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Optical transfer function in commutating and compensating cameras for high speed cinematography

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The calculations of the optical transfer function for two types of cameras for high speed cinematography: of the image commutation type and that of optical compensation of the film band motion by employing the prism are presented. The facts that the exit pupil changes its sizes during the exposure and that as the residual motion (so-called "kinematic aberration") of the image with respect to the light sensitive recording material occurs are encountered. The calculations have been performed by using the method of the double Fourier transform.

1. Introduction

Nowadays, the intensity distribution function in the point image, i.e., the point spread function and the optical transfer function (OTF) belong to most important measures of the imaging quality of optical systems. The numerical quality measures for concrete cases are based on these two general concepts [1]. Also the apodization problems play an essential role in the examinations of quality and its improvement possibilities in particular cases [1]-[3].

The systems in which the transmittivity of the pupil is a function of time, e.g., such ones in which the sizes of the pupil are subject to change during the image recording, may be considered as some special and singualar cases of apodization. The optical systems of commutating and compensating cameras for high speed cinematography may, in particular, be regarded as systems with time-apodization of the pupil [4], [5]. The pupil of rectangular shape (in most cases) is then first enlarged from zero to certain maximum value, then narrowed back to zero, either linearly as it is the case for commutating system, or harmonically as it is the case for compensating systems when the compensation is realized with the help of a rotating multi-sized prism [5]-[7].

In these cameras, apart from the said kinds of apodization there occurs also a residual movement of the image with respect to the recording medium, i.e., the so-called kinematic aberration [4], [5].

As it is well known, the OTF when starting with the pupil function of the system, may be determined in two ways: directly as the autocorrelation function of the latter and indirectly by determining first the image intensity distribution at the respective image point by using the Fourier transform technique, and next the transfer function by applying again the Fourier transform to this intensity distribution at the point image [8], [9].

Obviously, the OTF becomes time-dependent for the case when the pupil is time-apodized in one dimension only (for the direction in which the change of the rectangular pupil occurs).

As to the image to be recorded in the photographic emulsion, for example, not so much essential is the instantaneous intensity distribution depending on the actual pupil sizes as its integral over the whole exposure time which determines the irradiation, i.e., the total energy distribution established in the image during the recording.

In the present paper, the method of double Fourier transformation has been accepted, since in this way the average transfer function is obtained from the irradiated distribution in the image point. Moreover, due to the fact that the pupil is rectangular and changes only in one direction identical with that of the image motion, the separation of variables is possible, and the calculation of the OTF for this direction significantly simplified.

For the sake of simplification, it has been also assumed that the optical systems of cameras are free of aberrations.

2. Basic transformations

The irradiation distribution for the image is obtained, as mentioned above, by integrating the intensity distribution along the whole exposure time

$$H'_{\delta,j}(\bar{A}) = \int_{-1}^{1} I'_{\delta,j}(\bar{A}, t)\,dt \tag{1}$$

while the OTF, determined by Fourier-transform of the irradiation distribution, is

$$d(\bar{\alpha}) = \frac{\lambda^2}{8\pi^3} FT^{-\bar{A}\bar{\alpha}}[H_{\delta,j}(\bar{A})] \tag{2}$$

where: t – time normed to half of the total recording time of exposure time $2T$, $t = t'/T$,

$\bar{A}(A_x, A_y)$ – parametrized vector of the point position

$$\bar{A} = \frac{k\bar{a}}{z_0} = \frac{k\bar{a}'}{z_0+z}, \tag{3}$$

$k = 2\pi/\lambda$, λ – light wavelength,

z_0 – distance of the Gaussian plane from the exit pupil,

z – defocusing value,

$\bar{a}(a_x, a_y)$ – point position vector in the distorted object plane [10], i.e., in the imaging plane by perfect optical system (both aberration-and diffraction-free system) for the transversal magnification normed to unity.

$\bar{a}'(a'_x, a'_y)$ – point position vector in the defocused image plane,

$\bar{\alpha}(\alpha_x, \alpha_y)$ – parametrize circular frequency in object and image structures in arbitrary direction defined by the components α_x and α_y.

As it is well known, the intensity distribution is given by the formula

$$I'_{\delta,j}(\bar{A}) = V'_{\delta,j}(\bar{A})\, V'^{*}_{\delta,j}(\bar{A}) \tag{4}$$

where $V'_{\delta,j}(\bar{A})$ is the complex amplitude distribution. The index δ in these and other formulas is used to demonstrate that the point object is considered, index j – that the amplitude and intensity of the wave incident on the input pupil is normed to unity, and index $'$ – that the respective magnitudes concern the image, while * denotes the complex conjugate.

In the case when the point object (and its image by the same means) is located not on the system axis but at the point determined by the vector $\bar{A}_p$, then, instead of $\bar{A}$ in (4), the following formula should be assumed:

$$\bar{C} = \bar{A} - \bar{A}_p = \frac{k}{z_0}(\bar{a} - \bar{a}_p) = \frac{k}{z_0 + z}(\bar{a}' - \bar{a}'_p) \tag{5}$$

where $\bar{a}_p(a_{px}, a_{py})$ and $\bar{a}'_p(a'_{px}, a'_{py})$ denote the real positions of the object and the image, i.e.,

$$I'_{\delta,j}(\bar{A}) = V'_{\delta,j}(\bar{C})\, V'^{*}_{\delta,j}(\bar{C}). \tag{4a}$$

The amplitude and phase distributions are defined by the Fourier transforms of the pupil function

$$V'_{\delta,j}(\bar{C}) = \frac{1}{\lambda(z_0+z)} FT^{-\bar{C}\bar{\varrho}}[V_z(\bar{\varrho})] = \frac{1}{\lambda(z_0+z)} \iint_{-\infty}^{\infty} V_z(\bar{\varrho}) \exp(-i\bar{C}\bar{\varrho})\, d\bar{\varrho}, \tag{6}$$

while the pupil function is of the form

$$V_z(\bar{\varrho}) = T(\bar{\varrho}) \exp(ikp\varrho^2) \tag{7}$$

where: $\bar{\varrho}(\varrho_x, \varrho_y)$ is a vector defining the position of the point in the exit pupil, p – defocusing coefficient

$$p = \frac{1}{2}\left(\frac{1}{z_0+z} - \frac{1}{z_0}\right) \tag{8}$$

(z – defocusing value), $T(\bar{\varrho})$ is the pupil transmittance.

The pupil transmittance for the points $\bar{\varrho}$ located inside the pupil is equal to unity and equal to zero for the outside points

$$T(\varrho_x, \varrho_y) = \begin{cases} 1 & \text{for } |\varrho_x| \leqslant b\varrho_{0x}, \quad |\varrho_y| \leqslant \varrho_{0y} \\ 0 & \text{for } |\varrho_x| > b\varrho_{0x}, \quad |\varrho_y| > \varrho_{0y} \end{cases} \tag{9}$$

where $b = b(t)$ is the function defining the motion of the pupil edges ($b \leqslant 1$; this direction has been defined as the x axis), $2\varrho_{0x}$, $2\varrho_{0y}$ — the greatest width and height of the pupil, respectively.

By substituting (7)–(9) into (6) the integration may be reduced to the region within which the pupil transmittance is different from zero. Then we obtain

$$V'_{\delta,j}(\bar{C}) = \frac{1}{\lambda(z_0+z)} \int_{-b\varrho_{0x}}^{b\varrho_{0x}} \int_{-\varrho_{0y}}^{\varrho_{0y}} \exp[ikp(\varrho_x^2+\varrho_y^2)] \exp[-i(C_x\varrho_x + C_y\varrho_y)]\, d\varrho_x d\varrho_y$$
$$= \frac{1}{\lambda(z_0+z)} \int_{-b\varrho_{0x}}^{b\varrho_{0x}} \exp[i(kp\varrho_x^2 - C_x\varrho_x)]\, d\varrho_x \int_{-\varrho_{0y}}^{\varrho_{0y}} \exp[i(kp\varrho_y^2 - C_y\varrho_y)]\, d\varrho_y. \quad (10)$$

By introducing the normed variables in the pupil plane

$$r_x = \varrho_x/b\varrho_{0x}, \quad r_y = \varrho_y/\varrho_{0y} \quad (11)$$

which results in a respective change of the integration limits to -1 and $+1$, and denoting

$$G_{0x} = kpb^2\varrho_{0x}^2, \qquad G_{0y} = kp\varrho_{0y}^2,$$
$$H_x = b\varrho_{0x}C_x = b\varrho_{0x}(A_x - A_{px}), \qquad H_y = \varrho_{0y}C_y = \varrho_{0y}(A_y - A_{py}) \quad (12)$$

the expression (10) may be transformed to the forms:

$$V'_{\delta,j} = \frac{b\varrho_{0x}\varrho_{0y}}{\lambda(z_0+z)} \int_{-1}^{1} \exp[i(G_{0x}r_x^2 - H_xr_x)]\, dr_x \int_{-1}^{1} \exp[i(G_{0y}r_y^2 - H_yr_y)]\, dr_y, \quad (13)$$

$$V'_{\delta,j} = \frac{b\varrho_{0x}\varrho_{0y}}{\lambda(z_0+z)} D_x D_y \quad (13a)$$

where

$$D_u = \int_{-1}^{1} \exp[i(G_{0u}r_u^2 - H_ur_u)]\, dr_u, \qquad u = x, y. \quad (14)$$

After some further transformations taking account of the Euler formulae we obtain

$$D_u = 2(P_u + iQ_u) \quad (15)$$

where

$$P_u = \int_0^1 \cos(H_ur_u)\cos(G_{0u}r_u^2)\, dr_u,$$
$$Q_u = \int_0^1 \cos(H_ur_u)\sin(G_{0u}r_u^2)\, dr_u, \qquad u = x, y. \quad (16)$$

So

$$V'_{\delta,j} = \frac{4\varrho_{0x}\varrho_{0y}}{\lambda} \frac{b}{z_0+z}(P_x+iQ_x)(P_y+iQ_y),$$
$$I'_{\delta,j} = C_1C_2(P_x^2+Q_x^2)(P_y^2+Q_y^2), \quad (17)$$
$$H'_{\delta,j} = C_1 \int_{-1}^{1} C_2(P_x^2+Q_x^2)(P_y^2+Q_y^2)\, dt$$

where

$$P_x = \int_0^1 \cos(H_x r_x)\cos(G_{0x} r_x^2)\,dr_x, \qquad Q_x = \int_0^1 \cos(H_x r_x)\sin(G_{0x} r_x^2)\,dr_x,$$
$$P_y = \int_0^1 \cos(H_y r_y)\cos(G_{0y} r_y^2)\,dr_y, \qquad Q_y = \int_0^1 \cos(H_y r_y)\sin(G_{0y} r_y^2)\,dr_y, \tag{18}$$

$$C_1 = \left(\frac{4\varrho_{0x}\varrho_{0y}}{\lambda}\right)^2, \qquad C_2 = \left(\frac{b}{z_0+z}\right)^2. \tag{19}$$

Since the rectangular pupil changes only in one direction (x axis) along which there occurs also the residual image motion with respect to the recording medium, the image quality is here the lowest and hence, it is reasonable to calculate the transfer function for this direction (i.e., for different values of circular frequencies α_x for $\alpha_y = 0$).

The transfer function in the x axis direction is normed to the zero frequency in this direction, thus according to (2) it has the form

$$d_n(\alpha_x, 0) = \frac{d(\alpha_x, 0)}{d(0, 0)} = \frac{\int\int H'_{\delta,j}(A_x, A_y)\cos(\alpha_x A_x)\,dA_x\,dA_y}{\int\int H'_{\delta,j}(A_x, A_y)\,dA_x\,dA_y}, \tag{20}$$

or by introducing the radiation distribution in the image of a line parallel to y axis

$$H'_{l,j}(A_x) = \int H'_{\delta,j}(A_x, A_y)\,dA_y \tag{21}$$

takes the form

$$d_n(\alpha_x, 0) = \frac{\int H'_{l,j}(A_x)\cos(\alpha_x A_x)\,dA_x}{\int H'_{l,j}(A_x)\,dA_x}. \tag{22}$$

3. Results of numerical calculations and conclusions

The calculations of the transfer function have been performed for both the optical systems of commutating camera W-1 and the compensating camera of the prism compensation described in paper [4]. In the program for computer, it has been assumed that the calculations of the integrals are performed by Simpson method and the integration limits in Eqs. (21) and (22) contain about 8 points, where the $\mathrm{sinc}^2 A_x$ function (representing the intensity distribution in the image of diffraction point) becomes zero for the moment ($t = 0$) in which the width of the pupil is maximal and equal to $2\varrho_{0x}$. (For the narrower integration ranges, especially distinctly for the lowest frequencies, the influence of the irradiation function [11] truncation on the course of the transfer function was visible).

In the commutating camera the function of the pupil edge motion has the form [7]

$$b = \begin{cases} \dfrac{1-|t|}{1-N}, & 1 \geqslant |t| > N, \quad N \neq 1 \\ 1 & |t| \leqslant N. \end{cases} \tag{23}$$

Figure 1 shows the graphs of this function for three values of parameter: $N = 1$, 0.5 and 0. $N = 1$ means that the pupil is opened and closed with infinite speed and stays totally open during the whole time $2T$, ($t = 2$). The value of $N = 0$ corresponds to the pupil which starts to close immediately reaching the greatest

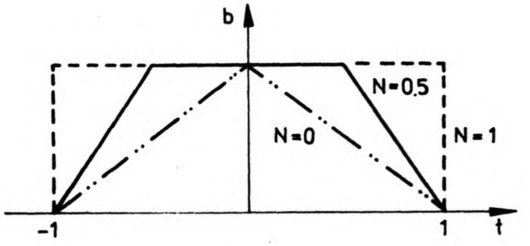

Fig. 1. Pupil motion function

width $2\varrho_{0x}$. The kinematic aberration in this camera means that during the work of the pupil there occurs a time-dependent defocusing (z) together with the change of the parametrized coordinate

$$A_{px} = \frac{kd_1}{z_0} t, \quad z = d_2 t, \quad (A_{py} = 0) \tag{24}$$

while the values $d_1 = 0.0091$ and $d_2 = 0.0882$ correspond to the kinematic aberration $A_k = 0.0188$. The remaining parameters for this camera are: $z_0 = 1000$, $\varrho_{0x} = 3.81$, $\varrho_{0y} = 15.24$, $\lambda = 0.5 \times 10^{-3}$.

Figure 2 shows the graphs of the normed transfer function vs the circular frequencies normed to the limiting frequencies $\alpha_{\lim} = 2\varrho_{0x}$, $\alpha_n = \alpha_x/\alpha_{\lim}$. The curves **a**, **b** and **c** correspond to the above kinematic aberration and to the respective values 1, 0.5, 0 of the parameter N. Curves **d** and **e** drawn for the sake of

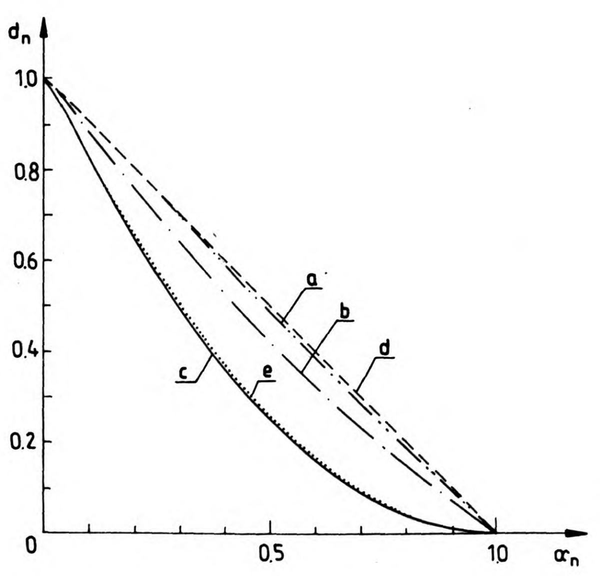

Fig. 2. OTF in the commutating camera: **a**, **b**, **c** – for the pupil motion function parameters $N = 1$, 0.5 and 0

comparison correspond to the motionless well-focused image ($d_1 = d_2 = 0$, $A_k = 0$) and to $N = 1$ and $N = 0$, respectively. The spatial frequency ($\tilde{a}' = 15.2\ \mathrm{mm}^{-1}$) corresponds to the limitting circular frequency ($\alpha_n = 1$, $\alpha_{\lim} = 7.62$).

In the compensating prism camera the pupil motion function shows the harmonic dependence on time [7]

$$b = \cos\left(\frac{\pi}{2}t\right) \tag{25}$$

due to steady rotational motion of the compensator.

In the most common case of the image recorded on a film placed directly on the driving drum ([4], p. 89–96) the following substitutions in Eqs. (8), (12) and (19) have to be done:

$$z_0 = z_{0,0} - x_1,$$

$$z_0 + z = z_{0,0} + x_2,$$

$$x_1 = d\left[1 - \cos(d_2 t) - n - \sqrt{n^2 - \sin^2(d_2 t)}\right],$$

$$x_2 = r_F\left[1 - \cos(d_3 t)\right],$$

$$A_{px} = \frac{kd_1 t}{z_{0,0} - x_1}.$$

For the camera, described in paper [4] p. 95, of the same kinematic aberration as the one mentioned above the constant values are amounting to: $z_{0,0} = 50$, $d = 45$, $n = 1.7581$, $r_F = 25$, $d_1 = 0.0091$, $d_2 = 0.0502$, $d_3 = 0.0195$, $\varrho_{0x} = 4.5$, $\varrho_{0y} = 4.5$.

Figure 3 presents the graphs of the transfer function for this camera. The curve **a** refers to the pupil opened and shut with infinite speed (as it is the case of the commutating camera for $N = 1$), while curve **b** corresponds to pupil working according to (25). The curves **c** and **d** correspond to the motionless focused image ($d_1 = d_2 = d_3 = 0$, $A_k = 0$) and to the pupil of the infinite opening and closing speeds – **c** or to that working according to (25) – **d**.

When analysing the curves in Figs. 2 and 3, it may be stated that in the case of both the cameras the values of frequencies of the transfer functions are lowered due to kinematic aberration (and defocusing). While in the commutating camera practically all the frequencies are transferred up to the limiting frequency in the compensating camera, a rapid decrease of the transferred frequencies down to zero takes place for the frequencies much less than the limiting frequency. There appears also a contrast reversal (several times in the case **a**). It should be noticed, however, that the highest spatial frequency transferred without the contrast reversal is much greater in the compensating camera than that in commutating one ($43.25\ \mathrm{mm}^{-1}$ and $15.2\ \mathrm{mm}^{-1}$).

This fact, as well as the differences in the courses of curves in Figs. 2 and 3, may be explained when noting that the aperture of the commutating camera is much less than the one of the compensating camera, being so small that both the courses of the curves of the irradiation distribution in the point image [7] and that of the curves of the transfer function are predominantly influenced by the diffraction of light. The influence of the kinematic aberration is almost negligible.

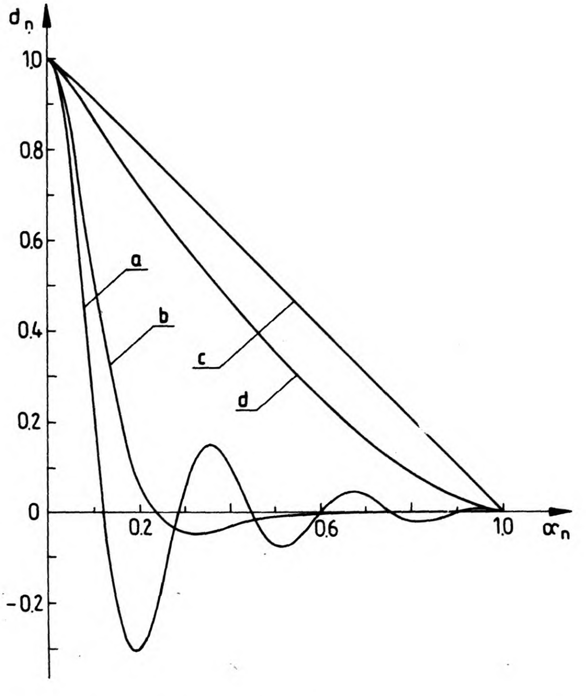

Fig. 3. OTF in the compensating camera: **a** – for rectangular pupil function ($N = 1$), **b** – for a cosinusoidal pupil motion function, **c** and **d** as well **a** and **b** without kinematic aberration

This is also confirmed by the course of the curves **d** and **e** in Fig. 2 (for the motionless focused image, i.e., when the kinematic aberration is absent, $A_k = 0$) being very close to the corresponding curves **a** and **c** (taking account of the motion and image defocusing). An inverse situation occurs in the compensating camera. Due to high apertures appearing in cameras of this kind the influence of diffraction is negligible. Then, the only factor practically limiting the frequency transfer is the kinematic aberration. The above conclusions are confirmed by the courses of the curves of the irradiation distribution in the point image in both the cameras as well as by the analysis of the resolving power given in [7].

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Оптическая функция переноса в коммутационных и компенсационных камерах для скоростной кинематографии

Проводятся расчеты оптической функции переноса в коммутационных и компенсационных камерах для скоростной кинематографии. Учитываются изменения размеров выходного зрачка во время экспозиции, а также остаточный сдвиг изображения относительно пленки („кинематографическая аберрация”). Расчеты выполняются с применением двухкратного преобразования Фурье.