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Consequence of mode nonorthogonality in lasers

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The effect of the mode nonorthogonality on the laser operation of Fabry–Perot/distributed feedback (FP/DFB) lasers is presented. We use the semiclassical analysis based on the Fokker–Planck equation corresponding to the set of coupled Langevin equations. Numerical results of the solution of the single-mode operation obtained for DFB laser with nonvanishing end reflectivity and the complex coupling coefficient reveal the difference between the standard approach (for orthogonal laser modes) and the realistic model (mode nonorthogonality included).

1. Introduction

A fundamental source of quantum noise in laser devices is spontaneous emission from upper level atoms. The standard analysis of spontaneous emission in a laser cavity or in any other system involving interaction between atoms and electromagnetic radiation leads to a general principle that the rate of spontaneous emission is always equal to a signal energy of one extra photon per mode.

However, it has been shown that the nonorthogonal nature of the laser modes can enhance a coupling of the spontaneous emission from the active medium to the lasing modes [1]–[5]. This leads to the excess-spontaneous emission or the so-called excess-noise factor.

In general, the theory of the laser operation taking into account quantum noise requires the stochastic approach. In the semiclassical formalism this analysis usually includes the solution of the appropriate Fokker–Planck equation corresponding to the set of coupled Langevin equations. Using this formalism, a single and two mode operation of FP/DFB laser has been investigated (see, for example, [6]–[9]). Moreover, in all approaches presented [6]–[9], the orthogonality of the laser modes has been assumed.

In this paper, we extend a theory based on the Fokker–Planck equation to take into account the nonorthogonal nature of the laser modes. We obtain the stationary and nonstationary solution of the single-mode operation.

2. Stationary solution of Fokker–Planck equation

We start with the single mode Langevin equations for the electric field E,

$$\frac{dE}{dt} = (\tilde{a} - \beta|E|^2)E + q(t), \tag{1}$$

$$\frac{dE^*}{dt} = (\tilde{a}-\beta|E|^2)E^*+q(t), \tag{1}$$

where $\tilde{a}$ is the net gain coefficient, β is the self-saturation parameter and $q(t)$ is a Langevin noise term representing spontaneous emission fluctuations. In contrast to the standard approach [6]–[9], it is possible to show that this term, including nonorthogonal nature of the laser modes, is a correlated Gaussian process with [10]

$$\langle q(t)q(t')\rangle = \langle q^*(t)q^*(t')\rangle = 0,$$
$$\langle q(t)q^*(t')\rangle = 4K\delta(t-t'), \tag{2}$$

where K is the Petermann's excess-noise factor dependent on the spatial distribution of the laser modes given by [1], [2]

$$K = \int_v dsdz\Phi_N(s,z)\Phi_N^*(s,z) \tag{3}$$

where $\{\Phi_N(s,z)\}$ are the adjoint functions representing a set of eigenfunctions propagating in the opposite or reverse direction along the same system, and $s = (x,y)$ and v is the volume.

It is worth noting that for orthogonal laser modes [6]–[9], we have $\langle q(t)q^*(t')\rangle = 4\delta(t-t')$.

Defining $p(E,E^*t)$ as the probability density of finding the field being characterized by the amplitude E at time t, one can then replace the nonlinear equations (1) with the associated Fokker–Planck equation of the following form

$$\frac{\partial p}{\partial^2 t} = -\frac{\partial}{\partial E}(Ap)-\frac{\partial}{\partial E^*}(A^*p)+4D_{ij}\frac{\partial^2 p}{\partial E\partial E^*} \tag{4}$$

where A, A^* represent the drift coefficients:

$$A = (\tilde{a}-\beta|E|^2)E,$$
$$A^* = (\tilde{a}-\beta|E|^2)E^*, \tag{5}$$

and D_{ij} is the diffusion matrix.

For orthogonal laser modes, the off-diagonal elements of the diffusion matrix are equal to unity and the on-diagonal elements are equal to zero. In the case of real laser structures with nonorthogonal eigenmodes, it is possible to show that the off-diagonal elements of the diffusion matrix correspond to the excess-noise factor K, given by Eq. (3) with finite, usually greater than unity value. In this case the stationary solution of Eq. (4) takes the form

$$p(\tilde{I}) = Q^{-1}\exp\left[\left(\frac{1}{2}\tilde{a}\tilde{I}-\frac{1}{4}\beta\tilde{I}^2\right)K^{-1}\right], \tag{6}$$

and the second cumulant equal to the variance or mean squared deviation of the intensity is given by

$$\sigma = \langle\tilde{I}^2\rangle-\langle\tilde{I}\rangle^2 = 1/\beta[2-2\tilde{a}/(KQ)-4/Q^2] \tag{7}$$

where $\bar{I} = |E|^2$ is the normalized intensity of the laser mode as follows:

$$\langle \bar{I} \rangle = \tilde{a}/\beta + 2K/(\beta Q) \tag{8}$$

and Q is the normalization constant,

$$Q = \sqrt{\pi K/\beta}\exp[\tilde{a}^2/(4K\beta)]\mathrm{erfc}(-\tilde{a}/2\sqrt{K\beta}). \tag{9}$$

3. Nonstationary solution of Fokker–Planck equation

The nonstationary solution is necessary in order to calculate the correlation function of the amplitude and of the intensity fluctuation. For further considerations it is convenient to introduce the polar coordinates ($E = re^{i\varphi}$). Then the Fokker–Planck equation (4) is transformed into

$$\frac{\partial p(r,\varphi,t)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}[(\tilde{a}-\beta r^2)r^2 p] + K\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right] + \frac{1}{r^2}\frac{\partial^2 p}{\partial \varphi^2}. \tag{10}$$

Since $p(r,\varphi,t)$ is a periodic function in φ and since the time derivative is of the first order, we can make the ansatz

$$p(r,\varphi,t) = \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} A_{nm} f_{nm}(r) e^{in\varphi} e^{-\lambda_{nm}t}. \tag{11}$$

Putting this expansion into Eq. (10), we obtain the equation for the eigenfunctions f_{nm}, which is a recognized Sturm–Liouville equation

$$(hf'_{nm})' - q_n f_{nm} + \frac{\lambda_{nm}}{K} h f_{nm} = 0 \tag{12}$$

where $h(r)$ and $q_n(r)$ are given by:

$$h(r) = r\exp\left[\frac{1}{K}\left(\beta\frac{r^4}{4} - \tilde{a}\frac{r^2}{2}\right)\right], \quad q_n(r) = \left[\frac{1}{K}(2\tilde{a} - 4\beta r^2) + \frac{n^2}{r^2}\right]h(r). \tag{13}$$

Using a variational method it is possible to obtain the lower order eigenvalues λ_{10} and λ_{01} that allow us to get an approximate value for the correlation function of the amplitude $g(\tau)$

$$g(\tau) = \langle r(t+\tau)e^{-i\varphi(t+\tau)} r(t) e^{i\varphi(t)}\rangle \approx g(0)\exp(-\lambda_{10}\tau), \tag{14}$$

and the correlation function of the intensity fluctuation $K(\tau)$

$$K(\tau) = \langle (r^2(t+\tau) - \langle r^2 \rangle)(r^2(t) - \langle r^2 \rangle)\rangle \approx K(0)\exp(-\lambda_{01}\tau). \tag{15}$$

The Fourier transform of the amplitude correlation function gives the lineshape in frequency domain with the linewidth $\Delta\nu$. The linewidth is proportional to the eigenvalue λ_{10} and in the case of more realistic system (mode nonorthogonality included) depends on the excess-noise factor.

3. Numerical results

We obtained the laser characteristics revealing the effect of the mode non-orthogonality for DFB lasers having nonvanishing end reflectivity and for DFB lasers with the complex coupling coefficient. For simplicity, we assumed parameter β to be normalized to unity.

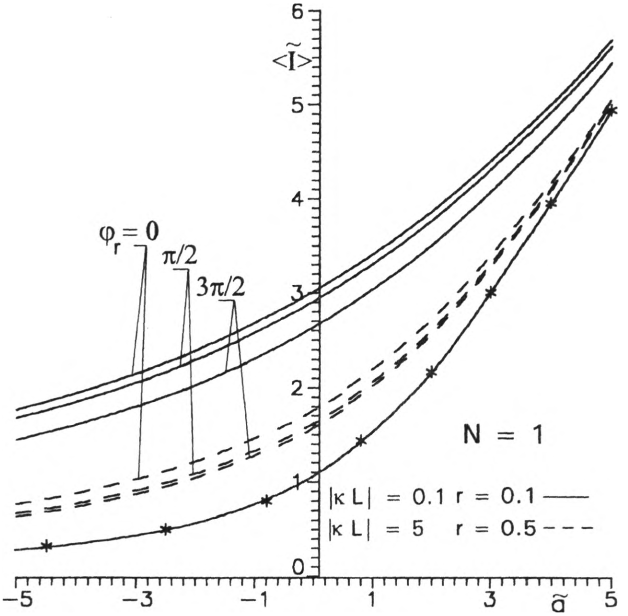

Fig. 1. Mean normalized laser intensity $\langle \tilde{I} \rangle$ as a function of the normalized pump parameter $\tilde{a}$ for the coupling coefficient $|\kappa L| = 0.1$ and the amplitude of the end reflectivity $r = 0.1$ (solid curves) and for $|\kappa L| = 5$ and $r = 0.5$ (dashed curves), with the phase of the end reflectivity φ_r, as a parameter. The asterisks curve is obtained for orthogonal modes

Figures 1 and 2 show the mean laser intensity $\langle \tilde{I} \rangle$ and variance σ, *i.e.*, the intensity fluctuations, as a function of the normalized pump parameter $\tilde{a}$. As can be seen, when the laser operates below and slightly above threshold, an increase of the phase of the end reflectivity causes a decrease of the mean laser intensity $\langle \tilde{I} \rangle$ and the intensity fluctuations σ are minimized. Moreover, at a given pump parameter $\tilde{a}$, the effect of the phase on the mean laser intensity $\langle \tilde{I} \rangle$ and on the intensity fluctuations σ becomes smaller for the stronger coupling strength (*i.e.*, higher $|\kappa L|$ and r, see Figs. 1 and 2, dashed curves). Simultaneously, the laser characteristics tend to that obtained for orthogonal modes (Fig. 1 – asterisks line). Furthermore, the difference between two laser models (*i.e.*, for orthogonal and nonorthogonal modes) diminishes with increasing pump parameter $\tilde{a}$. However, in strong pumping region (Fig. 2), for the laser intensity fluctuations an inversion in the laser characteristics is observed.

The effect of the mode nonorthogonality on the eigenvalue as well as the linewidth is shown in Fig. 3 for DFB laser with complex coupling coefficient. The

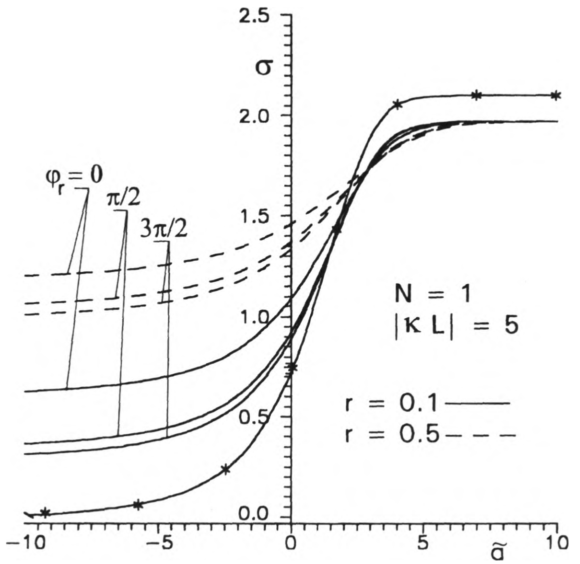

Fig. 2. Variation of the intensity fluctuations σ with normalized pump parameter $\tilde{a}$, for the amplitudes of the end reflectivity $r = 0.1$ (solid lines) and $r = 0.5$ (dashed lines), with the phase of the end reflectivity φ_r, as a parameter. The coupling coefficient is $|\varkappa L| = 5$. The asterisks curve is obtained for orthogonal modes

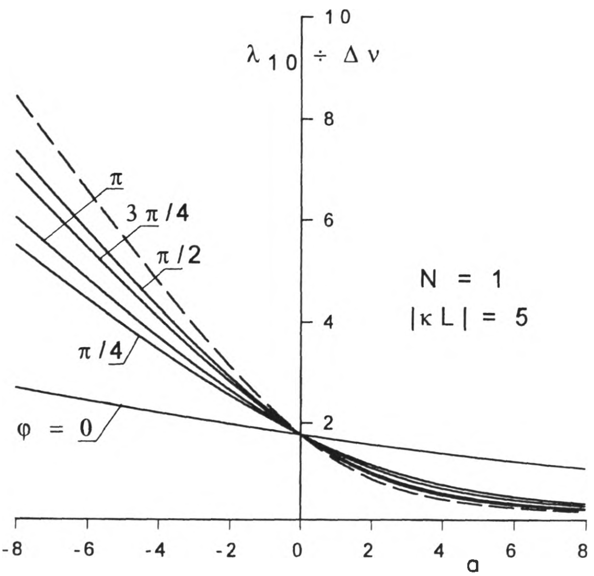

Fig. 3. Eigenvalue λ_{10} and laser linewidth *vs.* the normalized pump parameter $\tilde{a}$, for the coupling strength $|\varkappa L| = 5$, with the phase of the complex coupling coefficient φ as a parameter. The dashed curve is obtained for orthogonal modes

dashed curve is obtained in the case of orthogonal laser modes. As we can see, when the laser operates below threshold the phase of the coupling coefficient equal to zero (pure index coupling) provides the minimal values of the eigenvalue and the light coherence is the greatest. However, for above threshold operation we observe an inversion in the laser characteristics and the influence of the phase of the coupling coefficient becomes less important.

In this paper, the influence of the mode nonorthogonality on the laser operation has been investigated. Using the stochastic approach based on the Fokker–Planck equation it was possible to obtain the mean laser intensity and the variation of the intensity fluctuations (steady-state solution of the single-mode operation) and the correlation functions that determine the laser linewidth as well as the light coherence (nonstationary solution).

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