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Laser Doppler anemometry with selection of optical signal coherent component

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The method of selection of optical signal coherent component in laser Doppler anemometry has been presented. The possibility of improving the measurement accuracy and getting the information on the spatial structure of flows in laser Doppler anemometry with coherent signal selection has been discussed

1. Introduction

As it is known, laser Doppler anemometry (LDA) methods based on selection of the Doppler frequency shift in a coherent light scattered by the moving medium being investigated have been widely applied in contemporary hydro- and aerodynamics. Accuracy of such measurements is limited by the existence of the so-called phase background noise resulting from the superposition of signals from several scattering particles which are simultaneously present within the range of a probe optical field. With a high concentration of scattering particles the optical signal in LDA is the superposition of coherent and incoherent components in which frequency spectra are overlapped in the real fringe schemes [1–3]. Another problem is the expansion of the functional possibilities of LDA to determine the scatter sizes and concentration simultaneously with their velocity measurement. A velocity measurement technique based on the separation of a coherent component of the optical signal in LDA has been proposed in [4]. The estimations made show that LDA operation in the coeherent regime removes some limitations of the phase noise and expands functional possibilities of measurements. This technique consists in the formation, in the medium under investigation, of a superposition of coherent light fields with an angle spectrum diversity, and in optical mixing of light signals scattered by different local structures of the medium. Before photoblending, a preliminary space-frequency filtration of optical signals has been carried out by means of the Fourier filter matched with a spatial distribution of scattering structures. The case of scatterer motions in the plane orthogonal to the optical axis of the Fourier analyzer has been investigated [1]. The subject of this paper is to analyse the optical signal space-time structure with regard to the particle movement in the volume in which the probe optical field is localized.

2. Results and discussion

Referring to Figure 1, a differential scheme of the laser Doppler anemometer is shown, wherein the sounding optical field is formed by a coherent superposition of the two laser beams crossing under the 2α angle

$$E(x) = \exp[-\varepsilon(x-\alpha z)^2]\exp(j\alpha kz)\exp(j\Omega t)+\exp[-\varepsilon(x-\alpha z)^2]\exp(-j\alpha kz) \quad (1)$$

where $\varepsilon = w_0^{-2}$, w_0 is a throat radius of the Gaussian beam (we consider the case of beams crossing the region of throats localization, the wave fronts are supposed to be flat), Ω is the known difference of laser beam frequencies, e.g., given by a single-side band acousto-optical modulator, k is wave number.

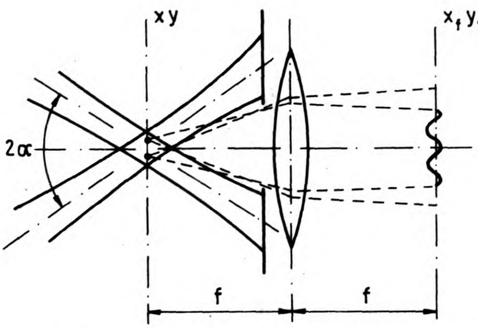

Fig. 1. Formation of coherent component of LDA optical signal

Let this field be crossed by a flow of scattering point particles. For simplicity we shall confine ourselves to the case of equal particle velocities. We shall investigate an optical signal formation and transformation in the xz plane which does not change the principle aspect of the problem. A pulse response of the objective, which describes in the Fourier-plane x_f the optical signal being formed from the point source with xz coordinates, is written in the form

$$h(x_f, x) = \frac{1}{j\lambda f}\exp\left(-j\frac{kz}{2f^2}x_f^2\right)\exp\left(-j\frac{k}{f}x_f x\right). \quad (2)$$

Here λ is laser wavelength, f – focal distance of the objective.

The field diffracted by particles is presented in the Fourier plane by the superposition integral

$$E(x_f) = \iint_{-\infty}^{\infty} E(x, z)\sum_{n=1}^{N}\delta(x-x_n)\delta(z-z_n)h(x_f, x)\,dx\,dz.$$

In this integral

$$\sum_{n=1}^{N}\delta(x-x_n)\delta(z-z_n)$$

is the two-dimensional matrix function describing the scattering particle in the xz plane, $x_n = v_x(t-t_n)$, $z_n = v_z(t-t_n)$, v_x and v_z are particle velocities along the x and

z axes, respectively. Taking advantage of the filtering properties of δ function we obtain

$$E(x_f) = \frac{1}{j\lambda f}\sum_{n=1}^{N}\exp\left(-j\frac{kz_n}{2f^2}x_f^2\right)\exp\left(-j\frac{k}{f}x_f, x_n\right)$$
$$\times\{\exp[-\varepsilon(x_n-\alpha z_n)^2]\exp(j\alpha kx_n)\exp(k\Omega t)$$
$$+\exp[-\varepsilon(x_n+\alpha z_n)^2]\exp(-j\alpha kx_n)\}. \quad (3)$$

The resulting light field intensity (3) is the sum of the pedestal I_p, incoherent I_{ic} and coherent I_c components. The pedestal corresponds to the speckle pattern formed by means of light scattering by particles without regard to the cross interference from laser beams

$$I_p = \frac{2}{\lambda^2 f^2}\sum_{n=1}^{N} A_n \operatorname{ch}(4\alpha\varepsilon x_n z_n) + \frac{2}{\lambda^2 f^2}\sum_{\substack{n,m\\ n>m}}^{N} A_n A_m \{\operatorname{ch}\gamma_{nm}\cos(\alpha k x_{nm})\cos$$
$$\times[k(a_{nm}x_f + b_{nm}x_f^x)] + \operatorname{sh}\gamma_{nm}\sin(\alpha k x_{nm})\sin[k(a_{nm}x_f + b_{nm}x_f^2)]\}$$

where: $\gamma_{nm} = 2\alpha\varepsilon(x_n z_n + x_m z_m)$; $a_{nm} = x_{nm}/f$; $b_{nm} = z_{nm}/2f^2$; $A_n = \exp[-\varepsilon(x_n^2 + \alpha^2 z^2)]$. The pedestal respresents an incoherent sum of speckle fields formed by scattering particles from each individual illuminating laser beam. The pedestal includes a component defined by an independent contribution of each scattering particle and by the result of mutual cross interference of N-particles found simultaneously at the measuring volume.

An incoherent component I_{ic} is formed due to the interference of light fields scattered by each separate particle from different illuminating laser beams

$$I_{ic} = \frac{2}{\lambda^2 f^2}\sum_{n=1}^{N} A_n^2\cos[\Omega t + \omega_D(t-t_n)] \quad (4)$$

where $\omega_D = 2\alpha k v_x$.

Usage of the term "incoherent" is associated here with the analogy (4) to the structure of the signal received at scattering of the raster projection by the particle flow in incoherent light. This signal corresponds to the narrow-band process discretized by a random flow of point particles. Owing to the narrow-band nature, this process frequency is determined by the sum

$$\omega_{ic} = \Omega + \omega_D + \tilde{\omega}_{ic}$$

where

$$\tilde{\omega}_{ic} = \frac{4\tilde{\omega}_D^2 \sum\limits_{\substack{n,m\\ n>m}}^{N} t_{nm} A_n^2 A_m^2 \sin\omega_D t_{nm}}{\pi^2 M^2 \sum\limits_{n,m}^{N} A_n^2 A_m^2 \cos\omega_D t_{nm}} \quad (5)$$

Here $\tilde{\omega}^2 = (v_x^2 + \alpha v_z^2)/\Lambda^2$, Λ is an interference fringe size in the sounding field, M – number of interference fringes in the probe field. This component, called a phase of background noise, occurs due to the superposition of signals from several particles found simultaneously at the measuring volume and is of a random nature. In media forming a multiparticle Doppler signal, the phase noise is the principle factor limiting the measurement accuracy. According to (5) a relative error of velocity measurements, which is due to the phase noise, is inversely proportional to the square of the number interference fringes in the probe field

$$\tilde{\omega}_{\text{ic}}/\omega_{\text{D}} \simeq Q_{\text{ic}}/M^2$$

where Q_{ic} is a dimensionless coefficient determined by the statistics of a space particle distribution.

Turn now to the coeherent component I_{c}. It appears as a result of the cross interference of light fields scattered by various particles from different laser beams

$$I_{\text{c}} = \frac{2}{\lambda^2 f^2} \sum_{\substack{n,m \\ n>m}}^{N} A_n A_m \Bigg\{ \text{ch}\beta_{nm} \cos\left[k(a_{nm} x_f + b_{nm} x_f^2)\right] \times \cos\left[(\Omega + \omega_{\text{D}})t - \frac{1}{2}\omega_{\text{D}}(t_n + t_m)\right] + \text{sh}\,\beta_{nm} \sin\left[k(a_{nm} x_f + b_{nm} x_f^2)\right] \sin\left[(\Omega + \omega_{\text{D}})t - \frac{1}{2}\omega_{\text{D}}(t_n + t_m)\right]\Bigg\} \tag{6}$$

where $\beta_{nm} = 2\alpha(x_n z_n - x_m z_m)$. From (6) is is apparent that the amplitude and phase of the coherent component of the resulting optical field provide functions of the spatial frequency $k(a_{nm} + b_{nm} x_f)$ which contains stationary ka_{nm} and linear frequency modulation $kb_{nm} x_f$ components.

Let us perform a photoelectric transformation of a coherent component of the optical field by the photodetector with an aperture $2d$ and transformation coefficient ϱ. Then the photocurrent may be represented by the expression

$$i_{\text{c}} = \varrho \int_{-d}^{d} I_{\text{c}}\, dx_f. \tag{7}$$

Substituting (6) into (7) and performing integration in a stationary phase approximation for

$$x_{nm} < -\frac{d}{f} z_{nm}, \qquad x_{nm} > \frac{d}{f} z_{nm}$$

we get

$$i_{\text{c}} = \varrho \frac{4d}{\lambda^2 f^2} \sum_{\substack{n,m \\ n>m}}^{N} A_n A_m \,\text{sinc}\left(\frac{ka_{nm} d}{\pi}\right) q_{nm}$$

where

$$q_{nm} = \left\{ \mathrm{ch}\, \beta_{nm} \cos(kb_{nm} d^2) \cos\left[(\Omega+\omega_{\mathrm{D}})\, t - \frac{1}{2}\omega_{\mathrm{D}}(t_n + t_m)\right] \right.$$
$$\left. + \mathrm{sh}\, \beta_{nm} \sin(kb_{nm} d^2) \sin\left[(\Omega+\omega_{\mathrm{D}})\, t - \frac{1}{2}\omega_{\mathrm{D}}(t_n + t_m)\right]\right\}. \quad (8)$$

The condition

$$|x_{nm}| > \frac{d}{f} |z_{nm}|$$

corresponds to the case when the size of the interference fringe (speckle size), forming as a result of cross interference of light waves scattered by the n-th and m-th particles of different incident beams, appears to be less that the photodetector aperture ($2d$). In other words, under this condition the maximum size of the coherence area is always less than the photodetector aperture. From (8) is follows that a coherent component of the photoelectric current is defined by the sum of N-terms, each of them including the factor $\mathrm{sine}(kx_{nm} d/\pi f)$.

An argument of this function is proportional to the ratio of the spacing between scattering particles x_{nm} and the size $\lambda f/2d$ of the space Fourier-image of the photodetector aperture diaphragm. It means that to separate the coherent component the photodetector aperture should have a maximum size which does not exceed the size of the least speckle in the Fourier-plane [4].

It is easy to see that the coherent signal amplitude appears here to be much smaller than the incoherent signal amplitude and a coherent mode of the speckle interferometer operation proves to be energetically disadvantageous.

Consider now the possibility of separation of a coherent component of the optical signal with optimal utilization of laser energy. Take into account the fact that according to (6) this component is the function of the spatial frequency in the Fourier-plane. Let us place a selecting filter with the transmission function $\cos Kx_f$ (one of the variant of such filter realization will be discussed later). In this case the expression for the photoelectric signal from the detector output takes the form

$$i_{\Phi} = \varrho \int_{-d}^{d} \cos(Kx_f)(I_{\mathrm{ic}} + I_{\mathrm{c}})\, dx_f.$$

After integration with regard to (4) and (8) for $x_{nm} < -\frac{d}{f} z_{nm}$, $x_{nm} > \frac{d}{f} z_{nm}$ we obtain:

$$i_{\Phi} = i_{\mathrm{ic}\Phi} + i_{\mathrm{c}\Phi},$$
$$i_{\mathrm{ic}\Phi} = 2d \sin(Kd/\pi)\, I_{\mathrm{ic}}, \quad (9)$$
$$i_{\mathrm{c}\Phi} = \frac{2d}{\lambda^2 f^2} \sum_{\substack{n,m \\ n>m}}^{N} A_n A_m \left\{ \mathrm{sinc}\left[\frac{d}{\pi}(K - ka_{nm})\right] + \mathrm{sinc}\left[\frac{d}{\pi}(K + ka_{nm})\right] \right\} q_{nm}$$

(integration of a coherent component is performed in a stationary phase approximation).

As it follows from (9) a coherent component predominates in the structure of the output signal with a selecting spatial frequency filter. For this component a spatial filter frequency is connected with the particle spacing by the ratio

$$K-\frac{k}{f}|x_{nm}|=0. \tag{10}$$

An incoherent component integrated by the finite aperture of the selecting filter becomes negligibly small since $Kd \gg 1$.

Consider the coherent signal structure with a selecting filter

$$i_{c\Phi}=\varrho\frac{2d}{\lambda^2 f^2}\sum_{\substack{n,m\\ n>m}}^{N} A_n A_m q_{nm}\operatorname{sinc}\zeta_{nm} \tag{11}$$

where $\zeta_{nm}=\frac{1}{\pi}\left(K-\frac{k}{f}|x_{nm}|\right)$.

Let us transform the expression (11) to the form

$$i_{c\Phi}=B\cos\left[(\Omega+\omega_D)t-\Phi\right]. \tag{12}$$

Here

$$B=\frac{2d}{\lambda^2 f^2}\left\{\sum_{\substack{n,m,p,q\\ n>m,\,p>q}}^{N} A_n A_m A_p A_q Q_{nm} Q_{pq}\operatorname{sinc}\zeta_{nm}\operatorname{sinc}\zeta_{pq}\cos(\Phi_{nm}-\Phi_{pq})\right\}^{1/2},$$

$$Q_{nm}=\{\operatorname{ch}^2\beta_{nm}\cos^2(kb_{nm}d^2)+\operatorname{sh}^2\beta_{nm}\sin^2(kb_{nm}d^2)\}^{1/2},$$

$$\Phi_{nm}=\frac{1}{2}\omega_D(t_n+t_m)+\arctan\left[\operatorname{th}\beta_{nm}\tan(kb_{nm}d^2)\right],$$

$$\Phi=\arctan\left\{\sum_{\substack{n,m\\ n>m}}^{N} A_n A_m Q_{nm}\operatorname{sinc}\zeta_{nm}\sin\Phi_{nm}\Big/\sum_{\substack{n,m\\ n>m}}^{N} A_n A_m Q_{nm}\operatorname{sinc}\zeta_{nm}\cos\Phi_{nm}\right\}.$$

In practice the signal (12) can be made narrow-band by the choice of the carrier frequency Ω. From here the signal frequency is defined as a sum

$$\omega_c=\Omega+\omega_D+\tilde{\omega}_c \tag{13}$$

where the noise component $\dot{\omega}_c$ is a derivative of the phase. Performing differentiation of the phase Φ we get

$$\omega_c=\frac{4\tilde{\omega}_D^2}{\pi^2 M^2} \times \frac{\sum_{\substack{n,m,p,q\\ n>m,\,p>q\\ n>p,\,m>q}}^{N}(t_{np}+t_{mq})A_n A_m A_p A_q Q_{nm} Q_{pq}\operatorname{sinc}\zeta_{nm}\operatorname{sinc}\zeta_{pq}\sin(\Phi_{nm}-\Phi_{pq})}{\sum_{\substack{n,m,p,q\\ n>m,\,p>q}}^{N} A_n A_m A_p A_q Q_{nm} Q_{pq}\operatorname{sinc}\zeta_{nm}\operatorname{sinc}\zeta_{pq}\cos(\Phi_{nm}-\Phi_{pq})}. \tag{14}$$

For $z \to 0$ $Q \to 1$ and the expression for ω_c is reduced to the previously considered case of the scattering particle motion in one plane [4].

Let us represent the particle spacing as a sum of the mean value $\langle x_{nm} \rangle$ and fluctuation δx_{nm}

$$x_{nm} = \langle x_{nm} \rangle + \delta x_{nm}.$$

We choose a spatial period of the selecting filter from the condition

$$K - \frac{k}{f} \langle x_{nm} \rangle = 0.$$

Then in (11) $I_{nm} = kd\delta x_{nm}/f$ and, therefore, selection steepness is defined by the steepness of the main lobe of the function sinc $(kd\delta x_{nm}/\pi f)$. When

$$-\frac{d}{f} z_{nm} < x_{nm} < \frac{d}{f} z_{nm}$$

an optical signal spatial period in the Fourier-plane exceeds the $2d$-aperture and selection of a coherent component by means of a period filter appears to be nonefficient.

A noise frequency component (14) of the coherent signal of LDA arises, due to superposition of signals, from different pairs of particles existing simultaneously at the measuring volume. The nature of a phase noise of a coherent signal is defined by a random particle distribution over the space and dimensions which, as in the incoherent case limits the accuracy of measurement in media with a high concentration of scattering particles. The measurement error introduced by the phase noise is inversely proportional to the square of the interference fringe number in the probe field with the accuracy to the constant coefficient determined by the statistics of scattering particles. A coherent phase noise vanishes when no more than two particles are present simultaneously at the probe volume. This follows directly from expression (14) for $N = 2$. From here it follows that in media with the particle concentration corresponding to one pair coordinated with a filter per the measuring volume, velocity measurement by the coherent signal frequency is preferable because of the accuracy improvement due to phase noise suppression. For higher concentrations the phase noise decrease is achieved by the choice of the probe optical field of such structure that the average distance between the scattering particles be equal to an integral number of interference fringes. Since ω_c and $\tilde{\omega}_{ic}$ [4] are determined by means of weakly dependent combinations of parameters, they may be considered statistically independent in a sufficient practical approximation. This prooperty seems useful for statistical flow measurements.

The filter with a transmission function $\cos Kx_f$ may be realized, for example, in the optical scheme of LDA shown in Fig. 2 [5].

The setup comprises the bichromatic laser 1, the probe interference field shaper consisting of the successively placed matching objective 2, dispersion prism 3, objective 4, single-side band acousto-optical modulator-splitter 5, dispersion phase plate 6 placed in the way of one of the splittered beams and achromatic objective

7, achromatic spacer of the spatial Fourier spectrum 8 in the Fourier-plane of which a spatial-frequency periodic filter 9 is mounted, and the photodetector 10. An electric signal frequency meter 11 is connected to the photodetector. The

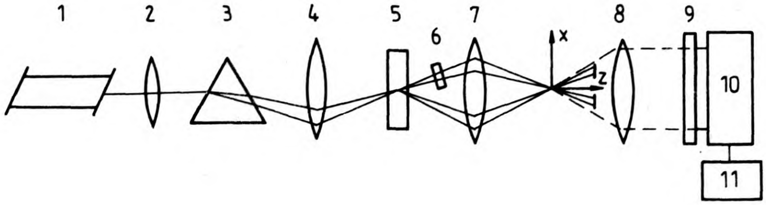

Fig. 2. LDA scheme with selection of coherent component optical signal

dispersive prism, objective, single-side band acousto-optical modulator and phase plate form together the laser beam splitter.

The spatial-frequency periodic filter is made in the form of a transparent plate with a dielectric multilayer coating periodically distributed in its plane. The coating is selective with respect to the wavelength of the incident radiation. For even half-periods it is made transparent only for the wavelength λ_1, but for odd ones – only for radiation with the wavelength λ_2 (e.g. blue and green emission lines of the argon laser, $\lambda_1 = 0.48$ µm and $\lambda_2 = 0.51$ µm, respectively).

A dispersion prism is used for correlation of laser lines with Bragg angles in the acousto-optical cell. The dispersive phase half-wave plate is mounted in the way of the splittered beams to provide the 180° phase shift between spatial-frequency structures of the formed interference probe fields with the same size of the fringes Λ.

In the setup the bichromatic laser, matching objectives, acousto-optical modulator-splitter and achromatic objective form in the investigated medium the superposition of the two interference probe fields differing in the laser spectrum.

Since the probe fields formed by different laser lines possess the antiphase spatial-frequency structure, coherent optical signals obtained under the scattering of these fields by moving particles are in the antiphase.

Hence we obtain that an alternative sequence of time phase 0 and π corresponds unambiguously to the sequence of spatial half-periods of the selective filter. The transfer function $\cos K x_f$ of the filter is thereby physically realized.

An incoherent optical signal integrated by the filter aperture practically vanishes since it averages over a large number of antiphase signals distributed over the aperture.

Functional possibilities of the LDA with a coherent optical signal selection can be extended to the measurement of linear sizes of scattering particles [6]. In the case of the tolerance control of linear sizes the spatial frequency of the selective filter correlates with the standard spacing between the front and back boundaries (edges) of the measuring object, while the filter steepness defined by its aperture correlates with this spacing tolerance. When the size of the sounding interference field along the x axis is known and defined precisely, the object size can be found by the formula

$$b = 2l - v\tau$$

where $2l$ is the interference field size along the x axis, v is the velocity calculated by the Doppler frequency of the coherent signal, τ – the time interval in which the front and back edges of the measuring object occur simultaneously in the

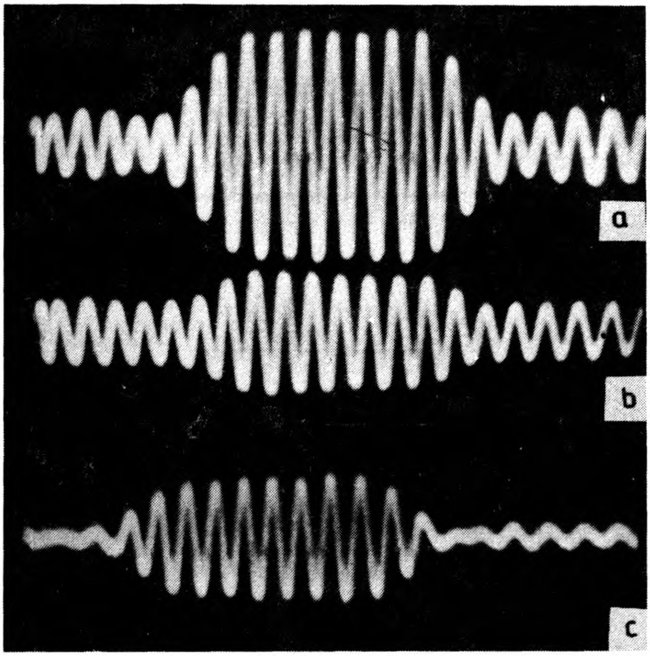

Fig. 3. Signal oscillograms: **a** – signal as a sum of coherent and incoherent signals, **b** – incoherent signal, **c** – coherent signal

limits of the sounding interference field (τ is a coherent signal duration), $v\tau$ – number of the coherent signal periods in the time interval τ. Signal oscillograms for the following parameters: $b = 150$ μm, $\Lambda = 16.6$ μm, $M = 20$ and $N = 2$, are given in Fig. 3.

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Лазерная доплеровская анемометрия с селекцией когерентного компонента оптического сигнала

Пространственно-временная структура оптического сигнала лазерного доплеровского анемометра (ЛДА) исследуется как суперпозиция когерентного и некогерентного компонентов с учётом движения рассеивающих частиц в измерительном объёме. Приводится бихроматическая схема ЛДА, в которой когерентный компонент подавляется. Обсуждается возможность применения лазерной доплеровской анемометрии с селекцией когерентной составляющей оптического сигнала для определения линейных размеров микрообъектов. Приводятся осцилограммы сигналов.