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Bending of thick holographic cylindrical lens

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A theoretical analysis of the light intensity across the aperture of the thick holographic cylindrical lens of the planar type has been performed and non-uniform distribution obtained. The non-uniformity is partly possible to be removed by bending the lens, which is also desired from the point of view of its imaging properties and Fresnel losses. Much more uniform distribution one can obtain by moon-like shapes of the lens.

1. Introduction

Holographic cylindrical lenses can be used as HOEs for transforming of free propagating beams and also as grating lenses on the surface of planar waveguide for transforming of guided modes. Extensive theoretical study of properties of volume (thick) holographic cylindrical lens was performed [1–3] using two-dimensional coupled-wave theory [4] as well as localized one-dimensional theory [5]. Grating lenses for guided-wave optics were experimentally realized firstly as chirp-gratings [6, 7] only, then also with inclined grating lines [8]. Two-dimensional coupled-wave theory was used by means of modal approach for analysis of the last type of planar grating lens [9]. All the analyses and experimental realizations maintained this type of optical element as a grating medium with straight and parallel boundaries. Small exception was made in [9], where one boundary is suggested as curved in order to compensate decreasing of coupling coefficient from the centre to the border of the element in the case of the polarization in the plane of incidence.

In this paper it will be shown that curved boundaries can be very useful for properties of that kind of optical element, particularly from the point of view of a uniformity of the beam but also for geometric optical advantages.

2. Distribution of the intensity across the aperture of the lens

Volume holographic cylindrical lens transforms a plane wave into a cylindrical wave or vice versa. This lens can be made by holographic exposure of a sensitive medium which fills a volume of planparallel slab or parallel strip. For the recording of the lens one must use waves of the same shape as these which will be used for reconstruction. In Fig. 1 the recording is sketched. One boundary of the recording medium lies in the coordinate plane xy and the other in the distance d_0 from this plane. The x-axis is parallel to the focal line of the cylindrical wave and

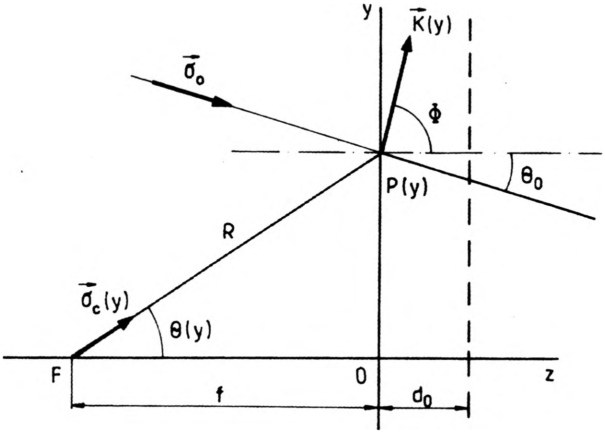

Fig. 1. Recording of the thick holographic cylindrical lens

the distance of the focal line from xy-plane is f. The angle of incidence of the plane wave is Θ_0. In a general point $P(y)$ the ray of the cylindrical wave having the angle of incidence $\Theta(y)$ interferes with the ray of the plane wave, and the grating vector $\vec{K}(y)$ in this point is given by the relation

$$\vec{\sigma}_c(y) = \vec{\sigma}_0 - \vec{K}(y) \tag{1}$$

where $\vec{\sigma}_c(y)$, $\vec{\sigma}_0$ are propagation vectors of the cylindrical and plane wave rays, respectively. ($|\sigma_c(y)| = |\sigma_0| = \beta = 2\pi/\lambda$, $|K(y)| = K = 2\pi/\Lambda$, where λ is the wavelength of the light and Λ is the period of the grating). Components of the vectors are:

$$\vec{\sigma}_0 = \{\beta \cos \Theta_0,\ \beta \sin \Theta_0\},$$
$$\vec{\sigma}_c(y) = \{\beta f/\sqrt{f^2+y^2},\quad \beta y/\sqrt{f^2+y^2}\},$$
$$\vec{K}(y) = \{K \cos \Phi\ \ K \sin \Phi\},$$

which substituted into Eq. (1) gives for the local period of the grating the expression

$$1/\Lambda^2(y) = (2/\lambda^2)[1-(f \cos \Theta_0 + y \sin \Theta_0)/\sqrt{f^2+y^2}], \tag{2}$$

and for the angle of the grating vector

$$\tan \Phi(y) = (y-\sqrt{f^2+y^2} \sin \Theta_0)/(f-\sqrt{f^2+y^2} \cos \Theta_0). \tag{3}$$

For simplicity we will suppose that the grating lines are straight because usually the condition $d_0 \ll f$ holds.

Applying localized one-dimensional theory of volume holograms [5] for our transmission case, we can obtain local diffraction efficiency in the form

$$\eta(y) = \sin^2 \{(\pi n_1 d_0/\lambda \sqrt{\cos \Theta_0}) \sqrt{1+(y/f)^2}\} \tag{4}$$

for on-Bragg case. Note that from the localized theory there cannot be done deep conclusions about exact intensity distribution across the aperture, because it in fact does not take into account an interaction between waves in the grating medium, and approximates the results for thickness approaching zero. But the locations of the minimum and maximum efficiency it can predict with good accuracy. For m-th maximum it holds that the argument in (4) must be equal to $(2m-1)(\pi/2)$ and for m-th minimum the expression $(m-1)\pi$ takes place.

For a moment let us assume on-axis case $\Theta_0 = 0$ and properly chosen parameters that on the axis m-th maximum of the diffraction efficiency be located. Then $m = 1/2 + n_1 d/\lambda$, and as an example, for the first maximum and $d_0 = 10\lambda$ there must be $n_1 = 0.05$. If m-th maximum should be on the axis then $n_1 d/\lambda = m - 1/2$, and $(m+M)$-th minimum for a point $P(y)$ can be obtained if

$$\tan \Theta = y/f = \{[2(m-1+M)/(2m-1)]^4 - 1\}^{1/2}. \tag{5}$$

As can be seen from the last expression, large variation in the intensity across the aperture of the cylindrical thick holographic lens can be expected. The table

Angles of minimum efficiency

$n_1 d/\lambda$	M = 1	2	3
1	63.6	80.8	85.3
10	24.9	40.9	50.2
100	8.1	13.9	17.9

summarises the angles Θ of the minimum efficiency for three values of the coupling constant $n_1 d/\lambda$ and three following orders. Naturally, this disadvantage in the distribution of the intensity can be compensated by the change of the shape of one boundary, similarly as sugested in [9] for another compensation. The condition is that the argument in (4) must not change with y-coordinate, from where

$$d(y) = d_0 [1 + (y/f)^2]^{-1/4}. \tag{6}$$

This rather complicated geometric shape can be replaced by parabolic if one takes into account very narrow vicinity of the central point only $y/f \ll 1$, when

$$d(y) \approx d_0 [1 - (y/2f)^2], \tag{6.1}$$

which can be rewritten as $d(y) \approx d_0 - y^2/2\varrho$, where $\varrho = 2f^2/d_0$. From (6.1) it follows that for instance for $f = 40$ mm and $d_0 = 1$ mm the radius of curvature $\varrho = 320$ mm, but for $d_0 = 3$ mm the radius is $\varrho = 106.7$ mm. Note that the function $d(y)$ is independent of Θ_0.

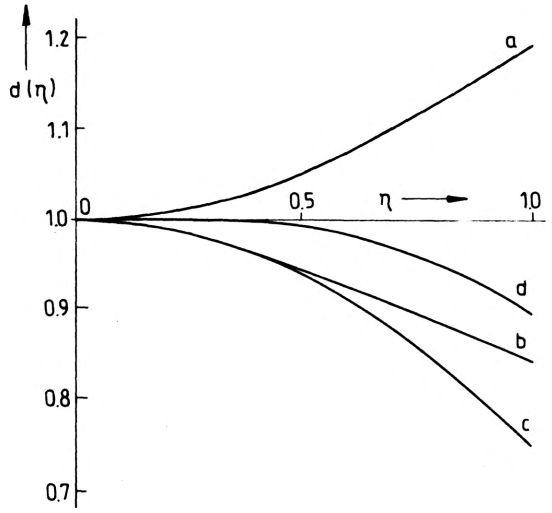

Fig. 2. Curves for compensation of the intensity distribution across the aperture by means of the change of shape of one boundary: curve **a** – dependence of the argument of the function of local diffraction efficiency (Eq. (4)) on the lateral coordinate, curve **b** – ideal shape of the boundary (Eq. (6)), curve **c** – approximate parabolic shape of the boundary (Eq. (6.1)), curve **d** – argument of the function of local diffraction efficiency for parabolic shape of the boundary

In most cases the shape (6.1) can replace (6) with good accuracy in much greater extent, as it is seen from Fig. 2 where three functions involved are plotted.

3. Bending of the lens

As it has been mentioned above a curved slab or strip of the grating medium can have advantages against the straight one. We expell from our consideration such bending when the convex side is directed to the focus and will treat the opposite case only due to its essential character. As an approach to the necessary demand of curving we study at first the case when interference fringes are perpendicular to the boundary and calculate the shape of the boundary. From (3) we see that a slope of the boundary curve is

$$dz/dy = (f - \sqrt{f^2+y^2}\cos\Theta_0)/(y - \sqrt{f^2+y^2}\sin\Theta_0), \tag{7}$$

and for the simplicity we consider again the case $\Theta_0 = 0$. Then

$$d\xi/d\eta = (1 - \sqrt{1+\eta^2})/\eta \tag{7.1}$$

where $\xi = z/f$ and $\eta = y/f$. Integrating (7.1) the shape of the boundary is of the form

$$\xi = \ln|\eta| - [\sqrt{1+\eta^2} - \ln|(1+\sqrt{1+\eta^2})/\eta|] + C, \tag{8}$$

when C is an integrating constant.

Using general relations of differential geometry we can also obtain the curvature of the curve (8) as dependence on η

$$\varrho = \frac{\sqrt{1+\eta^2}\,[\eta^2+(1-\sqrt{1+\eta^2})^2]^{3/2}}{\eta(1-\sqrt{1+\eta^2})} \tag{9}$$

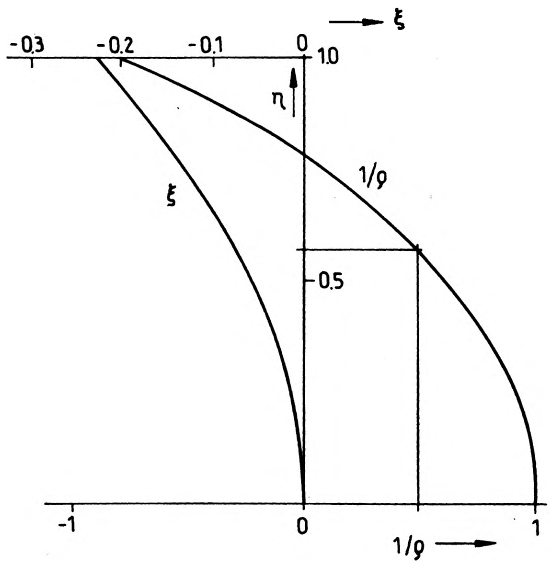

Fig. 3. Bending of the lens in such a manner the interference fringes are perpendicular to the boundary: curve ξ – shape of the boundaries, after Eq. (8), curve $1/\varrho$ – curvature of the curve ξ according to Eq. (9)

where $\varrho = r/f$. Figure 3 shows both last curves. From the curve for the curvature it is seen that the radius of curvature in the centre is equal to the focal length, then increases with increasing η and about $\eta \approx 0.6$ has the value $\varrho = 2$. Between $\eta = 0.78$–0.79 the curve has zero curvature and then the curvature is changed to opposite and the radius of curvature quickly decreases.

Naturally, this complicated shape must be replaced for practice by more simple one – circular or parabolic. In Fig. 4a absolute differences between the shape (8) and the circular shape $\Delta\xi = |\xi| - (\varrho - \sqrt{\varrho^2 - \eta^2})$ or the parabolic one $\Delta\xi = |\xi| - \eta^2/2\varrho$ are plotted for $\varrho = 2$, and in Fig. 4b relative differences for a sequence of

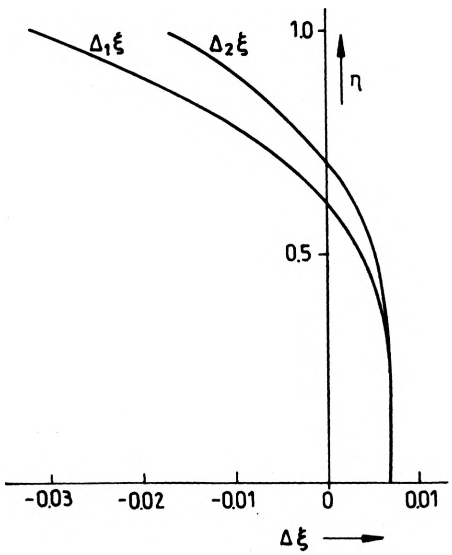

Fig. 4a. Differences between ideal shape of the boundaries of the bent lens after Eq. (8) and circular $\Delta_1\xi = |\xi| - (\varrho - \sqrt{\varrho^2 - \eta^2})$ or parabolic $\Delta_2\xi = |\xi| - \eta^2/2\varrho$ shapes if $\varrho = 2$

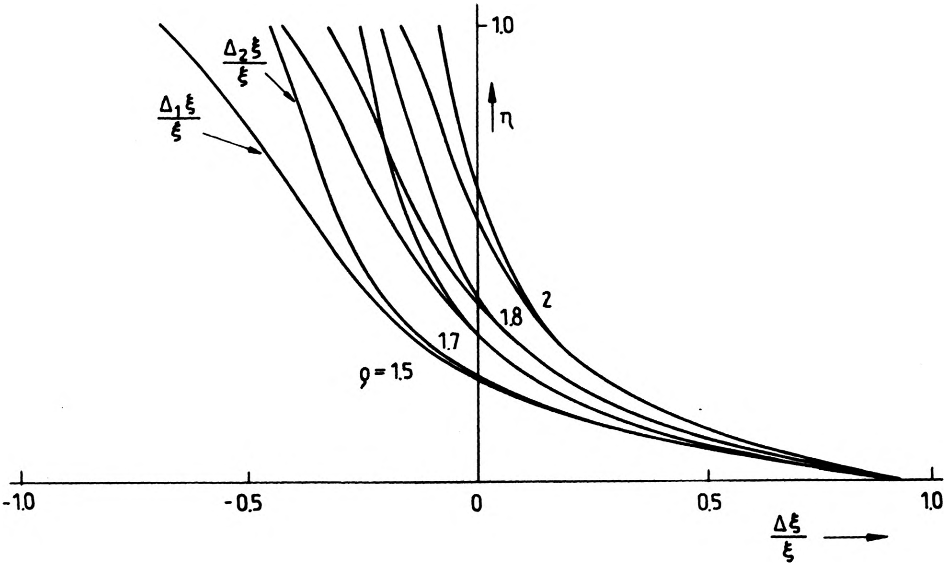

Fig. 4b. Relative differences $\Delta\xi/\xi$ for both cases in Fig. 4a and a sequence of curvatures ϱ

$\Delta\xi/\xi$ are plotted. It is seen that most convenient radius of curvature lies around a value $\varrho = 2$.

When the lens is bent, a point P, in which we have to consider the interference, does not lie on the y axis and relations for period (2) and angle (3) are no more valid. With the help of the sketch in Fig. 5 one can obtain new components of vectors which have more complicated form:

$$\vec{\sigma}_o = \{\beta[\sqrt{1-\eta^2}\cos\Theta_0 - \eta\sin\Theta_0],\ \beta[\sqrt{1-\eta^2}\sin\Theta_0 + \eta\cos\Theta_0]\},$$

$$\vec{\sigma}_c = \left\{\beta\frac{1-(\xi-1+\sqrt{1-\eta^2})^2+\eta^2-(1-\xi)^2}{2[(\xi-1+\sqrt{1-\eta^2})^2+\eta^2]^{1/2}},\right.$$

$$\left.\beta\frac{(1-\xi)\eta}{[\eta^2+(\xi-1+\sqrt{1-\eta^2})^2]^{1/2}}\right\}$$

where $\eta = y/r$, $\xi = f/r$ and for z-coordinate of the point P it holds $z_P = r-\sqrt{r^2-y^2}$.

The expression for the diffraction efficiency then takes a form

$$\eta^{\mathrm{circ}} = \sin^2\left\{\frac{2\pi n_1 d}{\lambda}\right.$$

$$\left.\times\frac{[\xi-1+\sqrt{1-\eta^2}]^{1/4}}{[1+(\xi-1+\sqrt{1-\eta^2})^2+\eta^2-(1-\xi)^2]^{1/2}[\sqrt{1-\eta^2}\cos\Theta_0-\sin\Theta_0]^{1/2}}\right\}, \tag{10}$$

which is much more complicated than the very simple form (4). Not only circular shape can be borne in mind for the shape of bent lens but also a parabolic one. In

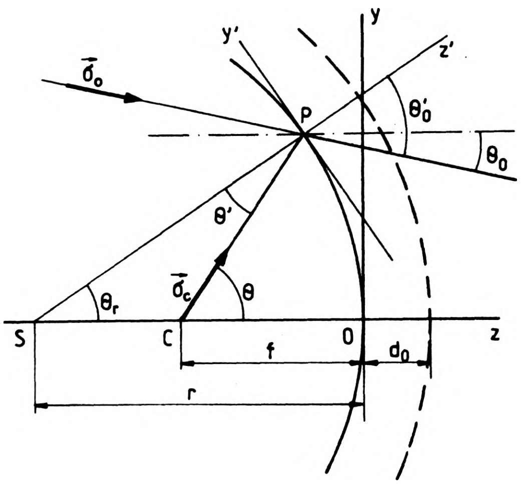

Fig. 5. Recording of the thick bent holographic cylindrical lens

this case the z-coordinate of the point P is $z_P = y^2/2r$ and in this sense the square roots $\sqrt{1-\eta^2}$ should be changed. Then the diffraction efficiency after simple algebraic alterations is

$$\eta^{\mathrm{par}} = \sin^2 \left\{ \frac{\pi n_1 d}{\lambda} \right.$$

$$\left. \times \frac{[\xi^2 + \eta^2(1-\xi) + \eta^4/4]^{1/4}}{[\xi + \eta^2(1-\xi)/2 + \eta^4/8] \times [(1-\eta^2/2)\cos\Theta_0 - \eta\sin\Theta_0]^{1/2}} \right\}. \tag{11}$$

When the lens is bent the maximum and minimum diffraction efficiencies must occur in another positions than in the case of straight boundaries. Figure 6 shows dependences of position of minimum efficiency on the curvature of the lens for some cases of the smaller M. It is seen that the shift to higher values of the

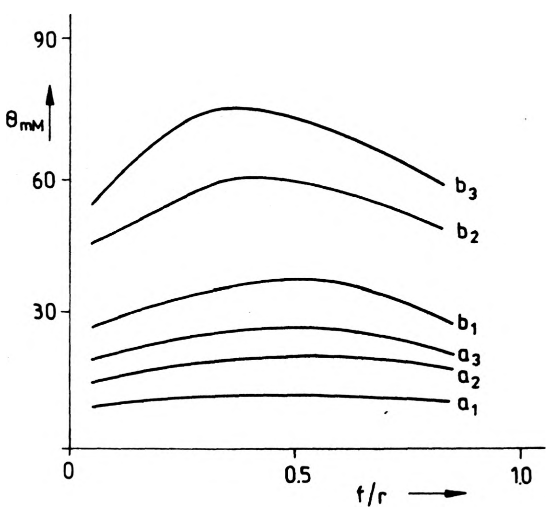

Fig. 6. Dependencies of position of minimum efficiency on the curvature of the lens for smaller m and M: curves **a** for $m \approx 100$ and $M = 1, 2, 3$; curves **b** for $m \approx 10$ and $M = 1, 2, 3$

transversal coordinate η for optimum curvature is not so much pronounced as one should want to expect. Therefore, the main result of bending can not be the removing of non-uniformity across the aperture which is due to non-uniformity of the diffraction efficiency. This must be done by a change of the thickness as in the case of planar lens.

If the diffraction efficiency ought to be uniform the argument in (10) or (11) must not change with η, and the thickness of the lens must change according to the expression

$$\frac{d(\eta)}{d_0} = \frac{[1+(\xi-1+\sqrt{1-\eta^2})^2+\eta^2-(1-\xi)^2]^{1/2}\times[\sqrt{1-\eta^2}\cos\Theta_0-\eta\sin\Theta_0]^{1/2}}{[(\xi-1+\sqrt{1-\eta^2})^2+\eta^2]^{1/4}} \tag{12a}$$

for the circular shape, and

$$\frac{d(\eta)}{d_0} = \frac{[\xi+\eta^2/2(1-\xi)+\eta^4/8]^{1/2}[(1-\eta^2/2)\cos\Theta_0-\eta\sin\Theta_0]^{1/4}}{[\xi^2+\eta^2(1-\xi)+\eta^4/4]^{1/4}} \tag{12b}$$

for the parabolic shape. These expressions are also very complicated and one must seek more simple form. This can be, for example, the subtraction of two circles of the same diameter but with shifted centres of curvature of circular shape, as it is seen in Fig. 7. The thickness of that moon-like section is changed according to the

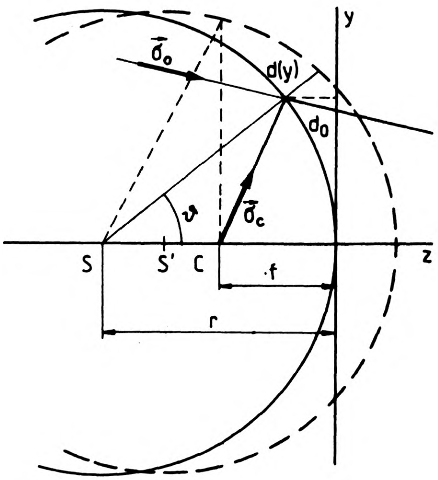

Fig. 7. Moon-like shape of the lens having boundaries of the same curvature

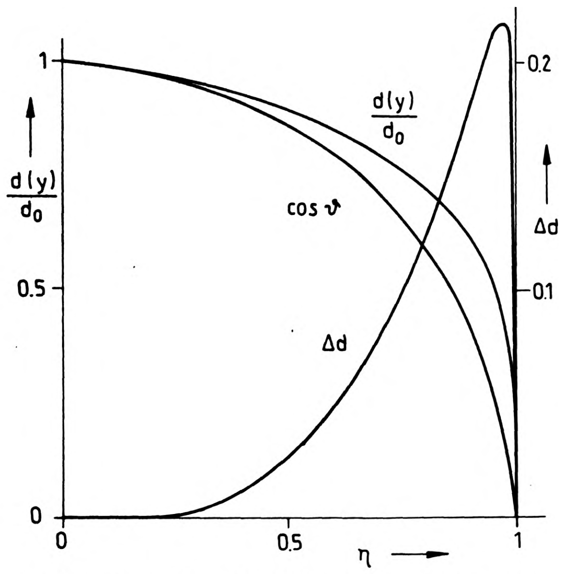

Fig. 8. Dependence of the thickness of bent lens on the lateral coordinate for the constant diffraction efficiency after Eq. (12) and approximate course of thickness for moon-like shape $d(\eta) = d_0 \cos\vartheta$. Curve Δd is the difference between both curves in magnified coordinate

dependence $d(\eta) = d_0 \cos \vartheta$, where $\vartheta = \arcsin(y/r)$, and in Fig. 8 both shapes and the difference between them are plotted in magnified coordinate for the case $\xi = f/r = 0.5$. It is seen that for cases important in practice, e.g., $\eta_{max} \approx 0.5$, these shapes can replace the theoretical shape very well. Naturally, one can also use circles with different radius for both boundaries of the element. But a choice of the radius of circles is limited by the inequality $r \leqslant r' < r + d_0$, and therefore an improvement is only very slight.

4. Conclusion

Non-uniform distribution of the intensity across the aperture of the thick holographic cylindrical lens can be compensated in the most part by the bending of the lens shape. This bending is realizable in the case of the lens in planar optics very easily by masking the grating. The distribution can still be improved when both boundaries of the shape have the same diameter.

Moreover, three other advantages follow from the bent shape of the element. The first of them is that Fresnel reflection loses on the boundaries can be smaller than in the case of straight boundaries because angles of incidence are smaller, especially in the off-axis part of the element. This fact has connection with other advantage: bent shape of the element generates more uniform distribution of a Gaussian beam which falls on the element from the concave side and is collimated by this element. It is due to replacement of tangens function by sinus function in the distribution of the intensity as dependence on the radius.

The third advantage is based on an improvement of aberrations if the optical element is bent. Optimum bending is under the condition $f/r = 0.5$, where an optical imaging is aplanatic. The same condition has been found in this paper for more uniform distribution of the light intensity across the aperture of the thick holographic lens.

This analysis has already been applied to calculate the performance of grating lenses to planar optics. Experimental work to verify the calculated results is currently being undertaken in our laboratory.

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Искривление толстой голографической цилиндрической линзы

Теоретически анализирована интенсивность света возле апертуры толстой голографической цилиндрической линзы планарного типа и получено ее неравномерное распределение. Эту неравномерность можно частично устранить искривлением линзы, которое желательно тоже из точки зрения изображающих свойств и френелевских затрат. Лучшей равномерности можно достичь лунарным видом линзы.