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Limiting properties of the autocompensating refractometric measurements based on total internal reflection

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The subject of this paper is to discuss the influence of absorption of the measured medium on the refractometric measurements based on the so-called critical (limiting) angle and realized by using photoelectric detector. The errors of most commonly used compensation refractometric method consisting in observing the light-shadow border of the light beam reflected from the surface of the continuous media at the vicinity of the limiting angle are analysed in detail. The limiting accuracy of this method has been determined for the case of most common measurements, used in industry. For the region of the most common applications the way of the optimum use of the method has been determined by distributing the systematic errors for the change of absorption amounting up to 2×10^{-3}. The formulae allowing an optimal design of the signal processing and balancing the autocompensative system are given.

1. Introduction

For many years the method of light refractive index based on the total internal reflection at the border of two continuous media has been applied in design of compensation analysers. The way of measurement is coming to determining the limiting angle and in consequence the quantity of one of the substances of the two-component mixture. This method and the way of measurements are mainly used for the opaque and turbid (due to suspension of colloids, for instance) fluids, since to the transparent fluids spectrometric and interferometric methods may be applied, as they assure high accuracy and good repeativity of the measurements.

The method based on the measurement of the limiting angle is practically the only one accessible so far for the opaque fluids, however, it appeared to be not univocal in the case of absorbing media. Since for the absorbing media (even slightly) the effect should not be described by Fresnel formulae, in spite of what was commonly done for many years in the past. The deviations from the Fresnel formulae appear distinctly, which should be taken into account in design and construction of analysers based on the total internal reflection, independently of whether the integrating, compensating, pulsed or polarization methods are used. In practice, the objects of measurements are fluids (pastes, pomaces, food dyers, petrol products and some pharmaceutical fluids). To this end several types of refractometric analysers have been built, some of which being designed to minimize the influence of absorption. The main point of interest, however, was to eliminate (or to compensate suitably) the external factors influencing essentially the measurement, i.e., the changes in sensitivity of photoelectric sensors, changes in emission of light source (information carrier) or to eliminate the influence of temperature on the changes in the refractive index [1–3].

The only known solutions taking account of the influence of medium absorption are the following: the polarization method [4], in which the influence of absorption is partly neglected by performing the measurement "before" the limiting angle and by the application of a conventional (changeable with the measurement range) reference to the limiting angle, and the method of a simultaneous measurement of the refractive index and the absorption coefficient [5], [6]. None of these methods, however, guarantee the construction of a simple measuring set-up giving univocal results of measurements.

2. Influence of the absorption of measured media

In the up-to-date elaborations and considerations, the deviations from the Fresnel formulae due to absorption have been usually neglected giving size to a number of errors, in particular in compensating system in which, among others, a false tracing point appears. The consequences of deviations from the Fresnel formulae largely concern also the classical Abbe refractometers if applied to measurements ot refractive index in absorbing media. The formulae for the intensity of light reflected from an absorbing medium were derived by LITTMANN [7] as applicable to the classical refractometry. The abbreviated procedure and the formulae obtained by Littmann are shortly presented below.

While considering reflection at the border of nonabsorbing and absorbing media, as it is in the case of refractometric analysis, we shall use the following notations:

i) nonabsorbing media

– (real) reflection angle – i,

– absorption index – $\varkappa$,

– (real) refractive index – $\hat{n}_1 = m$;

ii) absorbing media

– (complex) refractive angle – $\hat{i}_t$,

– (complex) refractive index – $\hat{n}_2 = n_2(1-j\varkappa_2)$,

– absorption coefficient – $\varkappa_2 = \varkappa = -\frac{\lambda_0}{4\pi n_2}\frac{\ln D}{d}$ (where D – transmission in the layer of the thickness d, λ_0 – wavelength of the light in vacuum),

– amplitude of the incident light – E,

– amplitude of the reflected light – R,

– intensity of the incident light – e,

– intensity of the reflected light – r.

The component vibrating in the parallel direction is denoted by index p, while that vibrating in the perpendicular direction – by index s.

The reflection law and the Fresnel formulae are the starting point, assuming that $E = \text{const} = 1$, $e = \text{const} = 1$:

$$\frac{\sin i}{\sin i_t} = \frac{\hat{n}_2}{\hat{n}_1} = \hat{n} \tag{1}$$

where $\hat{n} - \frac{n_2}{n_1}(1-j\varkappa_2) = n(1-j\varkappa_2)$,

$$\hat{R}_p = \frac{\hat{n}\cos_i - \cos i_t}{\hat{n}\cos i + \cos i_t}, \tag{2}$$

$$\hat{R}_s = \frac{\cos i - \hat{n}\cos i_t}{\cos i + \hat{n}\cos i_t}. \tag{3}$$

From Eq. (1) it follows that

$$\cos i_t = \frac{\sqrt{\hat{n}^2 - \sin^2 i}}{\hat{n}}. \tag{4}$$

By inserting (1) and (4) to (2) we obtain

$$\hat{R}_p = \frac{n^2(1-j\varkappa)^2\cos i - \sqrt{n^2(1-j\varkappa)^2 - \sin^2 i}}{n^2(1-j\varkappa)^2\cos i + \sqrt{n^2(1-j\varkappa)^2 - \sin^2 i}}. \tag{5}$$

Multiplying the Eq. (5) by its complex conjugate we get the intensity $r_p = \hat{R}_p \hat{R}_p$.

For the sake of simplification the substitutions

$$\hat{R}_p = \frac{(a-jb)-(c+jd)}{(a+jb)+(c+jd)} = \frac{(a-c)+j(b-d)}{(a+c)+j(b+d)} \tag{6}$$

may be used to obtain finally

$$r_p = \frac{(a-c)^2+(b-d)^2}{(a+c)^2+(b+d)^2}. \tag{7}$$

An analogical procedure gives the evaluation of

$$r_s = \frac{(\cos i - c)^2 + d^2}{(\cos i + c)^2 + d^2}. \tag{8}$$

If the light is unpolarized (natural)

$$r = \frac{r_p - r_s}{2},$$

hence the final formula for reflection is

$$r = \frac{1}{2}\frac{(a-c)^2+(a-b)^2}{(a+c)^2+(a+b)^2}+\frac{1}{2}\frac{(\cos i-c)^2+d^2}{(\cos i+c)^2+d^2} \tag{9}$$

where:

$$a = n^2(1-\varkappa^2)\cos i,$$
$$b = 2n^2\varkappa\cos i,$$
$$c = \left(\frac{p+q}{c}\right)^{1/2},$$
$$d = n^2\varkappa/c.$$
$$p = n^2(1-\varkappa^2)-\sin^2 i,$$
$$q = (p^2+4n^2\varkappa^2)^{1/2}$$

The formula (9) is accurate and valid for arbitrary i_n and $\varkappa$.

When considering the formula (9) for decaying absorption, i.e., for:

$$\varkappa = 0,$$
$$a = n^2\cos i,$$
$$b = d = 0,$$
$$p = n^2-\sin^2 i = c^2,$$

the Fresnel formulae for a nonabsorbing medium are obtained. Considering the same formulae for the case of weak absorption at the limiting reflection angle we get similarly, like in the case above:

$$a = n^2\cos i,$$
$$b = p = 0,$$

but

$$q = 2n^2\varkappa,$$
$$c = d = n\sqrt{\varkappa},$$

thus, the formula (9) for $\sin^2 i = n^2$ takes the following form:

$$2r = \frac{n^4\cos^2 i-2n^3\cos i\sqrt{\varkappa}+2n^2\varkappa}{n^4\cos^2 i+2n^3\cos i\sqrt{\varkappa}+2n^2}+\frac{\cos^2 i-2\cos i\, n\sqrt{\varkappa}+2n^2\varkappa}{\cos^2 i+2\cos i\, n\sqrt{\varkappa}+2n^2\varkappa}.$$

Taking into account the fact that $\cos^2 i = 1-\sin^2 i$, and restricting our considerations to the limiting angle, assuming further that $\varkappa \ll 1$, we obtain

$$r = \frac{n\sqrt{1-n^2}-2\sqrt{\varkappa}}{n\sqrt{1-n^2}+2\sqrt{\varkappa}}+\frac{1}{2}\frac{\sqrt{1-n^2}-2n\sqrt{\varkappa}}{\sqrt{1-n^2}+2n\sqrt{\varkappa}}, \tag{10}$$

which means that the deviation from the Fresnel formula occurs also for a weak absorption even when $\sin n = 1/n$ (otherwise r should be equal to unity).

The derivation of the formula (9) was given after Littman. This formula confirmed by experiment is convenient for calculations. The procedure used by Littman in order to derive this formula was not precise, the absorption coefficient was not exactly defined, and for calculations the concept of complex angle is used. Although the latter concept has no influence on the final form of the derived relation, it is of no physical meaning, and hence it may evoke some misconfidence. The Littman formulae may be derived rigorously, starting with fundamental relation concerning the electromagnetic wave propagation in continuous media [2].

Based on these formulae numerical calculations were performed and the reflection coefficient r vs the incidence angle at the vicinity of the limiting total reflection angle i_g was plotted for $\varkappa = 2\times10^{-3}$ (Fig. 1). This value is practically extreme with respect to the industrial fluids being the objects of measurement. The plot for $\varkappa = 0$ (according to the Fresnel formulae) is shown in the same figure.

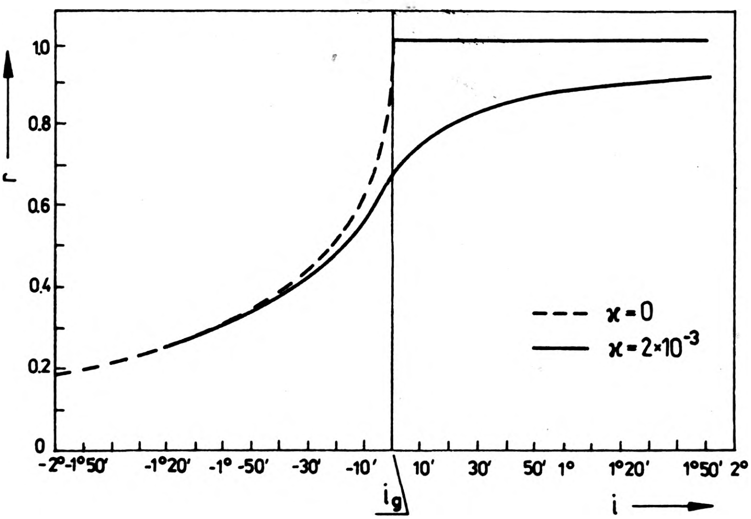

Fig. 1. Reflection coefficient vs the angle of nonpolarized light beam incidence on the border line of media $r = f(i)$. The beam falls on the border from the optically denser medium of $n = n_1/n_2 = 0.9$

Comparing the curves, it occurs that the problem of absorption cannot be neglected and should be taken into account in application of any measuring method.

3. Analysis of photoelectric accuracy of tracing refractometers

The refractometers based on the total internal reflection are most popular and often called the refractometers of critical angle. The angle is recorded geometrically as a border line between light and shadow, the position of which being the function of the refractive index of the fluid under test. The position of the border line is traced by a classical positioning servomechanism.

There also exists a convinction that this method allows to construct the most accurate refractometers. It is characteristic that according to many firms the sensitivity is a basic parameter, since it is really excellent, whereas the accuracy is often not defined. Many producers carelessly recommend these devices for the measurements of absorbing fluids, assuring that the absorption does not influence the measurement accuracy. The accuracy of such a refractometer, with respect to the measurement range, was analysed because of the change in the shape of curve $r = f(i)$ within the assumed interval observed both in the absorbing and nonabsorbing media.

From the discussion carried out earlier, it follows that, even in the case of the nonabsorbing fluids, such a device exactly measures the limiting angle in one point only (of the measurement range), while in all the remaining sites the tracing point is false because of the error due to the method itself. The knowledge of both the value and behaviour of this error is indispensable for optimal design of refractometric analysers.

In the analysis it has been assumed that the energy of both the incident and reflected beams (for $i > i_g$) is uniform, i.e., the change in beam energy in the region $i < i_g$ is a simple mapping of $r = f(n)$. It has been also assumed that the photoelectric sensors are equal and uniform, i.e., their surface sensitivity is the same at each point, and that photoelectric characteristics are linear. This may be exemplified, for example, by an ideal two-track photoresistor (of negligibly small interspace between the tracks) working in a classic system of Wheatstone bridge. Such a situation is shown in Fig. 2.

In this respect, the evaluation of maximal error and dispersion of errors due to the false tracking are essential. For this reason, the changes in the ratio of the areas covered by both the detectors (including the angular change of the "window

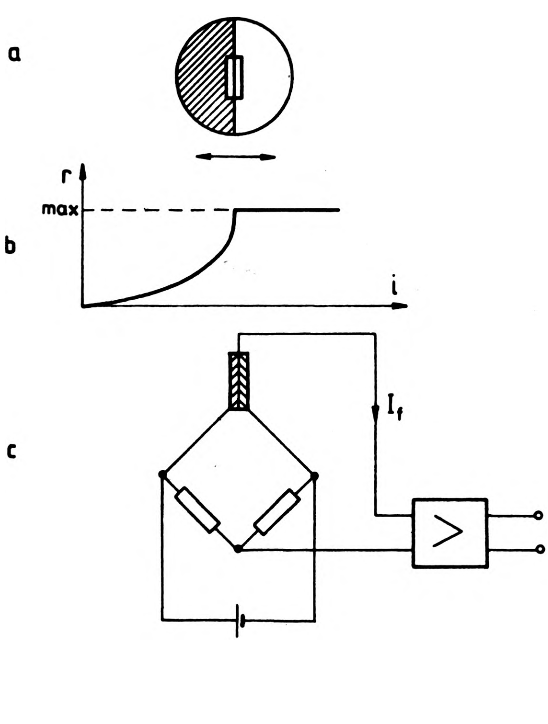

Fig. 2. Principle of signal processing $I_f = f(n)$. **a** – geometric image of the so-called critical angle refractometers, being the processing (tracking) basis, **b** – energy distribution in the image due to $r = f(i)$, **c** – photoelectric sensors (double photoresistor) in the Wheatstone bridge

capacity" of the detector) with changing angle i_g within the assumed measurement range have been examined. Such a procedure is possible due to the assumptions of linearities and uniformities. The ratio of areas is responsible for relative detuning of the Wheatstone bridge, directly proportional function (in the surroundings of equilibrium state) of which is the current (voltage) in the bridge diagonal deciding about further processing. It is also essential to know, how should the photoresistors "look" at the light-shadow border line, i.e., which of the manners shown in Fig. 3 is optimal and whether there also exists the optimum in other configurations.

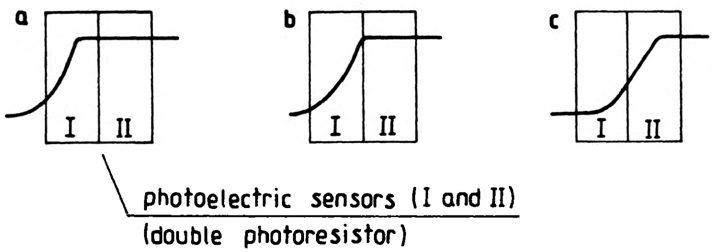

Fig. 3. Three most frequently encountered variations of the mutual position of sensors (double photoresistors) and the curve of energy distribution $r = f(i)$ at the surrounding of the angle i_g: light-shadow border line lies in the middle of the first (I) sensor (**a**), is covering the border of sensors (**b**), and lies in the middle of the second (II) sensor (**c**)

The formula (9) has been used for calculations assuming that: the measurement range is contained within $n_p = 1.333$ (H_2O) and $n_k = 1.3860$, since it is the one most frequently used in the food and sugar industry, for which the change from n_p to n_k corresponds to the shift of light-shadow border line by 35 mm in the image (detector) plane. The detector aperture (width of the photodetectors) is equal to 1 mm (the value most frequently used in practice). The calculations have been carried out for the relative refractive index $n = 0.89834$ corresponding (angularly) to one half of the measurement range. The shape of error curve in the vicinity of the angle i_g for the nonabsorbing fluids is shown in Fig. 4. The maximal error

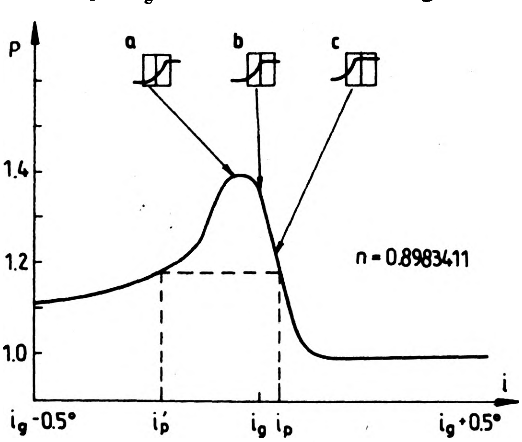

Fig. 4. Run of the ratio of the areas covered by the detectors depending on their position with respect to the limiting angle: light border in the middle of the first (**a**), between (**b**), and on the second sensor (**c**)

caused by the change in shape of the curve $r = f(i_g)$ within the measurement range amounts to 1.2×10^{-5}, while the relative error, referred to the accepted measurement range, is equal to $0.028\,^0/_0$. From the presented dependence it may be concluded that the relative error is by an order magnitude less than the inaccuracies of the most perfect photographic refractometers (of class 0.2), while the variant **a** in Fig. 3 is optimal due to position of the sensors with respect to the light-shadow border line. In this variant the sensitivity is highest when $25–50\,^0/_0$ of the first sensor area is illuminated with the reflected beam (the highest slope of the ratio of areas – Fig. 3). In variant **c** (Fig. 3) the sensitivity is lowest when one of the sensors operates exclusively in the reflected beam and the other – in the region $i < i_g$ at the light-shadow border line.

The systematic error for the beginning and the end of the measurement range in the case of a perfect balance is shown in Fig. 5 for the midpoint of the (angular) range as a function of the distance of the first detector centre from the position corresponding to the limiting angle. The dispersion of errors corresponds to the so-called false tracking at the range margins.

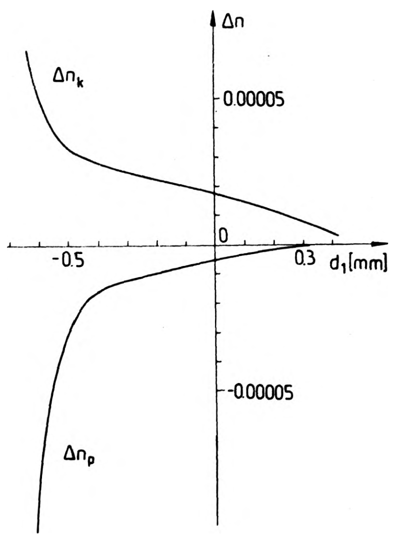

Fig. 5. d_1-distances of the first detector centre from the position corresponding to i_g (Δn_k – error at the end of measuring range, Δn_p – at the beginning of measuring range)

The error decreases monotonically when the detector is shifted toward the angle i_g. This procedure, however, may be used in a limited way, since in the extreme case the detector is positioned within the reflected beam which ends the measurement possibilities. As it follows from Fig. 5, the optimal position is when the first detector (inserted in the reflected beam, $i > i_g$) is illuminated in $25–50\,^0/_0$; this corresponds to the 0–25 mm distance of its centre from the position i_g (Fig. 5). Assuming for the sake of simplicity that the midpoint of the first detector covers the light-shadow border line (i_g) we can estimate the maximal dispersion of errors evaluated as being equal to 2.4×10^{-5}. By assuming that the system is not balanced for i_g corresponding to the angular centre of the range, but for i and the corresponding symmetric dispersion of the errors at its border lines, it may be estimated that for the nonabsorbing fluids the error should not exceed 1.2×10^{-5} in the absolute values. In Figure 6 the families of curves $P = f(i_g)$ for different detector widths are shown. If for the nonabsorbing fluids the reduction of the width of the photodetectors seems to be advantageous, this procedure being confirmed, in particular, by the relation $dP/di = f(i)$ (Fig. 7), then in the case of the absorbing fluids (Fig. 9) this procedure should take an opposite direction. At the width smaller than 1 mm (corresponding to the angle of about 0.0015°) the ratio of areas is too small, but it increases distinctly with the increasing width of the slit (this affects the dynamics of the device). Another very important conclusion follows from the shape of the curves, namely the same value of P occurs on both the sides of i_g (Fig. 6). Therefore, the construction of servomechamism system must guarantee the first readjustment at the same value of P for another value of i will

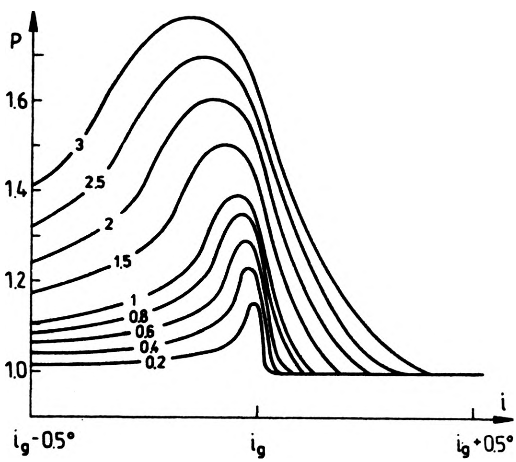

Fig. 6. Widths of detectors, in mm (for $\varkappa = 0$) vs the ratio of areas (of the light fluxes accepted by the detectors)

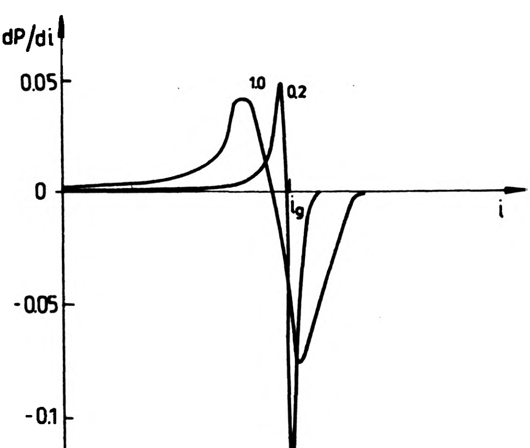

Fig. 7. Derivatives of the ratio of areas as a function of incidence angle

not cause the balance on the other side of i_g. This situation is shown in Fig. 4, where to the angle i_p within the optimal measurement region $0.3 < d < 0.5$, and corresponding to the limiting angle i_g and enlarged by the angle of false tracking (systematic error) corresponds to the angle i_p in the opposite side of i_g for which the ratio of area is the same. First readjustment at a rapid change of the refractive index in the measured medium cannot provoke the system to exceed the balancing element position corresponding to i_p. If it is the case, then the system will be balanced for i_p and the random error will be several times higher. This effect is especially dangerous for nonabsorbing fluids when a double detector is used. The double detector is often used because of its structural convenience, but we shall show below that by applying the method we obtain the solution which is not optimal for metrological reasons either.

4. Influence of the detector width

Figure 6 shows a family of curves representing dependence of the ratio of areas P on i for different widths of the photodetector tracks, while Fig. 7 represents the course of the derivative dP/di. It has been assumed that the detectors are in contact (two-track detector) and that the midpoint of the first detector lies on i_g ($d_1 = 0$). The reduction of photodetector width increases the maximal slope of the curve P, which should be advantageous for metrological reasons, but it is not since then, as already mentioned above, the value of P decreases and the danger of measurement uncertainty increases. The results of the errors of the refractive index measurements for different widths S of the detectors under the assumed working conditions are shown in Fig. 8. The theoretical error Δn decreases to zero with the detector width diminishing to zero. This, however, has no practical meaning since then the ratio of area P decreases to 1 and the measurement becomes impossible. From the course of the curve P for different width of the detectors under the same

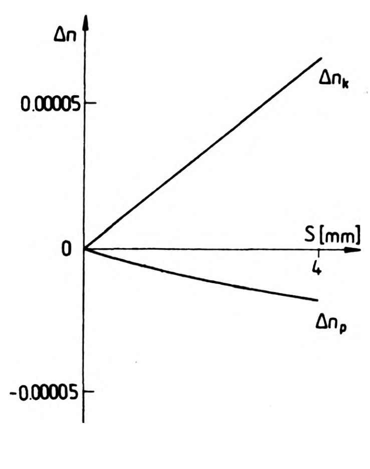

Fig. 8. Absolute error Δn for the beginning Δn_p and the end Δn_k of the measurement range vs the change in detectors width S

working conditions and the assumed limiting coefficient of absorption $\varkappa = 2 \times 10^{-3}$ (Fig. 9) it may be concluded that in contrast to the measurement of nonabsorbing fluids (for $\varkappa = 0$, Fig. 6) the decreasing width of photodetectors results in very disadvantageous flattening of the curve P.

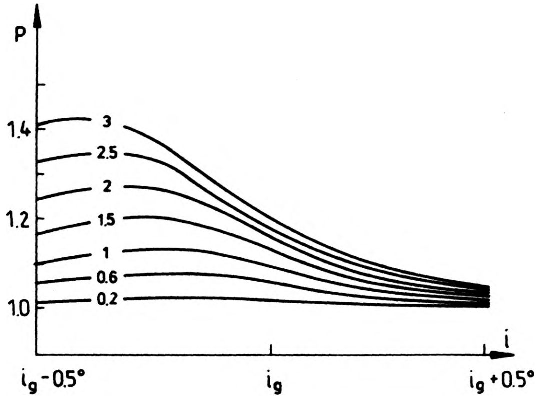

Fig. 9. Influence of the widths of detectors in mm (at $\varkappa = 2 \times 10^{-3}$) vs the ratio of areas (ratio of the fluxes accepted by the detectors)

5. The effect of detectors spacing (distance between their centres–axes)

The calculations were made for a constant detector width (1 mm) and constant position of the first detector centre on the limiting angle. From the family of curves (Fig. 10) it follows that the greater spacing of the detectors the better is the course of the curve P. Analogical calculations have been carried out for the assumed extreme value of absorption $\varkappa = 2 \times 10^{-3}$ (Fig. 11). Also in this case a greatest distance between the detectors is advantageous. Figure 12 shows the course of the curve P for widths of detectors 0.5 and 0.3 mm, at constant 3 mm

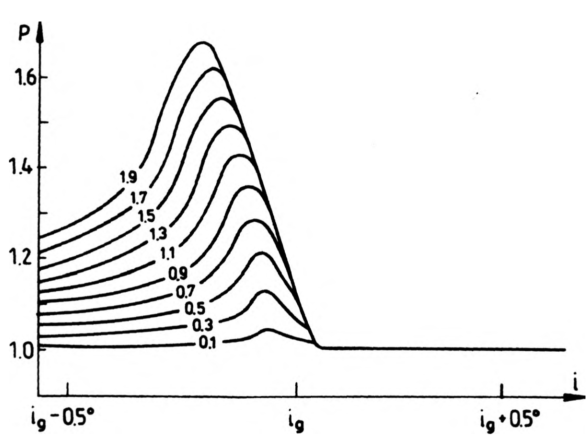

Fig. 10. Influence of the distances of detectors on the ratio of areas P, for $\varkappa = 0$

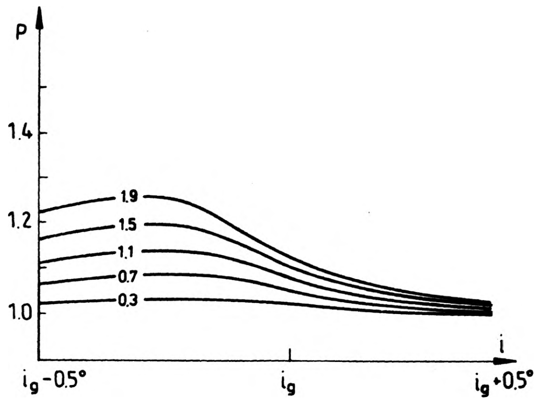

Fig. 11. Influence of the distances of detectors on the ratio of areas P, for $\varkappa = 2 \times 10^{-3}$

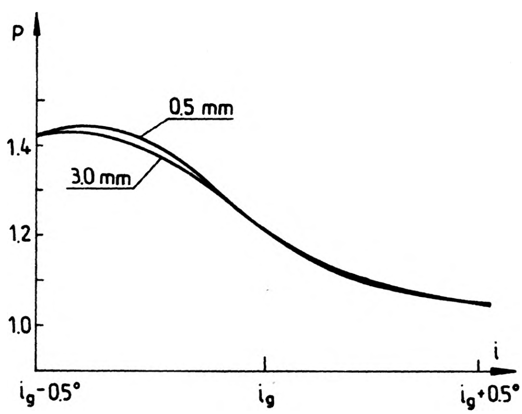

Fig. 12. Run of the dependence of the ratio of areas P at the constant spacing of detectors and the extreme widths

spacing, and absorption 2×10^{-3}. Under these conditions the width of detectors very slightly influences the run of the curve P. The lowest effect occurs at the vicinity of the limiting angle at the site of measurement.

6. Determination of the optimal positions of detectors

For this purpose the calculations have been performed for five different (from 1 to 3 mm) spacings of detectors at constant 1 mm detector width and constant position of the first detector centre on the limiting angle. Results are shown in Figs. 13–15. In all the plots the curves P corresponding to different coefficients of absorption are intersecting, forming a distinct narrowing. To the latter there corresponds the optimal position of detectors which means that the change in absorption coefficient has smallest effect on the error of limiting angle measurement. The optimal position of the first detector with respect to the limiting angle is

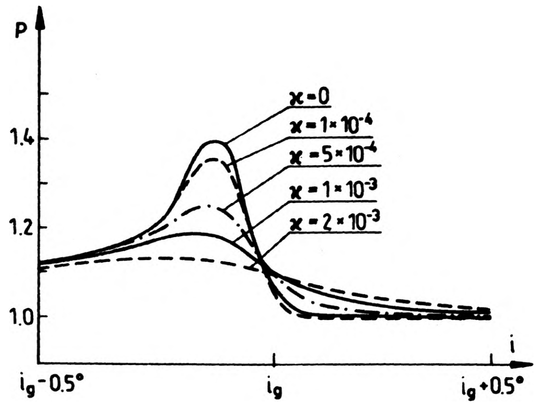

Fig. 13. Dependence of the ratio of area P for different absorptions at the 1 mm spacing of detectors

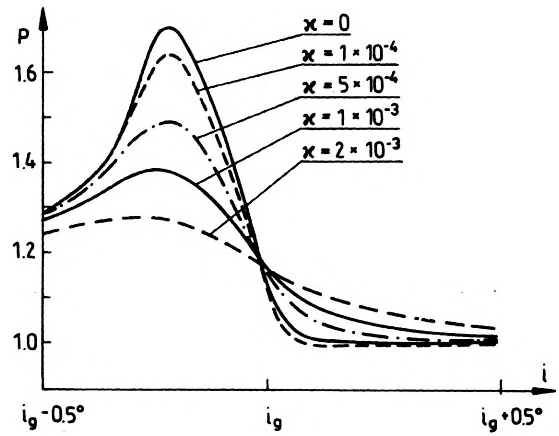

Fig. 14. As in Fig. 13, but at 2 mm spacing of detectors

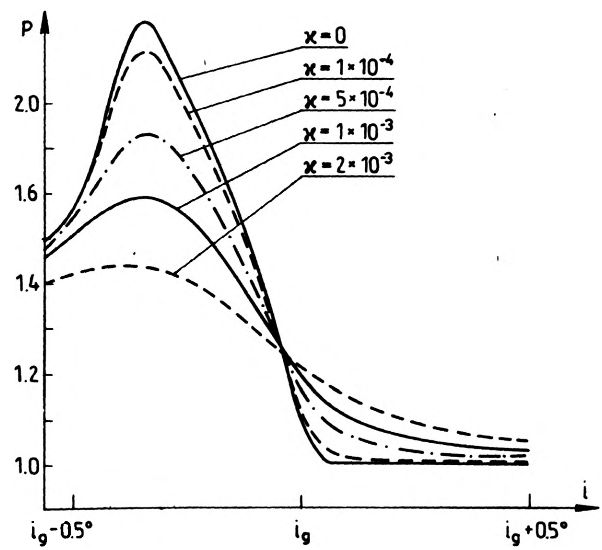

Fig. 15. As in Fig. 13, but at 3 mm spacing of detectors

shown in Fig. 16. By comparing the plots in Figs. 13, 14 and 15 it may be stated that the increase of the distance between the detectors is advantageous since then the ratio of area P as well as the slope of the curves at the vicinity of i_g increase.

In order to determine both the optimal positions of detectors and their influence on the error, the calculations have been performed for three values of the coefficient of absorption 10^{-3}, 2×10^{-3}, and 0, and for different spacings of detectors. The calculations are illustrated in Figs. 17–19. The relative errors of the refractive index measurement are plotted for the assumed minimal and maximal values of this coefficient (i.e., for the extreme values of the measurement range) and

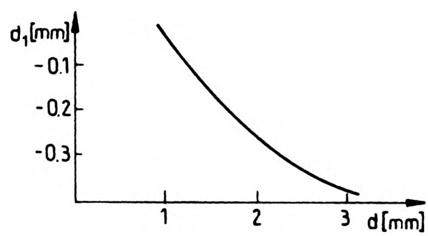

Fig. 16. Optimal distance of the first detector from the position corresponding to limiting angle (from the light-shadow border) depending on the spacing of detectors, for the detector width – 1 mm

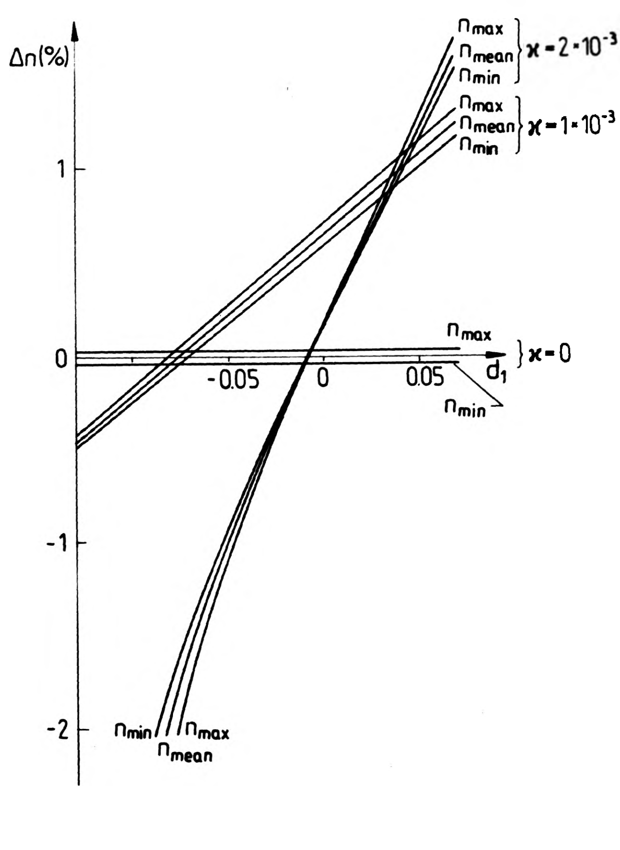

Fig. 17. Magnitudes of the errors depending on the position d_1 of the first detector at 1 mm spacing (width 1 mm), $d_{1\,\mathrm{opt}} = -0.03$

for the mean value. With the increasing distance between the detectors the errors and the effects of the absorption coefficient and position of detectors decrease, while that of the refractive index increases, espacially at higher values of absorption.

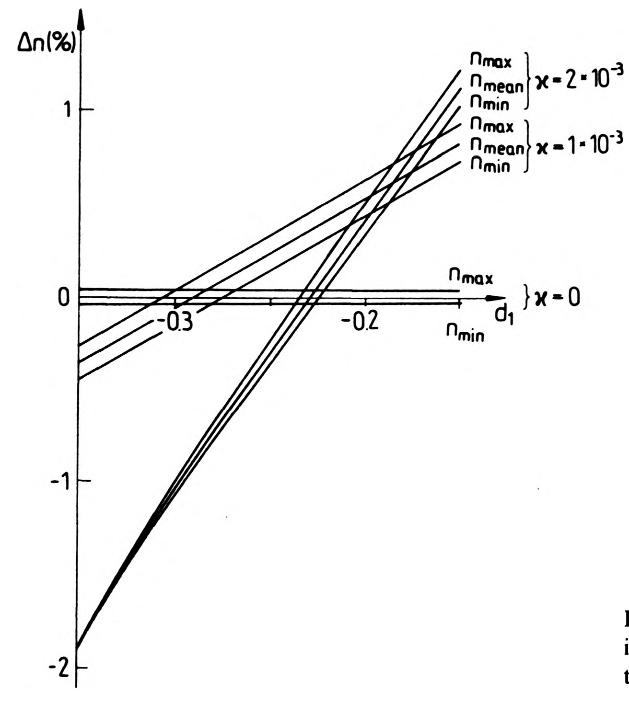

Fig. 18. Magnitudes of the errors depending on the position d_1 of the first detector (width 2 mm), $d_{1\,opt} = -0.25$

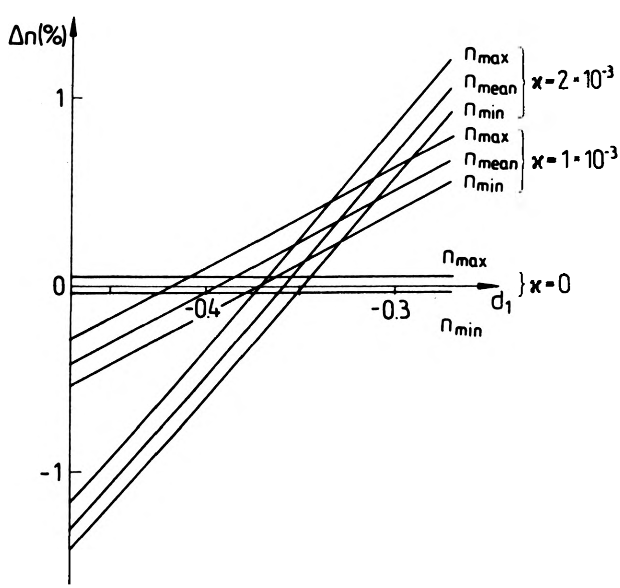

Fig. 19. Magnitudes of the errors depending on the position d_1 of the first detector at 3 mm spacing (width 3 mm), $d_{1\,opt} = -0.37$

Figures 20–22 represent the measurement errors (i.e., the systematic error, in other words, the error of the method) calculated for 1, 2 and 3 mm spacings of detectors, at the optimal position of the first detector determined previously for each case and for different values of absorption ranging within $0–2 \times 10^{-3}$. With the increasing distance between the detectors the values of the errors increase from about 0.45% (for 1 mm) to about 0.24% (for 3 mm). The error depends more to a greater degree on the coefficient of absorption than on the change in the reference index of the measured medium. The absolute value of the relative errors vs the distance of the detectors (at a constant width – 1 mm) is presented in Fig. 23.

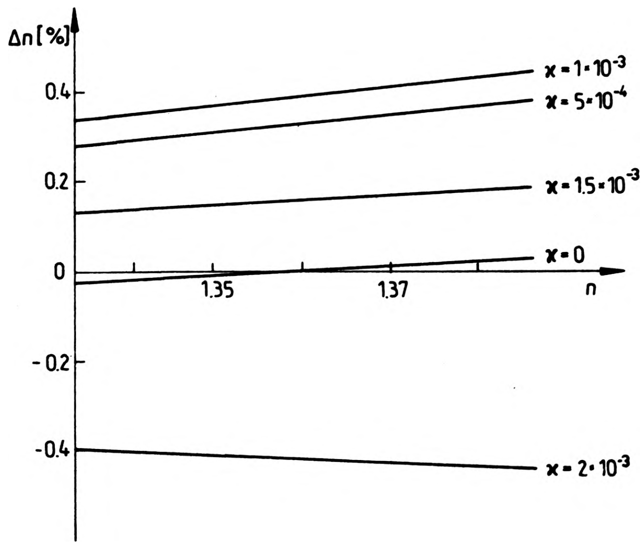

Fig. 20. Magnitudes of errors at optimal position of the first detector (width 1 mm), $d_{1\,opt} = 0.03$

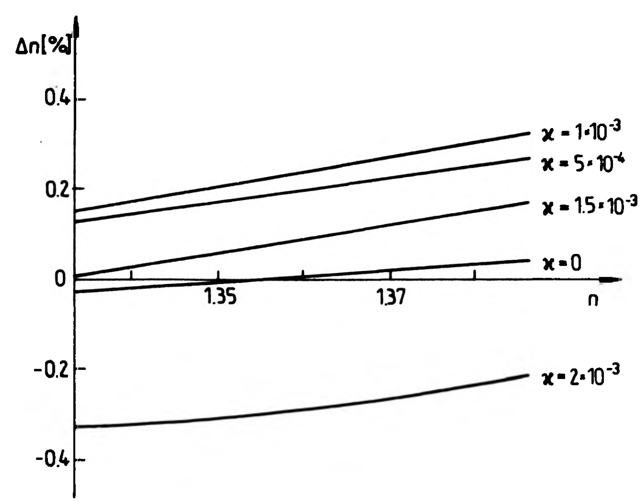

Fig. 21. Magnitudes of errors at optimal position of the first detector (width 2 mm), $d_{1\,opt} = 0.25$

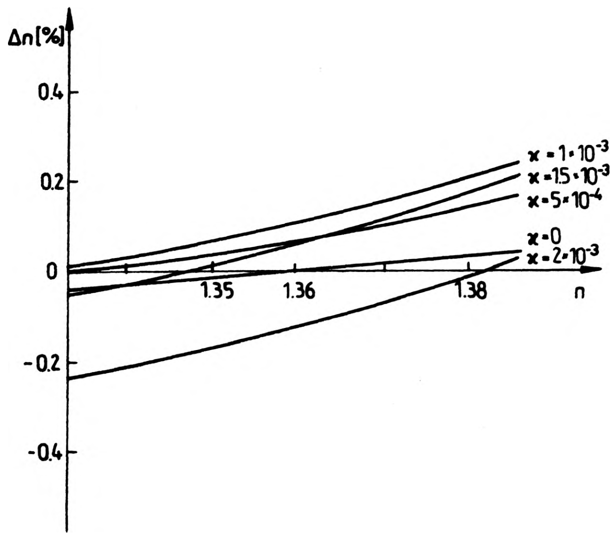

Fig. 22. Magnitudes of errors at optimal position of the first detector (width 3 mm), $d_{1\,opt} = 0.37$

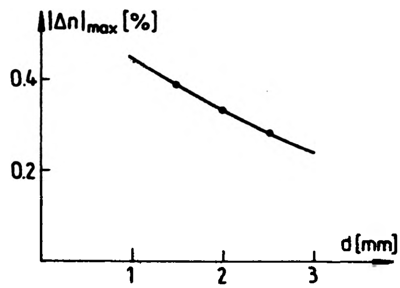

Fig. 23. Absolute value of the relative error as a function of spacing of detectors (at a constant 1 mm width)

7. Final remarks

In the work presented the examinations have been restricted to the measurement method itself in a given most commonly used, in the author's opinion, range of industrial measurements. With the increasing range of measurement the formulae will be changed and relative errors (due to the measurement method) will decrease.

The achievement of a suitable class* of device precision is a secondary task. The class of precision is determined by conditions defining this precision as well as by the changes in surrounding conditions and the perturbing factors. Determination of the errors of the method and of their optimal dispersion is of particular significance for the designers since it gives information how much may be gained from the given measurement method, and which precision class is achievable. In

* According to the class of precision defined for the analogue branch URS (system of automatic regulation of COMECON countries).

the discussed case at the maximal, about 0.24%, error of the method (under optimal conditions), by taking into account the errors appearing due to insufficient precision of the servomechanism (zone of insensitivity) as well as due to further signal processing of the signal, it may be estimated that the class of precision 1 is achievable, the class 0.6 is still possible, provided that the mechanical parts are produced with a particularly high precision and the potentiometers of highest (0.1) class are applied. The accuracy higher than 0.6% within the whole range is hardly realizable, and all the information assuring that such an accuracy is possible for the measurement range absorbing up to $\varkappa = 2 \times 10^{-3}$ are simply false.

Some authors [8] propose to classify the analysers by defining the measurement range up to $\varkappa = 2 \times 10^{-3}$ as the one of nonabsorbing fluids. This is risky since within this range the effect of the absorption on the measurement of n (or any related quantity) is significant and changes the inaccuracy due to the method itself, from 0.02 to 0.45% in optimal dispersion of errors. If a refractometer is scaled for a nonabsorbing fluid, and in particular at the end of the measurement range (which is most often the case, while scaling for H_2O) the error for $\varkappa = 2 \times 10^{-4}$ amounts to about 0.9%. This error is very great, hence it may be seen how important is its proper dispersion by optimal spacing and locating the detectors and by suitable scaling. The measurement accuracy achievable by the method discussed may increase by the factor of 4.

The presented plots (as well as the results of calculations according to the respective programs) allows us to design with respect to the measurements an optimal double detector of the optimal width and spacing of light sensitive tracks, and – moreover – to optimize the location and arrangement of the commercially avialable detectors.

8. Conclusion

The analysis has been performed assuming many simplifications. They, however, according to the author's opinion, do not influence essentially the character of errors and their optimal dispersion, as well as their (relative) values when the measurement range of n or any related quantity is widened or narrowed.

The error due to absorption could be diminished by applying the incident beam polarized in the plane perpendicular to the plane of incidence on the interface of the media (then the relation $r = f(n)$ runs more abruptly) and by applying the two-fold* reflection to multiply the relation $r = f(n)$ by itself.

References

[1] KOTECKI A., SZUKALSKI J., WAGNEROWSKI T., KRYSZCZYŃSKI T., Patent PRL No. 55086, kl 42 h, 36, 1968.

[2] SZUKALSKI J., Doctor's Thesis, Warsaw Technical University, 1966.

* The application of a greater than two-fold reflection makes the construction of analyser more complex and introduces other errors; these methods, therefore, were no more used in the industrial measurements as early as in 1970s.

[3] SZUKALSKI J., JASNY J., Polish Research Work in the Field of Industrial Refractometry, JMEKOV 1970, Versailles, Acta JMEKO B-202, PO-294, Ed. MTESZ Budapest 1972.
[4] JASNY J., Doctor's Thesis, Warsaw Technical University, 1968.
[5] SUCHAŃSKI J., Doctor's Thesis, Warsaw Technical University, 1980.
[6] SUCHAŃSKI J., SZUKALSKI J., Patent PRL No. 122654, Cl. GO1W71/17, GO1NZ1/63, 1984.
[7] LITTMANN H., Ann. Phys. **36** (1940), 5-38.
[8] LEIKIN M. V., MOLOCHNIKOV B. I., MOROZOV V. N., SHAKARYAN E. S., *Otrazhatelnaya refraktometriya* (in Russian), Ed. Mashinostroenie, Leningrad 1983.

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Предельные возможности автокомпенсационных рефрактометрических измерений, основанных на использовании явления полного внутреннего отражения

Предметом статьи является обсуждение и дискуссия над влиянием абсорбции измеряемой среды на рефрактометрические измерения, основанные на так называемом критическом углу, реализованные фотоэлектрическим способом. Проведен подробный анализ ошибок наиболее распространенного автокомпенсационного рефрактометрического метода, заключающегося в измерении-прослеживании границы света и тени пучка, отраженного от сплошной среды в области критического угла. Определены теоретические границы возможности (точности) метода в области наиболее часто применяемых в промышленности измерений. По отношению к области частого применения был определен способ оптимального использования метода посредством разложения систематических ошибок с изменениями абсорбции к величине 2×10^{-3}. Представлены зависимости, позволяющие оптимально запроектировать-сконструировать переработку сигнала и уравновешивания автокомпенсационной системы.