (SHB) effect and waveguide effects characteristic of planar and fibre structure. Moreover, our approach includes nonvanishing end reflectivity, which should be taken into account in real structures. The laser characteristics are obtained for planar and fibre Nd^{3+}:YAG lasers, which are one of the excellent candidates for a light source for optical communications operating at 1.064 μm and 1.3 μm wavelengths being within the low loss range of the optical fibres.

The 3dB modulation bandwidth can be expressed by the frequency Ω and the damping rate λ of the relaxation oscillation in the following way [1]:

$$f_{3dB} = \frac{1}{2\pi}\sqrt{\Omega^2-\lambda^2+2\sqrt{\Omega^2(\Omega^2+\lambda^2)+\lambda^4}}. \tag{1}$$

Hence, by determining λ and Ω, it is possible to obtain complete relation for 3dB modulation bandwidth as a function of system parameters. In the first DFB laser model with pure modulation of the index coefficient, the relaxation oscillations parameters are determined from the rate equations [2]. To solve the rate equations, it is necessary to define total intensity in the active medium $I_{nmq}(r,t)$ and the amplitudes of the two counterrunning waves R_q, S_q. Additionally, the description of the laser mode intensity includes a "coherent" term, which is responsible for the spatial hole burning effect [3]. Moreover, we assume that R_q, S_q are proportional to the threshold field distribution [3] and they have real time amplitude $A(t)$. In the real structures, we should expect non-zero end reflectivity, because of the discontinuity of the optical medium.

Similar as in [2], [3], using a small-signal perturbation solution of the laser rate equations, it is possible to obtain the expressions relating the frequency Ω and the damping rate λ to the system parameters in the following form:

$$\lambda = \frac{\gamma}{2\tau_Q}\frac{1}{(\tau\Omega)^2}\{\int dV|E_{nm}|^4[|f_{Rq}|^2+|f_{Sq}|^2+\eta(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})]^2\}\frac{1}{C}, \tag{2}$$

$$\Omega^2 = \frac{\gamma}{2\tau_Q}\left\{\int dV\frac{|E_{nm}|^4[|f_{Rq}|^2+|f_{Sq}|^2+\eta(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})]^2}{1+\gamma|E_{nm}|^2[|f_{Rq}|^2+|f_{Sq}|^2+\eta(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})]}\right\}\frac{1}{C} \tag{3}$$

where:

$$C = \frac{n_f}{n_{(eff)nm}L}\left\{\int dV\frac{|E_{nm}|^2[|f_{Rq}|^2+|f_{Sq}|^2+\eta(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})]}{1+\gamma|E_{nm}|^2[|f_{Rq}|^2+|f_{Sq}|^2+\eta(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})]}\right\}\frac{1}{C'} \tag{3}$$

$$\gamma = \frac{P_{out}}{P_s\mathcal{N}}, \quad \mathcal{N} = |\sinh\gamma_q L|^2|1-re^{i\varphi}|^2,$$

$E_{nm}(s)$ is appropriate field vector of planar or fibre unperturbed waveguide, η is the parameter which is 0 for envelope field approximation and 1 for including SHB effect, f_R, f_S are the longitudinal field distributions of the q-th laser mode, τ_Q is the cavity lifetime [3], P_{out} is the output power of nmq-th laser mode, P_s is the saturation power, c is the free-space velocity of light, $n_{(eff)nm}$ is the effective index of nm-th waveguide mode, n_f is the waveguide refractive index, α_L is the normalized output loss, τ is the spontaneous lifetime, L is the laser structure length, γ_q is the complex propagation constant and $re^{i\varphi}$ is the complex end reflectivity coefficient.

Up to this point, we have presented the dynamic model of DFB lasers with pure index modulation [4]. Generally, however, in real structures mixed case (modulation of the refractive coefficient and gain) occurs and the coupling coefficient is complex quantity [5], [6]. We consider two cases of the coupling, gain coupling and loss coupling, resulting in the complex coupling coefficient.

In general, both sets of the coupled laser rate equations for a loss and a gain coupling differ between each other. However, one of the coupled rate equations, describing the behaviour of the inversion density N has the same form for both kinds of coupling [7].

The difference appears in equations for the complex amplitudes R_q and S_q. In the case of the loss coupling they can be written in the following form [4]:

$$\frac{dR_q}{dz}+\frac{n_f}{c}\frac{dR_q}{dt}+(-\alpha_0 N+\alpha_L-i\delta)R_q=-\varkappa_{\text{Im}}S_q+i\varkappa_{\text{Re}}S_q, \tag{4}$$

$$-\frac{dS_q}{dz}+\frac{n_f}{c}\frac{dS_q}{dt}+(-\alpha_0 N+\alpha_L-i\delta)S_q=-\varkappa_{\text{Im}}R_q+i\varkappa_{\text{Re}}R_q \tag{5}$$

where $\alpha_0 N$ describes the gain of the active medium, δ is the deviation from the resonant (Bragg) frequency [4], $\varkappa$ is the complex coupling coefficient $\varkappa=|\varkappa L|e^{i\varphi_{1,g}}=\varkappa_{\text{Re}}+i\varkappa_{\text{Im}}$.

As we can notice, in this case an imaginary part of the coupling coefficient $\varkappa_{\text{Im}}$ does not depend on the laser mode intensity. The coupling strength is constant during the laser operation, and is determined by the geometry of the structure.

A different situation we observe in the case of the gain coupling. The coupling strength, relating to the periodical gain modulation depends on the mode intensity and it saturates with increasing light power in the laser cavity. Thus, the coupled equations for the field amplitudes have the following form [7]:

$$\frac{dR_q}{dz}+\frac{n_f}{c}\frac{dR_q}{dt}+(-\alpha_0 N+\alpha_L-i\delta)R_q=\frac{\alpha_0 VN}{2}S_q+i\varkappa_{\text{Re}}S_q, \tag{6}$$

$$-\frac{dS_q}{dz}+\frac{n_f}{c}\frac{dS_q}{dt}+(-\alpha_0 N+\alpha_L-i\delta)S_q=\frac{\alpha_0 VN}{2}R_q+i\varkappa_{\text{Re}}R_q \tag{7}$$

where V is the visibility of the gain grating induced externally (gain modulation). As we can notice, in contrast to the previous case the imaginary part of the coupling coefficient equals $\varkappa_{\text{Im}}=-\alpha_L VN/2$, depends on the laser intensity in similar way as the inversion density N.

Using the linear, small-signal perturbation solution of the coupled laser rate equations for the loss coupling, and for the gain coupling respectively, similar as in [2], [3], it is possible to obtain the general expressions for damping rate coefficient λ and frequency Ω of the relaxation oscillations for both cases of the coupling.

Thus, for the loss coupling, these quantities can be expressed in the following way:

$$\lambda=\frac{1}{\tau}\frac{\int dz(|f_{Rq}|^2+|f_{Sq}|^2)^2}{\int dz\dfrac{(|f_{Rq}|^2+|f_{Sq}|^2)^2}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}}, \tag{8}$$

$$\Omega^2=\frac{c}{n_f}\frac{\gamma}{\tau}\frac{\displaystyle\int dz\frac{(|f_{Rq}|^2+|f_{Sq}|^2)^2}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)^2}}{\displaystyle\int dz\frac{(|f_{Rq}|^2+|f_{Sq}|^2)}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}}$$

$$\times\left[\frac{2|\sinh\gamma_q L|^2+2\varkappa_{\text{Im}}\int dz(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})}{\int dz(|f_{Rq}|^2+|f_{Sq}|^2)}+2\alpha_L\right]. \tag{9}$$

For the gain coupling, the damping rate coefficient λ and the frequency Ω of the relaxation oscillations are derived as:

$$\lambda = \frac{1}{2}\,\frac{2\int dz(|f_{Rq}|^2+|f_{Sq}|^2+V\int dz(|f_{Rq}|^2+|f_{Sq}|^2)(f_{Rq}f^*_{Sq}+f_{Sa}f^*_{Sa})}{2\int dz\frac{(|f_{Re}|^2+|f_{Sq}|^2)^2}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}+V\int dz\frac{(|f_{Rq}|^2+|f_{Sq}|^2)(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}}, \tag{10}$$

$$\Omega^2 = \frac{\gamma}{\tau}\frac{c}{n_f}\,\frac{2\int dz\frac{(|f_{Rq}|^2+|f_{Sq}|^2)^2}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}+V\int dz\frac{(|f_{Rq}|^2+|f_{Sq}|^2)(f^*_{Rq}+f_{Sq}f^*_{Rq})}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}}{2\int dz\frac{(|f_{Rq}|+|f_{Sq}|^2)}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}+V\int dz\frac{(f_{Rq}f^*_{Sq}+f_{Sq}f^*_{Rq})}{1+\gamma(|f_{Rq}|^2+|f_{Sq}|^2)}}$$

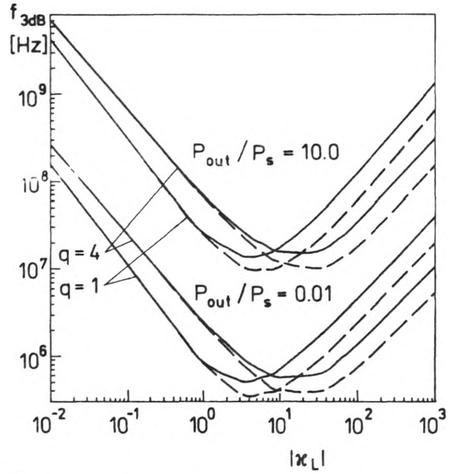
$$\times\frac{2|\sinh\gamma_q L|^2+2\alpha_L\int dz(|f_{Rq}|^2+|f_{Sq}{}^2)}{\int dz(|f_{Rq}|+|f_{Sq}|^2)}. \tag{11}$$

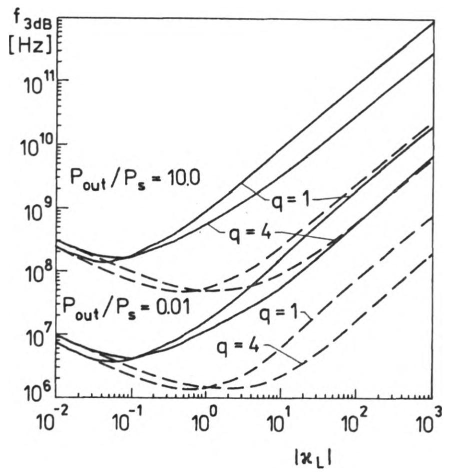
To obtain the characteristic of the 3dB modulation bandwidth, we use Equation (1) with appropriate relaxation oscillations equations for the cases of: pure modulation of the index coefficient (2) and (3), loss coupling (8), and (9), gain coupled distributed feedback lasers (10), and (11). The numerical results are obtained for the laser wavelength 1.06 μm and the spontaneous lifetime is $\tau = 260$ μs.

In the first DFB laser model with pure modulation of the index coefficient, in all figures, the dashed lines refer to the envelope field approximation and the characteristics revealing the SHB effect are plotted by solid line. In the case of the planar lasers the distributed losses are expressed by the mean-peak-valley height of the waveguide boundary surfaces $h = h_s = h_a$, normalized to the wavelength of the radiation [2]. The planar waveguide width is denoted by W. In the fibre structure the losses are expressed by the normalized distributed loss coefficient α_L. The radius of the fibre core is denoted by R.

Figures 1 and 2 show the dependence of modulation bandwidth on the normalized coupling coefficient $|\varkappa L|$ for the two different longitudinal modes of the fundamental planar (Fig. 1) and fibre (Fig. 2) transverse modes for two output power levels. In general, for both structures the modulation bandwidth increases with increasing output power. Additionally, all f_{3dB} plots exhibit a minimum in the presented range of the normalized coupling coefficient $|\varkappa L|$. This is caused by the fact that 3dB modulation bandwidth is practically determined by the relaxation oscillations frequency Ω, which is directly related to the small signal gain [8]. Thus, f_{3dB} characteristics should have the same character as the small signal gain characteristics. Therefore, we can say that the characteristics of the 3dB modulation bandwidth give evidence of the gain saturation.

Additionally, as we can notice for both structures, the spatial hole burning effect causes the increase of the modulation bandwidth f_{3dB} especially in strong coupling region. In this case, the energy stored in the laser structure is greater than in the structure with weak and moderate coupling and SHB effect is more evident.

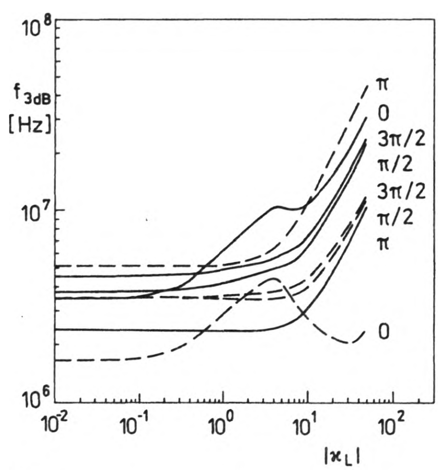


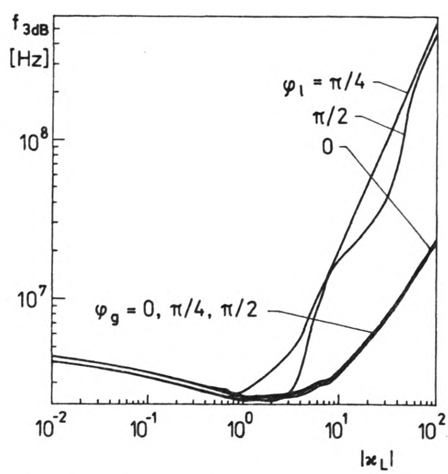


▲

Fig. 1. The f_{3dB} modulation bandwidth of the fundamental planar transverse mode as a function of the normalized coupling coefficient $|\varkappa L|$ with the normalized output power P_{out}/P_s and the longitudinal mode order q, as parameters (planar: TE_{0q}, $q = 1, 4$, $W = 1.46$ μm, $h = 0.01$, $\eta = 0$ – – –, $\eta = 1$ ——)

Fig. 2. The f_{3dB} modulation bandwidth of the fundamental fibre transverse mode as a function of the normalized coupling coefficient $|\varkappa L|$ with the normalized output power P_{out}/P_s and the longitudinal mode order q, as parameters (fibre: HE_{01q}, $q1, 4$, $R = 4.5$ μm, $\alpha_L = 1.0$, $\eta = 0$ – – –, $\eta = 1$ ——)





▲

Fig 3. The f_{3dB} modulation bandwidth of the first longitudinal mode of the fundamental planar transverse mode as a function of the normalized coupling coefficient $|\varkappa L|$ with the phase of the end reflectivity Φ, as a parameter (planar: TE_{01}, $W = 1.46$ μm, $h = 0.01$, $P_{out}/P_s = 1.0$, $r = 0.5$, $\varphi = 0, \pi/2, \pi, 3\pi/2$, $\eta = 0$ – – –, $\eta = 1$ ——)

Fig. 4. The f_{3dB} modulation bandwidth of the module of the complex coupling coefficient $|\varkappa L|$ with the phase of the coupling coefficient φ_1 for loss coupling and φ_g for gain coupling case, as parameters ($P_{out}/P_s = 1.0$, $\alpha_L = 0.1$, $q = 0$)

Moreover, the SHB effect shifts minima towards the smaller values of the coupling coefficient. Thus, the spatial hole burning effect acts similarly as losses in the structure.

Figure 3 shows the dependence of the 3dB modulation bandwidth on the normalized coupling coefficient $|\varkappa L|$ for the first longitudinal mode of the planar structure with the end reflectivity phase Φ, as a parameter. In general, the influence of the end reflectivity and SHB effect on modulation bandwidth is stronger for greater values of the coupling coefficient.

In the second DFB laser model with the complex coupling coefficient, the phase of the coupling coefficient is denoted by φ_1 for loss coupling case, and φ_g for gain coupling. Figure 4 shows the dependence of the modulation bandwidth on module of the complex coupling coefficient $|\varkappa L|$ for two kinds of coupling (loss and gain) for the coupling coefficient phase $\varphi_{1,g}$, as a parameter. In general, we observe the dynamic behaviour of the modulation bandwidth for loss and gain coupling. For the gain coupling an increase of the coupling coefficient causes smaller growth of the coupling strength than in the case of loss coupling. It is so because of the gain saturation effect. Finally, the modulation bandwidth has got greater values for loss coupling than for gain coupling.

In conclusions, we presented an analysis of the influence of the spatial hole burning effect and gain saturation effect on the 3dB modulation bandwidth in planar and fibre distributed feedback lasers as a function of the system parameters. We obtained an expression for the 3dB modulation bandwidth taking into account the waveguide effects, nonzero reflectivities of the structure as well as the SHB effect.

For the planar and fiber laser structures with pure modulation of the index coefficient, the spatial hole burning effect causes increasing values of the 3dB modulation bandwidth structures. However, the f_{3dB} modulation bandwidth is more sensitive to the SHB effect in the fibre waveguide structure than in the planar structure.

For the laser structures with complex coupling coefficient, the laser characteristics obtained reveal the influence of the gain saturation effect on the modulation bandwidth. In particular, as a result of the gain saturation, the difference between the loss coupling and gain coupling case appears in the moderate and strong coupling region.

References

[1] AGRAWAL G. P., Appl. Phys. Lett. **57** (1990), 1.
[2] SZCZEPAŃSKI P., MOSSAKOWSKA A., DEJNAROWICZ D., IEEE J. Lightwave Technol. **10** (1992), 220.
[3] MOSSAKOWSKA A., SZCZEPAŃSKI P., WOLIŃSKI W., IEEE J. Quantum Electron. **2** (1994), 230.
[4] KOGELNIK H., SHANK C. V., J. Appl. Phys. **43** (1972), 2327.
[5] KAPON E., HARDY A., KATZIR A., IEEE J. Quantum Electron. **18** (1982), 66.
[6] LOWERY A. J., NOVAK D., Electron. Lett. **29** (1993), 461.
[7] DULING N., RAYMER M. G., IEEE J. Quantum Electron. **20** (1984), 1202.
[8] SZCZEPAŃSKI P., SIKORSKI D., WOLIŃSKI W., IEEE J. Quantum Electron. **25** (1989), 871.