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Thickness of the layers of multilayer nonquarterwave interference filters controlled by direct level monitoring

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Analytical formulae are described for calculation of the thickness of nonabsorbing and homogeneous layers from direct level monitoring of nonquarterwave multilayer filters.

1. Introduction

A considerable interest in determining optical constants and thickness of thin-film optical layers has been observed recently [1–3]. This interest is due to the demand for an accurate monitoring of layers during production of nonquarterwave interference filters. The aim of this paper is to describe an analytical method for determining the thickness of nonabsorbing and homogeneous layers during filter monitoring. The calculations are based on the concept of optical admittance [4], [5].

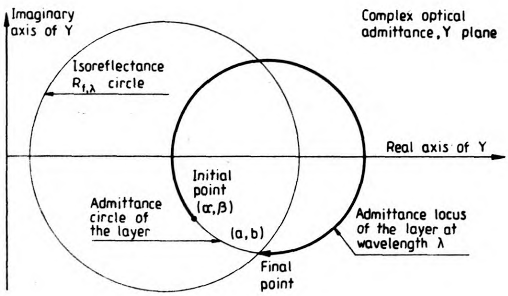
2. Analytical calculations

The monitoring data of the filter consist of the set of transmittance or reflectance values measured in vacuum during the filter production (Tab. 1). In the sequel the reflectance will be treated as a minitoring signal. The deposition of each layer is started at the initial reflectance $R_{i,\lambda}$ and is terminated when the reflectance reaches the final value $R_{f,\lambda}$. In terms of the optical admittance Y of the layer it means that

Table 1. Direct level monitoring data for 3-layer filter. Indices of substrate, H–layer, L–layer and medium: 1.515, 2.4956, 1.33 and 1.00, respectively

Layer		Lambda [nm]	Levels of monitoring signal				Number of turning points
			Transmittance		Reflectance		
Number	Type		Initial	Final	Initial	Final	
1	H	452	0.958	0.774	0.042	0.226	1
2	L	452	0.774	0.7643	0.226	0.2357	2
3	H	452	0.7643	0.9138	0.2357	0.0862	1

its admittance locus starts at the point (α, β) of complex plane Y corresponding to $R_{i,\lambda}$ and ends in the point (a, b), corresponding to $R_{f,\lambda}$ (Fig.). For the first layer, $R_{i,\lambda}$ is equal to the reflectance in vacuum of the bare substrate. In this case, the coordinates of the starting point of the admittance locus are $(n_s, 0)$, where n_s denotes the index of the substrate at wavelength λ. This case forms the initial condition for the whole analysis of the multilayer, because the starting (initial) point of the locus of the next layer is equal to the final point of the locus of the preceding layer.



Optical monitoring of the layer on a complex admittance plane

For the calculation of the coordinates of the final point of the admittance locus of the layer (in a general case) we shall proceeded in the way described by MACLEOD [6]. From the figure it can be seen that this point is one of two intersection points of two circles. The first circle is the isoreflectance one corresponding to $R_{f,\lambda}$ and the second is the admittance circle of the considered layer. Radii and centres (laying on the real axis of the complex plane Y) of these circles are given by the formulae [6]:

$$\varrho_R = [4R_{f,\lambda}/(1-R_{f,\lambda})^2]^{1/2}, \tag{1}$$

$$C_R = (1+R_{f,\lambda})/(1-R_{f,\lambda}), \tag{2}$$

$$\varrho_Y = \{[(\alpha^2+\beta^2+N^2)/(2\alpha)]^2 - N^2\}^{1/2}, \tag{3}$$

$$C_Y = (\alpha^2+\beta^2+N^2)/(2\alpha) \tag{4}$$

where N denotes the index of the layer at wavelength λ. Using Eqs. (1)–(4) it is easy to show that the coordinates of the final point of admittance locus of the layers equals (a, b) or $(a, -b)$, where

$$a = (C_R^2 - C_Y^2 + \varrho_Y^2 - \varrho_R^2)/[2(C_R - C_Y)], \tag{5}$$

$$b = \pm[\varrho_Y^2 - (a - C_Y)^2]^{1/2}. \tag{6}$$

Plus or minus sign for the coordinate b may be determined by taking into account the number of turning values of the reflectance measured during the layer monitoring.

Having radius, centre, coordinates of starting and final points of the admittance locus of the monitored layer its whole admittance locus can be constructed. In order to determine the thickness of the layer it is necessary to relate its optical or phase thickness to the coordinates of final point of its admittance locus. For this purpose we will use the analytical formula [7], which describes the relation between the phase thickness δ of the layer of given index N and the coordinates (α, β) and b of any initial and final points on the layer admittance locus

$$b = N\frac{(N^2-\alpha^2-\beta^2)\tan\delta+\beta N(1-\tan^2\delta)}{(N-\beta\tan\delta)^2+(\alpha\tan\delta)^2}. \tag{7}$$

Introducing the following auxilliary quantities

$$\sigma^2 = \alpha^2+\beta^2, \tag{8}$$

$$\varepsilon = (b\sigma^2/N^2)+\beta, \tag{9}$$

$$\zeta = (\sigma^2/N)-N-(2\beta b/N), \tag{10}$$

$$\eta = b-\beta, \tag{11}$$

Eq. (7) may be transformed into the form of the quadratic equation with respect to $\tan\delta$

$$\varepsilon\tan^2\delta+\zeta\tan\delta+\eta = 0. \tag{12}$$

Substituting

$$x = tan\,\delta \tag{13}$$

we find the following solutions:

$$x_1 = (-\zeta+\Delta^{1/2})/(2\varepsilon), \tag{14}$$

$$x_2 = (-\zeta-\Delta^{1/2})/(2\varepsilon) \tag{15}$$

where

$$\Delta = \zeta^2-4\varepsilon\eta. \tag{16}$$

Inserting Eqs. (9)–(11) into Eq. (14) and (15) we can derive the following formula:

$$\tan\delta_{1(2)} = \frac{(2\beta b/N)+N-(\sigma^2/N)\pm\Delta}{2[(b\sigma^2/\mathrm{N}^2)+\beta]} \tag{17}$$

where $\tan\delta_1$ and $\tan\delta_2$ corresponds to x_1 and x_2, respectively, and

$$\Delta = [(2\beta b/N)+N-(\sigma^2/N)]^2+4(\beta-b)[(b\sigma^2/N^2)+\beta]. \tag{18}$$

From the value of $\tan\delta$ we can calculate the optical thickness of monitored layer

$$\delta = k\pi+\mathrm{arc}\tan x \tag{19}$$

and its physical thickness

$$e = \lambda\delta/(2\pi N). \tag{20}$$

The value of integer k may be determined by taking into account the number of turning values of the reflectance measured during layer deposition as well as the sign and value of $\delta_{1(2)}$. Applying the above procedure to each subsequent layer of the filter we find the thicknesses of all deposited layers. This procedure was implemented to 8-bit computer.

3. Numerical example

As a simple example we will consider the data for single wavelength direct level monitoring of three-layer nonquarterwave filter designed for the increasing colour temperature of light source [7]. Suppose that the results of measurements of the monitoring apparatus during filter deposition are the same as those presented in Tab. 1. Below there are described calculations performed according to the procedure from Sect. 2 which allows us to determine the thickness of each deposited layer.

Radius and centre of the isoreflectance circle corresponding to the value 0.226 (final point of reflectance for the first layer) are equal to 1.2284 and 1.584, respectively. The same parameters for the optical admittance circle of this layer are 1.2979 and 2.8129. From Eq. (5) and (6) we find the coordinates (a, b) of the points of the intersection of these two circles. Since the measured reflectance passes through one turning point, then for the final point of admittance locus of the layer we take the negative value of b, thus $b = -1.10187$. After solving Eq. (12) we obtain for x_1 and x_2 -2.96591 and -0.91488, respectively. From Eq. (19) it follows that $\delta_1 = 1.896$ and $\delta_2 = 2.4006$. Again, in view of the fact that the deposition of the layer is terminated after the monitoring signal passes through one turning point, a higher value of δ is considered and the value of thickness e is calculated from Eq. (20). For the second and third layers the calculation procedures are similar. Results of all calculations are presented in Tab. 2. If several

Table 2. Calculated values of the physical thickness of layers

Layer		Thickness [nm]
Number	Type	
1	H	69.2
2	L	162.6
3	H	29.3

wavelengths are chosen for the monitoring program of the filter [8], then initial and final reflectances should be measured and stored for all wavelengths and for each layer. This procedure gives the input data for the reconstruction of admittance locus of each layer and for all the specified wavelengths, thus the possibility of determining the thicknesses of all the layers.

4. Conclusions

The paper presents the method for the determining the thickness of homogeneous and nonabsorbing layers of nonquarterwave multilayer filters from measurements made in vacuum during the filter monitoring. Although the method is illustrated by a single wavelength level monitoring it may be applied to direct or semi-direct multiwavelength monitoring, based on level or turning value measurements.

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Определение толщины слоев фильтров с нечетвертьволновыми оптическими толщинами при использовании фотометрического способа контроля нанесения слоев

Предположен аналитический метод определения толщины слоев многослоевых интерференционных фильтров во время их нанесения в вакууме при использовании фотометрического способа контроля оседания слоев. В методе базируют на аналитических формулах описывающих шкалу диаграммы полной оптической проводимости однородного непоглощающего слоя любой толщины нанесенного на любой многослой.