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Optical system testing by means of first and second derivatives of the wavefront aberration function*

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A modified method of a quantitative wavefront testing in the Talbot interferometer is proposed. The coefficients of the wavefront aberration function are found by interpreting fringes, the intensity distribution of which is proportional to the first and second derivative of the function. The setup with a spatial filtering for rapid measurements of derivatives of aberration functions is presented.

1. Introduction

The most common testing methods of optical system are the Foucault test, interferometry and the Ronchi test. The Foucault test is a very sensitive and convenient method. It is, however, rather a quantitative one, since the position of shadow is difficult to define. Interferometry, providing direct information about the shape of the tested wavefront, is on the one hand the most informative of all the methods used for testing spherical or astigmatic deviations. On the other hand, however, it is complex, expensive, sensitive to vibrations (not common path), and in order to obtain high accuracy of quantitative measurements the data should be reduced.

The third technique, i.e., the Ronchi test, is also a well-known and widely used method. Opticians willingly use it because of its simple setup parameters. However, the difficulty in interpreting the observed patterns quantitatively is its main disadvantage. Several methods for solving this problem are proposed, namely:

– comparison of the observed Ronchi pattern with the computed patterns, for selected aberrations [1],

– production of special Ronchi gratings with curved lines that give straight fringes when the wavefront is correct [2],

– purely qualitative interpretation of the Ronchi test relying on experience,

– semi-quantitative technique, based on simple analytical formulae com-

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bining the values of some basic types of aberration and the parameters of fringe patterns [3].

In this paper a modified method is proposed for a quantitative testing of wavefronts in the Talbot interferometer [4] (which may be treated as a special case of the Ronchi interferometer). It is by interpretation of fringes, the intensity distribution of which is proportional to the first and second derivatives of the phase function.

2. Analysis

Let us consider the optical system shown in Fig. 1. The grating is illuminated by a quasi-plane beam characterized by the wavefront aberration function $\exp[i\Phi(x, y)]$ in the grating plane. The distorted self-images are formed at the intervals $z = 2d^2/\lambda$ behind the grating. The distortion of the grating due to the aberration function may be checked by the following methods:

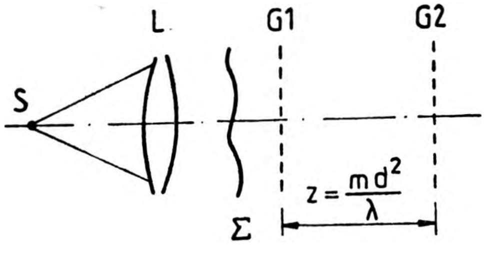


Fig. 1. Talbot interferometer arrangement used for testing the wavefront aberration function. G1, G2 – two identical Ronchi gratings. The M-th self-image of the grating G1 is detected by G2 (S – source) L – lens, Σ – aberrated wavefront,

i) The Moiré method, i.e., by inserting the detecting grating G2 in the plane of the M-th distorted self-image of the grating G1. The intensity distribution of the Moiré pattern is proportional to the first derivative $\partial\Phi(x, y)/\partial x$ of the aberration function in the direction perpendicular of the grating lines.

ii) Differentiation of the distorted self-image due to the lateral displacement of the grid G1 or a photographic plate (Fig. 2) implemented between the two exposures. The Moiré pattern visualizes the second derivative $\partial^2\Phi(x, y)/\partial x^2$ of the aberration function [4].

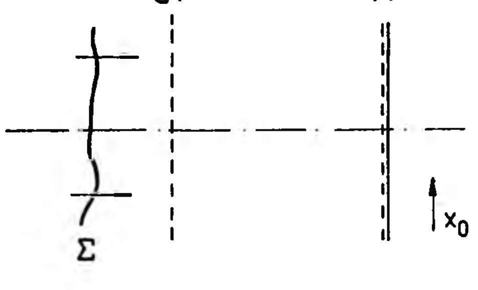


Fig. 2. Arrangement used for obtaining the second derivative of the wavefront aberration function by the double exposure technique. Photographic plate FP is placed in the M-th self-image of G1

The "Moiré method" is equivalent to the two-beam interference in the Ronchi interferometer [5], while the differentiation of the distorted self-image corresponds to the three-beam interference [6, 7].

Let us assume, for simplicity, that the linear diffraction grating has cosinusoidal amplitude transmittance (this assumption does not affect the generality of the approach). The grating is illuminated by the quasi-plane beam characterized by the wavefront aberration function $\exp[i\Phi(x, y)]$ in the plane of the grating; the rulings of grating are parallel to the y-axis. We assume that $\Phi(x, y)$ is a slowly varying function of the coordinates x, y and z. At a distance z from the grating the complex amplitude is given by

$$T(x, y, z) = A_0\exp[i\Phi(x, y)] + A_1\exp\left\{i\frac{2\pi}{d}\left[x - \frac{\lambda z}{2d} + \frac{d}{2\pi}\Phi(x-\Delta, y)\right]\right\} + A_{-1}\exp\left\{-i\frac{2\pi}{d}\left[x + \frac{\lambda z}{2d} - \frac{d}{2\pi}\Phi(x+\Delta, y)\right]\right\} \tag{1}$$

where $\Delta = (\lambda/d)z$ is the lateral displacement of ± 1 diffraction orders with respect to the zero-order beam, λ is the light wavelength.

Let us consider the intensity distribution at the distance $z = Md^2/\lambda$ corresponding to the M-th self-image plane. According to the analysis given by PATORSKI [4] the contrast of the fundamental harmonic of the intensity distribution is proportional to the first derivative $\partial\Phi(x, y)/\partial x$ of the aberration function. It may be visualized by putting the detecting grating G2 in the M-th self-image plane with the relative lateral shift x_0. In such a case the Moiré intensity distribution is described as follows:

$$I_M \pm \propto \cos\frac{2\pi}{d}\left[x_0 + \frac{d}{2\pi}\Delta\frac{\partial\Phi(x, y)}{\partial x}\right] \tag{2}$$

where $\Delta = (\lambda/d)z = Md$.

The localization of the Moiré fringes is given by the following equation:

$$\frac{2\pi}{d}\left[x_0 + \frac{d}{2\pi}\Delta\frac{\partial\Phi(x, y)}{\partial x}\right] = (2p+1)\pi. \tag{3}$$

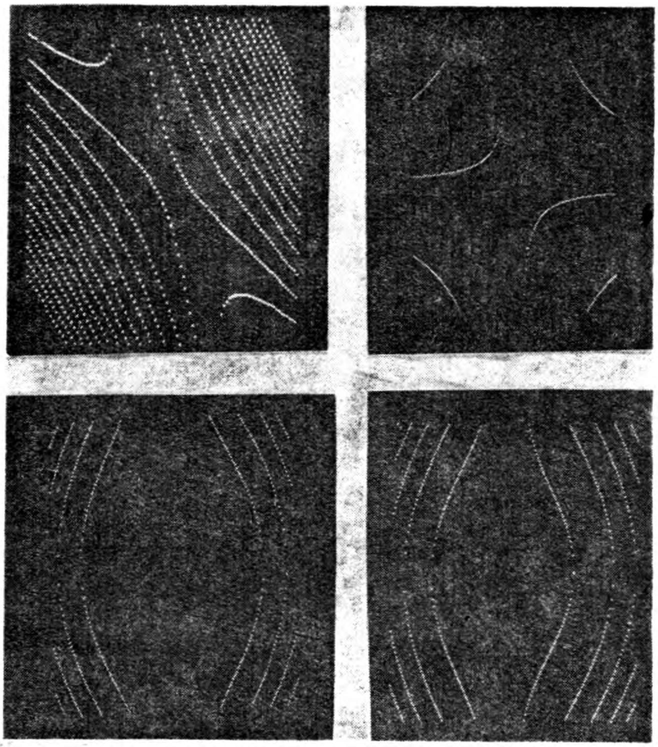
The aberration function may be expressed as a power series in both coordinates [6]. In this series the terms of degree smaller than 2 are omitted and the third-order aberration is considered only

$$\Phi(x, y) = w_{20}(x^2+y^2) + w_{40}(x^2+y^2)^2 + w_{22}xy + w_{30}x^3 + w_{21}x^2y + w_{12}xy^2 + w_{03}y^3 \tag{4}$$

where the terms of the second degree represent defocusing and the third-order astigmatism, third-degree terms represent coma and the term $w_{40}(x^2+y^2)^2$ represents third-order spherical aberration. While testing the above given function $\Phi(x, y)$ by means of the Moiré method we get the localization of the fringes in form of the following equation:

$$2w_{20}x+4w_{40}x(x^2+y^2)+w_{22}y+3w_{30}x^2+2w_{21}xy+w_{12}y^2 = \text{const}. \tag{5}$$

In order to interpret the fringe pattern we must find the coefficients w_{mn}. When one of the basic aberrations, e.g., pure coma or astigmatism, occurs, its value can be determined from simple analytical formulae combining the parameters of fringe patterns and the values of aberration [3]. Figure 3 shows



a b

c d

Fig. 3. Numerical simulation of the Ronchi pattern of the third-order coma: $w_{21} \neq 0$ (a), $w_{30} \neq 0$, $w_{21} \neq 0$, $w_{12} \neq 0$ (b), and the third-order spherical aberration: $x_0 = 0$ (c), $x_0 \neq 0$ (d)

examples of numerical simulation of the Moiré fringes obtained for the third-order coma (Figs. 3a, b) and third-order shperical aberration (Figs. 3c, d). Let us consider a complex aberration wavefront. Figure 4 shows examples of simulated fringes obtained for the wavefront suffering from the third-order aberrations. In general, in order to obtain the w_{mn} coefficients, a complex analysis of the fringe pattern is required [8–10]. Thus the second stage of optical system testing needs much more specialized and expensive equipment than that required at the first stage, during which the fringe pattern is obtained.

In order to apply the Ronchi test technique to quantitative measurements in a workshop the interpretation method of fringe pattern should be simplified.

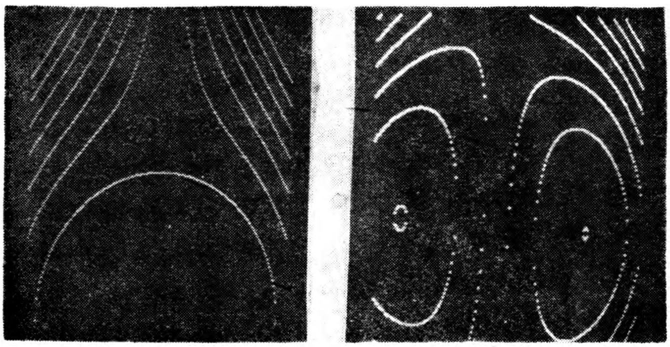


Fig. 4. Numerical simulation of the Ronchi pattern of the wavefront suffering from the third-order aberrations: $w_{40}, w_{21} \neq 0$

Here we propose to test the second deritative of the wavefront aberration function $\partial^2\Phi(x, y)/\partial x^2$. The localization of the Moiré fringes obtained after differentiation of the distorted self-image is given by

$$\frac{2\pi}{d} x_0 \left[1 + \frac{d}{2\pi} \Delta \frac{\partial^2 \Phi(x, y)}{\partial x^2}\right] = (2p+1)\pi \tag{6}$$

where x_0 is the relative lateral shift between the grating and the recording plate in the direction perpendicular to the grating lines.

Putting the second derivative of the function given in (4) the localization of the Moiré fringes may be expressed as

$$4w_{40}(3x^2+y^2)+6w_{30}x+2w_{21}y = \text{const.} \tag{7}$$

In this way the second-degree curves give information about the third-order coma and spherical aberration, so the equation is by two orders lower than this obtained in the Twymann-Green interferometers and one order lower than in shearing interferometry. Therefore, the values of the coefficients w_{40}, w_{30} and w_{21} may be determined by simple measurements of the parameters of fringe patterns. These valies may be used for further interpretation of the patterns obtained by the Moiré method. Figure 5 shows examples of the fringe patterns representing spherical aberration. In the case of coma, the Moiré fringes are straight and their inclination depends on the direction of the axis of symmetry of coma. The fringe pattern representing the complex aberration function is shown in Fig. 6. It should be mentioned, however, that the sensitivity of the method for testing the second derivative $\partial^2\Phi(x,y)/\partial x^2$ is much lower than that of the Moiré method. Therefore, it may be used, e.g., for the spherical aberration, when $w_{40} \geq 1\lambda$, and is recommended for testing optical systems with larger aberrations. In general, higher-order aberrations may be determined by detecting the higher-order derivative of the tested function. In such a case it is necessary to use large shear or dynamic way for detecting the second-order derivative.

Fig. 5. Numerical simulation of the fringe pattern representing the second derivative of the spherical aberration $w_{40} \neq 0$

a b

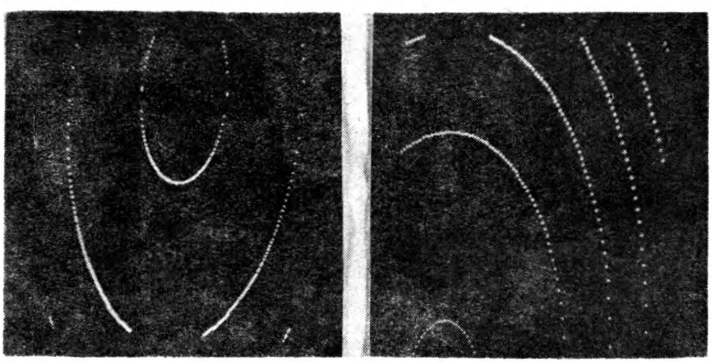


Fig. 6. Numerical simulation of the fringe pattern representing the second derivative of the wavefront aberration function: w_{40}, $w_{21} \neq 0$ (a), w_{40}, w_{21}, $w_{30} \neq 0$ (b)

3. Experimental setup

The dynamic detection may be obtained in the setup containing a spatial zero-order filtering (Fig. 7). Let us place the optical system behind the Talbot interferometer illuminated by a spatially coherent quasi-plane beam characterized by the wavefront aberration function. The Fourier spectra of double grating diffraction are formed at the rear focal plane of the first lens. The spatial filter selects beams in the $m+n = 0$ order direction, where m and n denote the diffraction order number at the first and second grating, respectively. The phenomenon may be simplified to the three beam interference ((0, 0), (+1, +1),

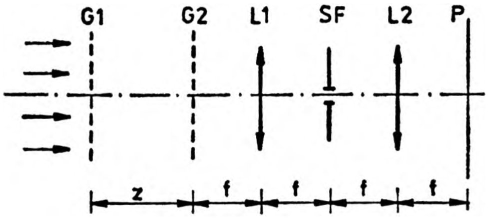


Fig. 7. Spatial filtering arrangement used for differentiation of a quasi-plane wavefront. Spatial filter SF selects the beam propagation direction. The first and second derivatives of the wavefront function are visualized at the output plane P conjugate to the second grating plane

(−1, −1)) which at the plane P forms the fringe pattern giving the information about the second derivative of the wavefront being tested. Figure 8 shows examples of the second derivative of the wavefront with spherical aberration. The derivative was obtained by mechanical differentiation (Fig. 8a) and in the spatial filtering configuration of the Talbot interferometer (Fig. 8b).

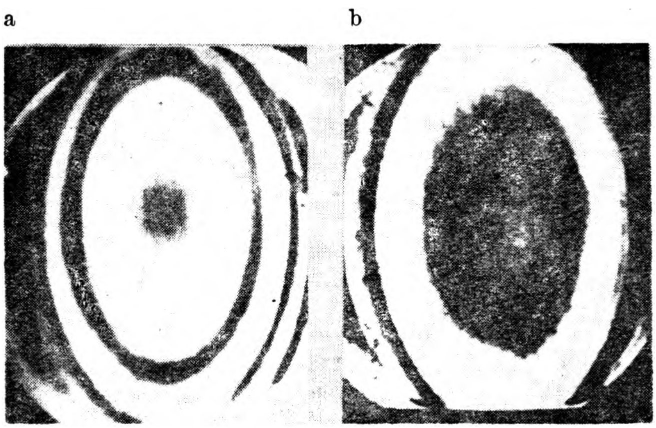


Fig. 8. Fringe pattern obtained by: double-exposure method and mechanical differentiation (a), in the spatial filtering configuration of the Talbot interferometer (b). The fringes depict the wavefront function with spherical aberration

Another advantage of the above setup is the possibility of testing the first derivative of the wavefront by the spatial filtering in the $m+n = 1$ order direction, which was done by a simple change of the spatial filter SF. The above technique is an improved (real-time) version of the shearing interferometry method in which the processing of negative is no longer required, but a good-quality optical processor is necessary.

4. Conclusions

Optical system testing by means of the second derivative allows some of the coefficients of the wavefront aberrations to be determined more rapidly and accurately than by first derivative (the Moiré method). The shape of the fringe does not depend markedly on the degree of defocusing and astigmatism, which makes the interpretation easily. The method is especially recommended for the workshop testing of optical systems with aberrations greater than 1λ.

The proposed setup with the spatial filtering seems to be very convenient for rapid measurements, in which quantitative results may be obtained.

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Испытание оптических систем с помощью производной первого и второго порядка функции аберрации

Предложен модифицированный метод количественного испытания в интерферометре Тальбота. Был найден коэффициент волновой функции аберрации с помощью соответствующей интерпретации линий, распределение интенсивностей которых пропорционально производной первого и второго порядка этой функции. Представлена система с пространственным фильтрованием для быстрых измерений производных функций аберрации.