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On the problem of diffractographic detection of linear dimension by means of spatial filtering

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The application of spatial filtering in measurements of linear dimensions has been discussed. The optimal parameters of amplitude spatial filter has been found. The calculated values have been compared with those obtained from an experiment.

1. Introduction

The magnitude of optical signal produced in diffractographic method, where slit is of width D, was detected by a rectangular space filter. The numerical calculation of the metrological properties as well as the results of measurements are presented. The experimental set-up is shown in Fig. 1. He-Ne LG–600 laser has been used. The light from He-Ne laser was directed on the slit (edges 2 and 3) of width D. One of the edges was movable (edge 2), the accuracy of its displacement by means of indicator Fimeter 200 being 0.2 μm. Lens of focal distance 787.8 mm was used to produce Fourier spectrum related to the slit of width D. In the focal plane of the lens rectangular amplitude filter Fi is placed. The filter influences the intensity of the light collected on the silicon fotocell

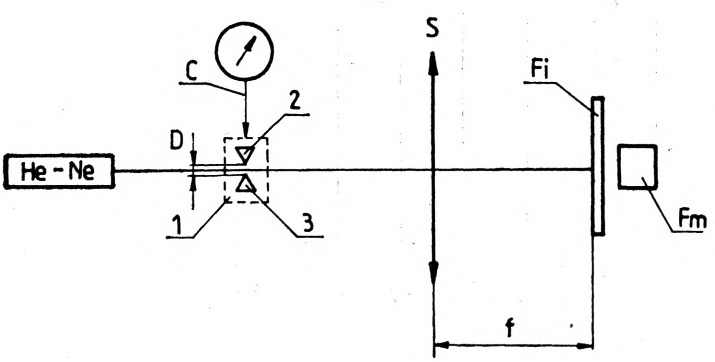

Fig. 1. Experimental set-up: 1 — slit, 2, 3 — edges, S — lens, Fi — rectangular amplitude filter, Fm — detector, C — indicator Fimeter 200

BPYP 07A. When width D of the slit is changed, the space distribution of diffraction fringes changes according to the formula (1)

$$I(\bar{x}, D) = A^2 D^2 \frac{\sin^2\left(2\pi \frac{D}{2} \bar{x}\right)}{\left(2\pi \frac{D}{2} \bar{x}\right)^2} \tag{1}$$

where: $\bar{x}$ — spatial frequency,
$I(\bar{x}, D)$ — light intensity,
A — light amplitude.
After rearrangement, for $D = D_0$ we get

$$I(\bar{x}, D_0) = A^2 \frac{\sin^2(\pi D_0 \bar{x})}{(\pi \bar{x})^2}. \tag{2}$$

2. Structure of the filter

The rectangular amplitude filter is composed of equally spaced transparent and non-transparent strips. The width and position of the strips are selected according to the following procedure. From Fig. 2 it can be seen that $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_6$ ... are the inflexion points, thus the points $\bar{x}_1, \bar{x}_3, \bar{x}_5$ are determined

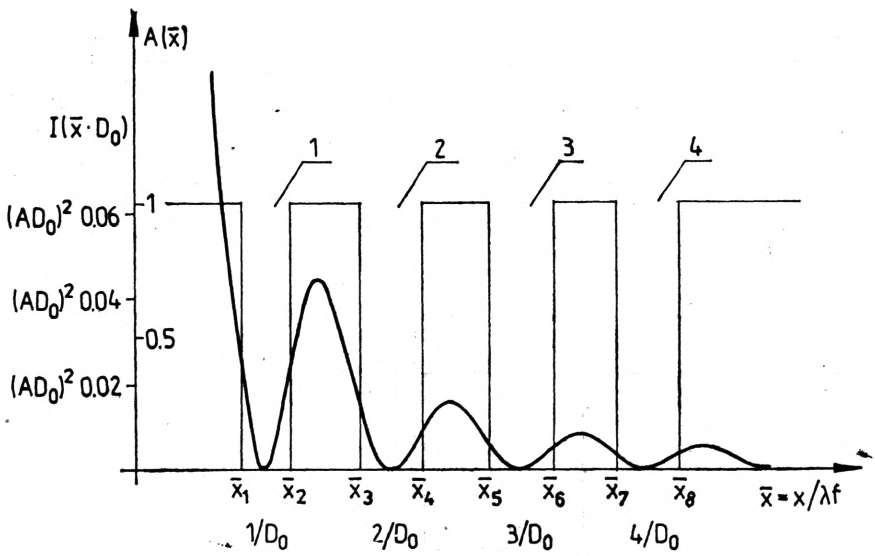

Fig. 2. Rectangular amplitude filter: 1, 2, 3, 4 — number of transparent part, $\bar{x}$ — spatial frequency, I — light intensity

according to the relations:

$$\int_{\bar{x}_1}^{1/D_0} I(\bar{x}, D_6)\,d\bar{x} = \int_{1/D_0}^{\bar{x}_2} I(\bar{x}, D_0)\,d\bar{x}, \tag{3a}$$

$$\int_{\bar{x}_3}^{2/D_0} I(\bar{x}, D_0)\,d\bar{x} = \int_{2/D_0}^{\bar{x}_4} I(\bar{x}, D_0)\,d\bar{x}, \tag{3b}$$

$$\int_{\bar{x}_5}^{3/D_0} I(\bar{x}, D_0)\,d\bar{x} = \int_{3/D_0}^{\bar{x}_6} I(\bar{x}, D_0)\,d\bar{x}. \tag{3c}$$

Half of filter Fi consists of nontransparent strips placed between 0 and $\bar{x}_1$, between $\bar{x}_2$, $\bar{x}_3$ and between $\bar{x}_4$, $\bar{x}_5$, and so on. So the transparent part marked as number 1 is positioned between $\bar{x}_1$ and $\bar{x}_2$, the one marked as number 2 is between $\bar{x}_3$, $\bar{x}_4$, the one marked as number 3 between $\bar{x}_5$, $\bar{x}_6$, and so on (see fig. 2).

The position of the strips is chosen so as to produce minimal optical signal at slit width $D = D_0$. For slit width $D_0 = 138$ μm the parameters of the filter were determined numerically. The value of the function $I(\bar{x})$ was computed for the range $1/\lambda f \leqslant x \leqslant 60/\lambda f$ (where λ — wavelength) in $0.2/\lambda f$ steps, the positions of $\bar{x}_2, \bar{x}_4, \bar{x}_6 \ldots$ points were then calculated with the accuracy of $10^{4-}/\lambda f$. The values of $\bar{x}_1, \bar{x}_3, \bar{x}_5, \ldots$ points were obtained by numerical integration of integrals (3) with the accuracy of $10^{-2}/\lambda f$.

3. Properties of rectangular filter

The response of the system S to D ranging from 138 μm to 209 μm was determined numerically for different parts of the filter (number 1, 2, ...), and their combinations. The calculation was carried out according to the following relations:

$$S_1(D) = 2\int_{\bar{x}_1}^{\bar{x}_2} I(\bar{x}, D)\,d\bar{x}, \text{ for strips number 1}$$

$$S_2(D) = 2\int_{\bar{x}_3}^{x_1} I(\bar{x}, D)\,d\bar{x}, \text{ for strips number 2}$$

$$S_{1-2}(D) = 2\left(\int_{\bar{x}_1}^{\bar{x}_2} I(\bar{x}, D)\,d\bar{x} + \int_{\bar{x}_3}^{\bar{x}_4} I(\bar{x}, D)\,d\bar{x}\right), \text{ for strips number 1 and 2,}$$

and so on.

In general, it may be written as

$$S_{P-N}(D) = 2\sum_{k=P}^{N}\int_{\bar{x}_{2k-1}}^{\bar{x}_{2k}} A^2 \frac{\sin^2(\pi D\bar{x})}{(\pi\bar{x})^2}\,d\bar{x}.$$

All these relations are non-linear with respect to D, since the second derivative

$$S''_{P-N}(D) = 2A^2\sum_{k=P}^{N}\frac{\sin(2D\pi\bar{x})}{D\pi}\Bigg|_{\bar{x}_{2k-1}}^{\bar{x}_{2k}}$$

is always nonzero.

To find the filter arrangement, for which the relation S vs D would be sufficiently close to the linear relation, the following investigations were performed. The maximal sensitivities of the system $(\Delta S/\Delta D)_{max}$ calculated for different arrangements of the filter are shown in Table 1.

Table 1

$S(D)$ [μW/μm]	S_1	S_{1-2}	S_{1-3}	S_{1-4}	S_2	S_{2-3}	S_{2-3}	S_3	S_{3-4}
$(\Delta S/\Delta D)_{max}A^2$	103	144	165	180	59	95	120	41	70

The range of the variable D related to the quasilinear part of S vs D characteristics was determined by calculating the relative sensitivities $\eta = (\Delta S/\Delta D)/(\Delta S/\Delta D)_{max}$. It has been assumed that for quasilinear range relative sensitivity must be less than 20 % (e.g., $\eta \leqslant 20\,\%$). Relative sensitivity vs D, calculated for the filter consisting of differently marked strips, is shown in Fig. 3. The characteristics marked with S_1 and S_2 are related to strip number 1 and 2, respectively, that marked with S_{1-2} to both strips, number 1 and 2, and so on. The calculation has shown that the characteristic S_1 posseses the largest range of quasilinear part, namely from 158 to 184 μm, as shown in Fig. 3c. It was necessary to verify whether the width of the transparent strip influences the linearity of $S(D)$ curve. Having moved the points $\bar{x}_{2k}$ close to the inflexion point it is possible to make the transparent part broader or thinner. The point $\bar{x}_{2k}$ was moved in steps $0.2/\lambda f$, the points $\bar{x}_{k2-1}$ were then calculated according to the Eqs. 3a,b,c. The calculated maximal sensitivity is shown in Table 2.

Table 2

$S(D)$	S_{1a}	S_{1b}	S_{1c}	S_{1d}	S_{1e}
$(\Delta S/\Delta D)_{max}A^2$ [μW/μm]	105	108	103	91	71

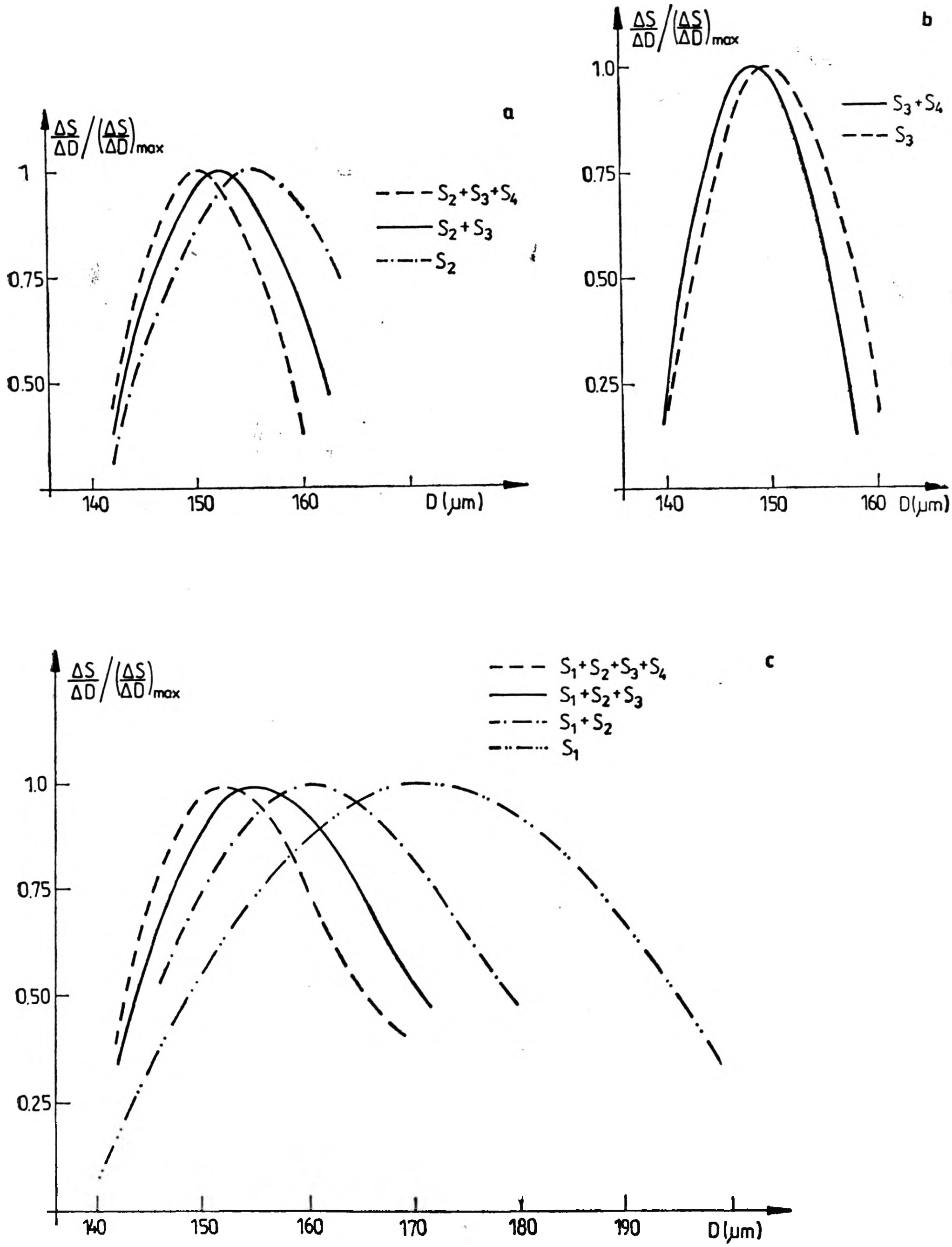

Fig. 3. Relative sensitivity for displacement as a function of number and order of the rectangular transparent parts of the filter: **a** — for S_2, S_{2-3}, S_{2-4}; **b** — for S_3, S_{3-4}; **c** — for S_1 S_{1-2}, S_{1-3}, S_{1-4}, (S_i — optical signal from particular transparent part of the filter)

On the assumption that the relative sensitivity η is less than 20 %, the range is quasilinear (see Fig. 4). From this figure it is apparent that the width of the transparent part does not affect the quasilinearity range. The influence of D_0 on quasilinearity range was chacked, too. The results are presented in Table 3. It can be seen from this table that this range is virtually independent of D_0, as shown in Fig. 5. Hence, it can be concluded that the rectangular amplitude

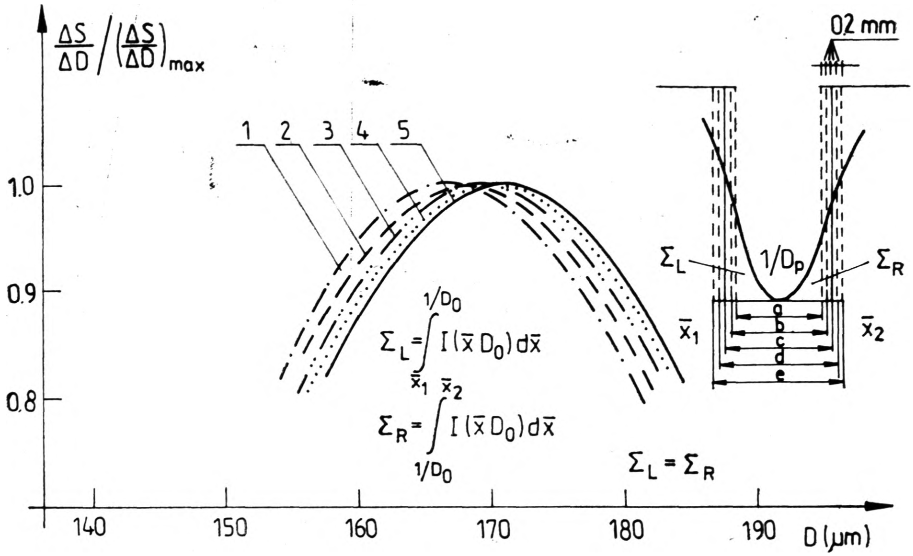

Fig. 4. Relative sensitivity for displacement as a function of the width of the transparent part number 1 (curve 1 – for a, 2 – for b, 3 – for c, 4 – for d, 5 – for e)

Table 3

$S(D)$	$S_{1c\,128}$	$S_{1c\,138}$	$S_{1c\,148}$
$(\Delta S/\Delta D)_{\max} A^2$ [$\mu W/\mu m$]	103	103	104

filter of number 1, and type c has the optimal parameters with respect to the linearization of the system response. The influence of the accuracy of filter dimensions as well as the change of laser beam parameter on the error of slit width D measurement was investigated. This accuracy depends on the dislocation of the laser beam axis. For the condition $S_{1c}/S_{1c138} = 2.356$ the optical signal S_{1c} was detected. For 32 measuring points the relative error was 4 %. This, after considering the experimental calibration curve gives for measurement error $\Delta D = \mp 2$ μm. The application of more stable laser would result in decrease of the above error which can be considered as the maximal error of the method. The arrangement precision of the filter and other optical elements

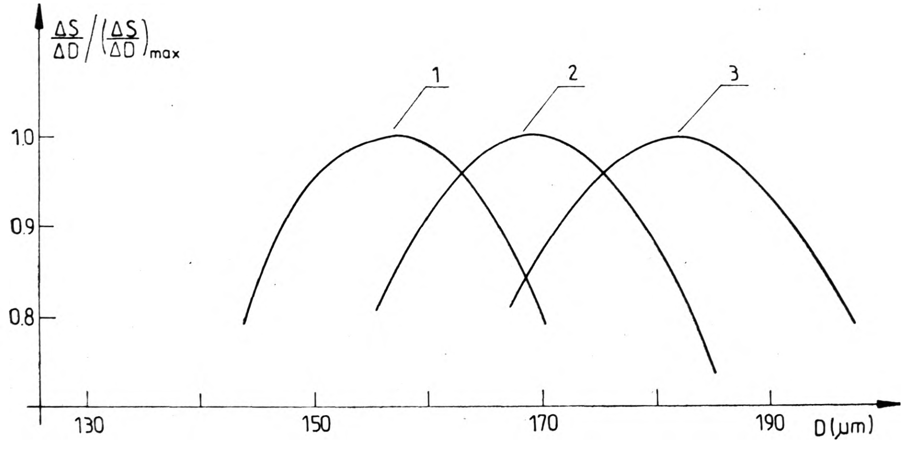

Fig. 5. Relative sensitivity for displacement vs the width of the initial slit D_0 taken as a parameter (curve 1 – for $D_{01} = 128$ μm, 2 – for $D_{02} = 138$ μm, 3 – for $D_{03} = 148$ μm)

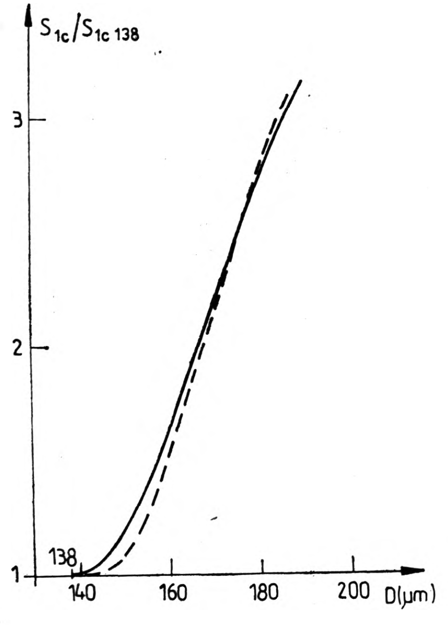

Fig. 6. Experimentally determined calibration curve for base slit $D_0 = 138$ μm (———) compared with the calculated calibration (— — — —)

would result in the reduction of systematical error which is corrected in the course of the system calibration. The response was investigated experimentally for the initial slit width $D_0 = 138$ μm. The results are shown in Fig. 6.

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Проблема детекции линейных размеров при помощи пространственной фильтрации

В настоящей работе предложен метод пространственной фильтрации измерения линейных размеров. Приведены оптимальные параметры для амплитудного, прямоугольного фильтра. Сопоставлены полученные результаты измерения линейного размера с численными результатами, полученными теоретическим путём.