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Wave-aberration of an optical system with small decentrations. Vector approach

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A new vector method is proposed for analysis of small decentrations on the wave-aberration of an optical system. The wave-aberration introduced by an element of the system, including decentration aberrations, can be interpreted as a "distorter". The mathematical description of such a distorter, which enables us to analyse the decentrations and their influence on the image quality, is based on the methods of image synthesis under Fresnel approximation. The wavefront variance in the exit pupil of the optical system suffering from spherical aberration and the coma of decentration is evaluated as an example. The effect of defocusing on the variance is also discussed.

1. Introduction

For an infinitely large and aberration-free optical system the field distribution on the object sphere of reference is reproduced on the proper image sphere, the only changes being field amplitude and scale [1]. The elements which disturb these conditions in the system are referred to as distorters [1]. In the Fresnel diffraction approximation an infinitely large and aberration-free optical element is a purely quadratic phase corrector (not a distorter). Thus, the aberrational disturbance introduced by any element of the optical system may be interpreted as the distorter.

The real optical system is a combination of quadratic phase correctors and distorters. It is shown in [1] that these two sets can be treated separately. The aim of this paper is to determine a distorter that would represent the contribution into the wave-aberration due to decentration of any element (such as lens or a diaphragm) of the system. A set of such distorters, describing decentration aberrations due to production tolerances of the system elements, can be then transferred to one space of the system (by means of the one-space method [1]). Only small decentrations due to production tolerances along with the existence of primary aberrations of the corresponding centred system are considered.

2. Basic formulae

The function describing the primary wave-aberration of i-th element of optical system can be written in the vector form as

$$\Phi = w_{20i}\,\bar{\varrho}_i^2\,\bar{a}_i^2 + w_{40i}\,\bar{\varrho}_i^4 + (w_{11i}\,\bar{a}_i^2 + w_{31i}\,\bar{\varrho}_i^2)(\bar{\varrho}_i\,\bar{a}_i) + w_{22i}(\bar{\varrho}_i\,\bar{a}_i)^2 \,* \tag{1}$$

where (see, also, Fig. 1): $\bar{\varrho}_i$ – localization vector of a point in the aperture, $\bar{a}_i$ – localization vector of a point in the image plane of i-the element, w_{mni} – aberration coefficients of wave-aberration of the i-th element of the optical system [2].

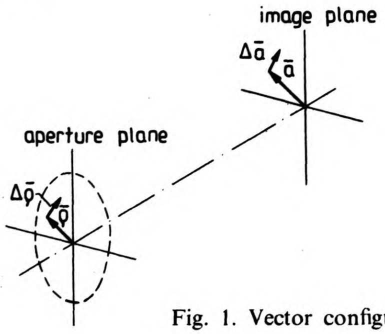

Fig. 1. Vector configurations in the aperture and image planes of a decentred lens

The decentration of the i-th element generates the displacement of the aberration function in the aperture, vector $\overline{\Delta\varrho_i}$, and the shift of the image point, vector $\overline{\Delta a_i}$ (i.e., a new centre of the reference sphere). By substituting the vectors $\bar{\varrho}_i$ and $\bar{a}_i$ with $\bar{\varrho}_i + \overline{\Delta\varrho_i}$ and $\bar{a}_i + \overline{\Delta a_i}$ in Eq. (1) we can write

$$\begin{aligned}\Phi_d = {} & w_{20i}(\bar{\varrho}_i + \overline{\Delta\varrho_i})^2(\bar{a}_i + \overline{\Delta a_i})^2 + w_{40i}(\bar{\varrho}_i + \overline{\Delta\varrho_i})^4 \\ & + [w_{11i}(\bar{a}_i + \overline{\Delta a_i})^2 + w_{31}(\bar{\varrho}_i + \overline{\Delta\varrho_i})^2][(\bar{\varrho}_i + \overline{\Delta\varrho_i})(\bar{a}_i + \overline{\Delta a_i})] \\ & + w_{22i}[(\bar{\varrho}_i + \overline{\Delta\varrho_i})(\bar{a}_i + \overline{\Delta a_i})]^2. \end{aligned} \tag{2}$$

Leaving only the terms with $\overline{\Delta\varrho_i}$ and $\overline{\Delta a_i}$ taken up to the second order and omitting index i we get

$$\begin{aligned}\Phi_d = {} & w_{20}\,\bar{\varrho}^2\,\bar{a}^2 + w_{40}\,\bar{\varrho}^4 + (w_{11}\,\bar{a}^2 + w_{31}\,\bar{\varrho}^2)(\bar{\varrho}\bar{a}) + w_{22}(\bar{\varrho}\bar{a})^2 \\ & + (\bar{\varrho}\overline{\Delta\varrho})[2w_{20}\,\bar{a}^2 + 4w_{40}\,\bar{\varrho}^2] + (\bar{\varrho}\overline{\Delta a})[w_{11}\,\bar{a}^2 + w_{31}\,\bar{\varrho}^2] \\ & + (\bar{a}\overline{\Delta\varrho})[w_{11}\,\bar{a}^2 + w_{31}\,\bar{\varrho}^2)] + (\bar{a}\overline{\Delta a})\,2w_{20}\,\bar{\varrho}^2 \\ & + (\bar{\varrho}\overline{\Delta\varrho})(\bar{\varrho}\bar{a})\,2w_{31} + (\bar{\varrho}\overline{\Delta a})(\bar{\varrho}\bar{a})\,2w_{22} + (\bar{a}\overline{\Delta\varrho})(\bar{\varrho}\bar{a})\,2w_{22} \\ & + (\bar{a}\overline{\Delta a})(\bar{\varrho}\bar{a})\,2w_{11} + (\overline{\Delta\varrho})^2[w_{20}\,\bar{a}^2 + 2w_{40}\,\bar{\varrho}^2] \end{aligned}$$

* Hereafter the notations $\bar{a}\bar{b}$ are used for scalar product.

$$+(\overline{\Delta\bar{\varrho}\Delta a})[w_{11}\bar{a}^2+w_{31}\bar{\varrho}^2]+(\overline{\Delta a})^2 w_{20}\bar{\varrho}^2$$
$$+(\overline{\bar{a}\Delta\varrho})^2 w_{22}+(\overline{\bar{a}\Delta\varrho})(\overline{\bar{a}\Delta a})2w_{11}+(\overline{\Delta\varrho})^2(\overline{\varrho\bar{a}})w_{31}$$
$$+(\overline{\Delta\varrho\Delta a})(\overline{\varrho\bar{a}})2w_{22}+(\overline{\Delta a})^2(\overline{\varrho\bar{a}})w_{11}$$
$$+(\overline{\bar{\varrho}\Delta\varrho})^2 4w_{40}+(\overline{\bar{\varrho}\Delta\varrho})(\overline{\bar{\varrho}\Delta a})2w_{11}+(\overline{\bar{\varrho}\Delta a})^2 w_{22}$$
$$+(\overline{\bar{\varrho}\Delta\varrho})(\overline{\bar{a}\Delta\varrho})2w_{31}+(\overline{\bar{\varrho}\Delta a})(\overline{\bar{a}\Delta\varrho})2w_{22}+(\overline{\bar{\varrho}\Delta\varrho})(\overline{\bar{a}\Delta a})4w_{20}$$
$$+(\overline{\bar{\varrho}\Delta a})(\overline{\bar{a}\Delta a})2w_{11}. \tag{3}$$

When only a single element of the system is decentred we can assume that $\overline{\Delta a} = m\overline{\Delta\varrho}$, where m is a coefficient related to the magnification of the image and the magnification of the aperture of the i-th element. If the decentred element is a lens, then $m \neq 0$, and if it is a diaphragm, then $m = 0$. Now, Eq. (3) can be rewritten in the following form:

$$\Phi_d = \Phi(\overline{\Delta\varrho} = 0)+\Phi(\overline{\Delta\varrho})+\Phi(\overline{\Delta\varrho^2}) \tag{4}$$

where

$$\Phi(\overline{\Delta\varrho} = 0) = w_{20}\bar{\varrho}^2\bar{a}^2+w_{40}\bar{\varrho}^4+(w_{11}\bar{a}^2+w_{31}\bar{\varrho}^2)(\overline{\varrho\bar{a}})+w_{22}(\overline{\varrho\bar{a}})^2, \tag{4.1}$$

and

$$\Phi(\overline{\Delta\varrho}) = (\overline{\bar{\varrho}\Delta\varrho})[2w_{20}\bar{a}^2+4w_{40}\bar{\varrho}^2+m(w_{11}\bar{a}^2+w_{31}\bar{\varrho}^2)]$$
$$+(\overline{\bar{a}\Delta\varrho})[w_{11}\bar{a}^2+w_{31}\bar{\varrho}^2+2w_{20}\bar{\varrho}^2 m]$$
$$+(\overline{\bar{\varrho}\Delta\varrho})(\overline{\varrho\bar{a}})[2w_{31}+2mw_{22}]+(\overline{\bar{a}\Delta\varrho})(\overline{\varrho\bar{a}})[2w_{22}+2mw_{11}], \tag{4.2}$$

$$\Phi(\overline{\Delta\varrho^2}) = (\overline{\Delta\varrho})^2[w_{20}\bar{a}^2+2w_{40}\bar{\varrho}^2+m(w_{11}\bar{a}^2+w_{31}\bar{\varrho}^2+w_{20}\bar{\varrho}^2 m^2]$$
$$+(\overline{\Delta\varrho})^2(\overline{\varrho\bar{a}})[w_{31}+2mw_{22}+m^2 w_{11}]$$
$$+(\overline{\bar{\varrho}\Delta\varrho})(\overline{\bar{a}\Delta\varrho})[2w_{31}+2mw_{22}+4mw_{20}+2m^2 w_{11}]$$
$$+(\overline{\bar{a}\Delta\varrho})^2[w_{22}+2mw_{11}]+(\overline{\bar{\varrho}\Delta\varrho})^2[4w_{40}+2mw_{31}+m^2 w_{22}]. \tag{4.3}$$

It can bee seen from Eq. (4) that the wave-aberration function of the system with a single decentration can be interpreted as the sum of the aberration of the contred system $\Phi(\overline{\Delta\varrho} = 0)$, the first-order aberration of decentration $\Phi(\overline{\Delta\varrho})$ and of the second-order aberration of decentration $\Phi(\overline{\Delta\varrho^2})$. The first term relates to the aberrations of the centred system, and the two next ones to the decentration of i-th element of an optical system. The interpretation of the decentration aberrations can be given in a simple way using geometrical optics approach.

3. Geometrical interpretation

From Equation (4) one can calculate the ray aberrations for a system with a decentred element. The vectors can be determined in the rectangular coordinate system as $\bar{\varrho}(X, Y)$, $\bar{a}(X_1, Y_1)$ and $\overline{\Delta\varrho}(X_D, Y_D)$. One can use the well-known relations [3]:

$$X_1 - X_1^* = \frac{\partial\Phi}{\partial X}\frac{R}{n}, \qquad Y_1 - Y_1^* = \frac{\partial\Phi}{\partial Y}\frac{R}{n} \tag{5}$$

where R is the radius of the reference sphere, and n denotes the refractive index of the medium in the image space.

Since only the decentration of a single element is considered in this moment, it may be assumed, for simplicity, that $Y_D = 0$. To express the function with the help of Eqs. (4) and (5) the formulae for both the components of the ray aberrations are given in Tab. 1. The equations shown in this table are analogous to those

Table 1. Ray-aberration of decentration generated by primary aberration of centred system

$(X_1 - X_1^*)\dfrac{n}{RX_D}$	$(Y_1 - Y_1^*)\dfrac{n}{RX_D}$	Interpretation (type of aberration)
$(X^2+Y^2)(4w_{40}+mw_{31})$	$2XY(4w_{40}+mw_{31})$	Coma of decentration
$+2XY_1(3w_{31}+2mw_{22}+2mw_{20})$	$+2YX_1(w_{31}+2mw_{20})$	Astigmatism and image
$+YY_1(2w_{31}+2mw_{22})$	$+XY_1(2w_{31}+2mw_{22})$	inclination
$+X_1^2(2w_{20}+3mw_{11}+2m_{22})$	$+X_1Y_1(2mw_{11}+2w_{22})$	Distortion of decentration
$+Y_1^2(2w_{20}+mw_{11})$		

mentioned by GUBEL [4], but instead of Seidel coefficients they include wave-aberration coefficients and linear ray coordinates. Using the arrangement of coefficients shown in Tab. 1, one can determine the aberrations of decentration being generated by the separate aberrations of the centred system. These aberrations, shown in Tab. 2, correspond to those discussed by MARECHAL [5] and the

Table 2. Aberrations of decentration generated by separate primary aberrations of centred system

Aberrations of a centred system		Generated decentration aberrations
Spherical aberration	w_{40}	Decentration coma
Coma	w_{31}	Decentration coma Decentration astigmatism Image inclination
Astigmatism	w_{22}	Decentration astigmatism Decentration distortion
Field curvature	w_{20}	Decentration distortion Image inclination
Distortion	w_{11}	Decentration distortion

other authors. The discussion of the types of decentration aberrations is well-known [4] and will not be repeated here.

Now, let us consider the function $\Phi(\overline{\Delta\varrho})^2$ in the form analogical to that given in Tab. 1. The corresponding result is given in Tab. 3. Such an expression does not have its counterpart in the literature. Its first term (the upper line in Tab. 3), being

Table 3. Ray-aberrations of decentration of second order of $(\overline{\Delta\varrho})^2$

$(X_1 - X_1^*)\dfrac{n}{RX_D^2}$	$(Y_1 - Y_1^*)\dfrac{n}{RX_D^2}$
$2X(6w_{40}+3mw_{31}+m^2 w_{20}+m^2 w_{22})$	$2Y(2w_{40}+mw_{31}+m^2 w_{20})$
$+X_1(3w_{31}+4mw_{22}+3m^2 w_{11}+4mw_{20})$	$+Y_1(w_{31}+2mw_{22}+m^2 w_{11})$

proportional to the aperture height, can be treated as a specific form of astigmatism (constant within the whole image plane). This term consists of all the aberration coefficients of the centred system, except for the distortion w_{11}. This fact is easily understood, for w_{11} is not a function of aperture height. The second term (the lower line in Tab. 3), being a form of the distortion of decentration, asymmetrical in the image plane, is generated by all the aberrations of the centred system, except for the spherical aberration.

Both the above mentioned aberrations depend on the squared decentration. Therefore, their values are small in comparison with the aberrations expressed in Tab. 1.

Equations (2)–(4) enable us to determine the decentration aberrations from the wave-aberration coefficients of the centred system without any additional meridional or pseudo-ray tracing [6], [7]. Moreover, the calculation of additional coefficients specific of decentration aberrations is not necessary any more. The mathematical formulae lead us quickly to the results which are in accordance with those obtained from a more involved analysis [4], [5]. They also give information about the decentration aberrations of second order of magnitude with respect to $(\overline{\Delta\varrho})^2$, see Tab. 3. In addition, they make it possible to evaluate the wavefront emerging from the optical system, by using the method described in [1].

4. System with decentrations

It is demonstrated in Sect. 3 that Eqs. (4), (4.1)–(4.3) of Sect. 2 permit us to make a clear geometrical interpretation of decentration aberrations. Obviously, in the real composed system with several decentrations the problem is more complicated. The effect of decentrations on the wave-aberration of the system can be described by means of the additional asymmetrical terms. They give an extra contribution to the wave-aberration of the corresponding centred system. The aberrational terms of the wave-aberration function, which is determined with respect to the proper reference sphere, may be treated as distorters [1]. Such distorters may be transferred to the image space of the system by taking into account the Fresnel diffraction approximation. In consequence, the composed optical system in the Fresnel approximation can be expressed as a set of the following components:

1. Purely quadratic phase correctors that describe focusing properties of all the elements.

2. Distorters due to aberrations of the corresponding centred system, which for the whole system may be described as

$$\Phi(\overline{\Delta\varrho} = 0) = \sum_{i=1}^{n} [w_{20i}\,\bar{\varrho}^2\,\bar{a}^2 + w_{40i}\,\varrho^4 + (w_{11i}\,a^2 + w_{31i}\,\varrho^2)(\varrho a) + w_{22i}(\bar{\varrho}\bar{a})^2]. \quad (6)$$

3. Distorters due to the decentrations of the first order which, after proper rearrangement of Eqs. (3) and (4), may be expressed as

$$\begin{aligned}\Phi(\overline{\Delta\varrho} \neq 0) = \sum_{i=1}^{n} \{ & w_{20i}[2(\bar{\varrho}\overline{\Delta\varrho_i})\,\bar{a}^2 + 2(a\overline{\Delta a_i})\,\varrho^2] + w_{40i}\,4(\varrho\overline{\Delta\varrho_i})\,\varrho^2 \\ & + w_{11i}[(\bar{a}\overline{\Delta\varrho_i})\,\bar{a}^2 + (\varrho\overline{\Delta a_i})\,\bar{a}^2 + 2(\bar{a}\overline{\Delta a_i})(\varrho\bar{a})] \\ & + w_{31i}[(\bar{a}\overline{\Delta\varrho_i})\,\bar{\varrho}^2 + 2(\bar{\varrho}\overline{\Delta\varrho_i})(\varrho a) + (\varrho\overline{\Delta a_i})\,\varrho^2] \\ & + w_{22i}[2(\bar{a}\overline{\Delta\varrho_i})(\bar{\varrho}\bar{a}) + 2(\bar{\varrho}\overline{\Delta\varrho_i})(\bar{\varrho}\bar{a})]\}. \end{aligned} \quad (7)$$

In Eq. (7), analogically as in Eq. (2), $\overline{\Delta\varrho_i}$ is the displacement (with arbitrary amplitude and azimuth) of aberration function in the aperture of i-th element, and $\overline{\Delta a_i}$ – shift of the image point caused by the decentration of i-th element.

In Section 3 it is mentioned that the squared decentration depending aberrations are small in comparison with those depending on the first order of decentrations. In practice, the decentration tolerances are small. Therefore, in the further considerations the quadratic terms of decentration aberrations are not taken into account.

The expression (7) should be taken into account twice. Firstly, when the effect of shift of the aberrated wavefront incident on i-th element (caused by the aberrations of the previous part of the system) is analysed. In such a case, the coefficients w_{mni} in Eq. (7) should be replaced by $\sum_{k=1}^{i-1} w_{mnk}$ and $\overline{\Delta\varrho_i} = (1-M_{ai})\,\overline{\Delta c_i}$, $\overline{\Delta a_i} = (1-M_{oi})\,\overline{\Delta c_i}$. Secondly, when the effects of first order aberrations introduced by i-th element are considered. In this case, the function of first order aberrations of i-th element is shifted with them by Δc_i from the optical axis of the system. By transforming this function to the exit pupil of the i-th element we obtain the expression (7), but with $\overline{\Delta\varrho_i} = -\overline{\Delta c_i}\,M_{ai}$, and for the image shift $\overline{\Delta a_i} = (1 - M_{oi})\,\overline{\Delta c_i}$ for both cases: $\overline{\Delta c_i}$ is a lateral shift of the i-th element, M_{ai}, M_{oi} – pupil and image magnifications of the i-th element, $\bar{\varrho}$, $\bar{a}$, $\overline{\Delta\varrho_i}$, $\overline{\Delta a_i}$ are normalized to the maximal pupil and image heights, respectively, in the image space of i-th element).

The wave-aberration function of the decentred system, as expressed by Eqs. (6), (7), permits us to apply the methods of image assessment known from the diffraction theory of imaging. It enables us to carry out the numerical evaluation of the intensity distribution in the diffraction pattern under Fresnel approximation or the calculation of the variance for the wavefront in the pupil (under Marechal approximation [3]).

5. Example

Let us consider a simple case, when the wave-aberration is represented by the spherical aberration and defocusing. For the aberrations of a centred system and assuming a single decentration we have $w_{40} \neq 0$, $w_{11} = w_{31} = w_{22} = w_{20} = 0$ and an additional term, representing the defocusing in form of $w_0 \bar{\varrho}^2$ (in the full Hopkins notation [2] this is ${}_0w_{20}\,\bar{\varrho}^2$). In this case, we can assume that only spherical aberration and decentration coma will occur in the image. Thus, the influence of defocusing will be discussed.

We can consider a simple optical system of two thin lenses positioned close to each other; the spherical aberration is assumed to be uncompensated for the system (Fig. 2). The aberrations for lenses A and B can be described as

$$\Phi_{\mathrm{A}} = w_{40\mathrm{A}}\,\bar{\varrho}^4, \tag{8}$$

and

$$\Phi_{\mathrm{B}} = w_p\,\bar{\varrho}^2 + w_{40\mathrm{B}}\,\bar{\varrho}^4. \tag{9}$$

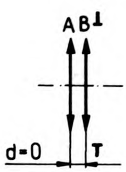

Fig. 2. System of two thin lenses with the longitudinal separation, selected as equal to zero, $d = 0$

In the last equation the defocusing has been taken into account. The lens A will behave as the decentred one, thus the wavefront behind this lens is shifted in the aperture by $\overline{\Delta\varrho}$. In such a case the wave-aberration function for lens A is

$$\Phi_{\mathrm{A}d} = w_{40\mathrm{A}}(\bar{\varrho} + \overline{\Delta\varrho})^4, \tag{10}$$

and for the whole system A+B (omitting the terms of $\overline{\Delta\varrho}$ of order higher than one)

$$\Phi = \Phi_{\mathrm{A}d} + \Phi_{\mathrm{B}} = w_p\,\bar{\varrho}^2 + (w_{40\mathrm{A}} + w_{40\mathrm{B}})\,\bar{\varrho}^4 + (\bar{\varrho}\overline{\Delta\varrho})\,4w_{40\mathrm{A}}\,\bar{\varrho}^2. \tag{11}$$

Assuming that the aberrations in the centred system are relatively well compensated we can write

$$|w_{40\mathrm{A}} + w_{40\mathrm{B}}| \ll |w_{40\mathrm{A}}|;\ |w_{40\mathrm{B}}|. \tag{12}$$

If the vector $\overline{\Delta\varrho}$ is described in polar-coordinate system, the variance of wavefront expressed by (11)

$$E = \frac{1}{\pi}\int\int \Phi^2 \varrho d\varrho d\theta - \frac{1}{\pi^2}\left[\int\int \Phi\varrho d\varrho d\theta\right]^2, \tag{13}$$

can be written as

$$E = \frac{1}{12}w_p^2 + \frac{1}{6}w_p(w_{40A}+w_{40B}) + \frac{4}{45}(w_{40A}+w_{40B})^2 + (\Delta\varrho)^2\, 2w_{40A}^2. \tag{14}$$

For determining the best image plane the condition $\partial E/\partial w_p = 0$ must be fulfiled, hence

$$w_{p\,opt} = -(w_{40A}+w_{40B}). \tag{15}$$

In this case, the best image plane is the same as that for a centred system. We can consider also the influence of the terms of $(\Delta\varrho)^2$, which were neglected in Sect. 4 (Eq. (7)). Then Eq. (10) yields

$$\Phi = w_p\bar{\varrho}^2 + (w_{40A}+w_{40B})\,\bar{\varrho}^4 + (\bar{\varrho}\overline{\Delta\varrho})\,4w_{40A}\,\bar{\varrho}^2 + (\bar{\varrho}\overline{\Delta\varrho})^2\,4w_{40A}$$
$$+(\overline{\Delta\varrho})^2\,2w_{40A}\,\bar{\varrho}^2. \tag{11a}$$

Taking into account Eqs. (11a) and (13), we express the variance as

$$E = \frac{1}{12}w_p + \frac{1}{6}w_p(w_{40A}+w_{40B}) + \frac{4}{45}(w_{40A}+w_{40B})^2$$
$$+(\Delta\varrho)^2\left(\frac{8}{3}w_{40A}^2 + \frac{2}{3}w_p\,w_{40A} + \frac{2}{3}w_{40A}\,w_{40B}\right) \tag{16}$$

instead of Eq. (11). In this case, for determining the best image plane we obtain for $w_{p\,opt}$: $\dfrac{\partial E}{\partial w_p} = 0$, hence

$$w_{p\,opt} = -(w_{40A}+w_{40B}) - (\Delta\varrho)^2\,4w_{40A}. \tag{17}$$

The variance expressed by Eq. (16) will be discussed in three cases: i) in the Gaussian image plane, ii) in the best image plane for the system without decentration, and iii) when the decentration is taken into account.

In the first case (Gaussian image plane) we have $w_p = 0$ and

$$E = \frac{4}{45}(w_{40A}+w_{40B})^2 + (\Delta\varrho)^2\left(\frac{8}{3}w_{40A}^2 + \frac{2}{3}w_{40B}\,w_{40A}\right). \tag{18}$$

In the second case (i.e., the best image plane, the influence of the decentration being not included)

$$w_p = -(w_{40A}+w_{40B}),$$
$$E = \frac{1}{180}(w_{40A}+w_{40B})^2 + (\Delta\varrho)^2\,2w_{40A}^2. \tag{19}$$

Finally, in the last case (decentration influence being considered)

$$w_p = -(w_{40\mathrm{A}}+w_{40\mathrm{B}})-(\Delta\varrho)^2\,4w_{40\mathrm{A}},$$
$$E = \frac{1}{180}(w_{40\mathrm{A}}+w_{40\mathrm{B}})^2+(\Delta\varrho)^2\,2w_{40\mathrm{A}}\left[1-\frac{2}{3}(\Delta\varrho)^2\right]. \tag{20}$$

Taking into account Eq. (14) and the fact that $\Delta\varrho \ll 1$, the following statements can be formulated:

i) Longitudinal shift of the image plane has a well-known influence on the part of the variance function related to the aberrations of a centred system.

ii) Displacement of the image from the Gaussian plane to the plane described by Eq. (19) reduces a part of the variance bounded with decentration aberrations by about 40%.

iii) As to the influence of decentration when the image plane is displaced (Eq. (20)), the observed contribution to the image quality degradation is very small if compared with the case expressed by Eq. (19). This is because $\frac{2}{3}(\Delta\varrho)^2 \ll 1$.

6. Summary

The expression for wave-aberration including the decentration errors has been derived. It makes it possible to describe the phase distorter of decentration of any lens in the optical system. The proposed analytical method enables us to make a simple classification of the decentration aberrations expressed with the help of aberration coefficients of centred system. The amount of calculations and data (for Seidel aberrations – five coefficients for every optical element) is reduced. Distorter of any type can be transferred through a focusing element from one space to another one without altering its influence on the imaging process. This is true, of course, under limitations of the Fresnel approximation and in the isoplanatic region of the object. After transferring all the distorters to one space of the system (the one-space method [1]), it is possible to determine the perturbation due to the diffraction phenomena in the whole system. Taking into account an effective series of distorters it is possible to determine field disturbances resulting from these centring tolerances, and to analyse the corresponding deterioration of the image quality. Alternatively, based on a given image quality criterion we can calculate the admissible decentration tolerances. A simple example of application of the method to the analysis of the interrelations between the aberrations of centred system and decentration has been shown.

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Волновая аберрация оптических систем с малыми децентрировками. Векторный анализ

Представлен новый, векторный метод анализа влияния небольших децентрировок на волновую аберрацию оптической системы. Волновая аберрация, введенная одним из членов оптической системы, в том и аберрация децентрировки может быть интерпретирована как „дистортер". Математическое описание такого „дистортера" дает возможность анализа децентрировок, как и их влияния на качество изображения при использовании методов синтеза изображения в приближении Френеля. Показан пример вычисления вариаций для оптической системы со сферической аберрацией и комой децентрировки. Проанализировано влияние дефокусировки в таком случае.