Optica Applicata, Vol. XVII, No. 3, 1987

On a modification of the Fialovszky method for the case of nonsymmetric tolerances

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There has been presented in the paper the problem of determining the manufacturing tolerances for the optical elements in the case of nonsymmetric tolerancing.

1. Introduction

The fundamental objection to the FIALOVSZKY method [1], [2] is that its model does not include the reality of the technological process, i.e., the real distributions occurring in the workshop practice. Each technological process is organized so that the worker can "safely" approach the nominal dimension. A typical example is a principle of maximum of the material and connected with it nonsymmetrical tolerancing, i.e., in depth of the material, e.g., 20.1–0.2 (instead of symmetric one: 20 ± 0.1). This leads to the probability density distributions in which the most probable dimensions occur in the vicinity of the upper deviation. Such distributions are usually described by a rectangular triangle in the first approximation. In the case of optical design parameters (radius of curvature, spacing, refractive index) the spacing tolerance (thickness) is typical, Fig. 1**b**. A special commentary is required for the case of curvature tolerancing of an optical system for which the verification of the surface dimension with the help of interference gauges requires one-sided deviations (two-point contact of a gauge with the optical elements, Fig. 2), although theoretically the deviations in plus and minus with respect to the nominal dimension are acceptable. This leads to the distributions indicated in Fig. 1**a**. The only distributions close to the normal ones are those of the refractive index. However, even here the slight shifts of the expected index value with respect to the one taken from the catalog are possible, which follows from the inaccurate consideration of the due correction during the glass stabilization process (see Table and Fig. 1**c**). The statistical studies carried out in the optical industry [3] confirm the above considerations. Therefore, there arises the necessity of realistic modifications of the Fialovszky method since a credulus application of its original form to determine the performance tolerances for optical elements may lead to an uncontrolled percentage of waster optical systems due to violating the admissible values of aberrations.

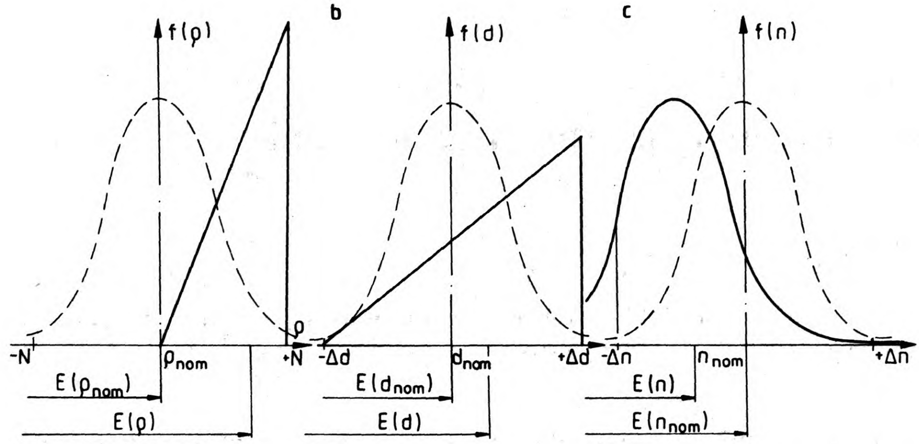

Fig. 1. Probability density distributions for design parameters of optical system: **a** – surface curvature, **b** – spacings (thickness), **c** – refractive index. The continuous line denotes the distributions appearing in the workshop practice while the broken line represents the theoretical distributions according to the Gauss curve. $E(\varrho_{\mathrm{nom}})$, $E(d_{\mathrm{nom}})$ and $E(n_{\mathrm{nom}})$ denotes the respective average values of curvatures, spacings, refractive indices for master case of Gaussian distribution, while $E(\varrho)$, $E(d)$ and $E(n)$ are the expected values (averages) for these parameters for the real systems

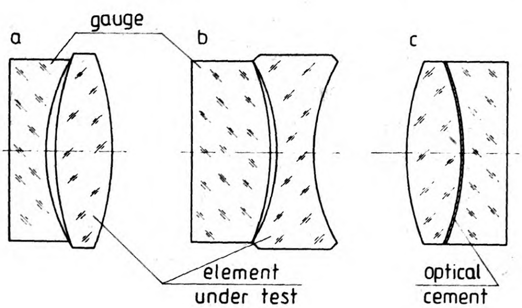

Fig. 2. Two-point contact of the optical surface for interference control (**a** – convex surface, **b** – concave surface), and for the cemented elements in an assembly (**c**)

Comparison of the typical shifts of the refractive index elaborated by the Hoya firm (for the stress relieving rate 1°/h) with statistical approximation of this value

Sort of glass	BK7	BaK4	SK10	SK4	F2	SF2
Statistical increment: $n \times 10^{-5}$	+70	+91	+104	+137	+41	+45
Increment acc. to Hoya firm: $n \times 10^{-5}$	+90	+80	+110	+110	+50	+60

2. Description of the method

In the new method a realistic probability model for the random variables occurring in the problem should be regarded, while the technical control procedure used by technologists and workers in the workshop should be retained not to perturbe the actual state of the art in this respect. Therefore, the denotations of the deviation on the technological documentation of optical systems as well as the controlling procedure should remain unchanged. The basis of the modified method is the statement proven in paper [4] that the distributions of aberrations of the optical system being a linear combination of partial aberrations are normal and of definite parameters. From this elementary properties of the independent random variables it follows that:

$$E(\Sigma x_i) = \Sigma E(x_i), \qquad \sigma^2(\Sigma x_i) = \Sigma \sigma^2(x_i).$$

Hence, the parameters of the normal distribution for the aberrations are defined univocally by the distributions of the random variables attributed to the design parameters of the system. In the normal distribution there exists a simple dependence between the tolerance T and the standard deviation σ: $T = 2t\sigma$, where t is a standardized variable of the normal distribution. In practice, the six-standard field of tolerance is widely applied (i.e., for $t = 3$), for which the probability of leaving the field of tolerance equals: $P(|t| > 3) = 0.0027$. In the Fialovszky method the same feature was attributed to the normal distribution of the performance deviations and the problem of tolerance determination was relatively simple. In the case of other types of distributions such a simple way is no more possible. For these types of distributions and especially for nonsymmetric ones the field of tolerances must be determined in another way. The following approach to such problems is practiced in the engineering works: the so-called master case is the starting point, in which both the components of the sum and the sum itself are of normal distributions. This theoretically most advantageous case has been considered in Fialovszky's work.

Now, if in the arbitrary case of summing up of the aberration sum components of different distributions the resultant distribution is also normal (central theorem of probability), then, it may be assumed that this is such a case as if the components of the sum were of normal distributions but of other distribution parameters. Thus, each distribution may be characterized by a coefficient λ, which says how much the field of tolerance for the given type of distribution differs from the field of tolerance for the corresponding component in the master case. Obviously, $\lambda = 1$ by definition for components of normal distributions. Therefore, consider first the master case. Since the tolerance for the j-th component is

$$T_j = 6\sigma(x_j), \qquad \text{then } \sigma(Z_i) = \sqrt{\Sigma a_{ij}^2 \sigma^2(x_j)} = \frac{1}{6}\sqrt{\Sigma a_{ij}^2 T_j^2}$$

where a_{ij} – influence of the j-th parameter on the i-th aberration.

The upper limit of the aberration tolerance amounts to

$$Z_{i\max} = Z_{i\text{av}} + 3\sigma(Z_i) = Z_{i\text{av}} + \frac{1}{2}\sqrt{\Sigma a_{ij}^2 T_j^2},$$

while the lower limit is

$$Z_{i\min} = Z_{i\text{av}} - 3\sigma(Z_i) = Z_{i\text{av}} - \frac{1}{2}\sqrt{\Sigma a_{ij}^2 T_j^2}.$$

Hence, the field of tolerances for the aberration T_i amounts to

$$T_i = Z_{i\max} - Z_{i\min} = \sqrt{\Sigma a_{ij}^2 T_j^2}, \tag{1}$$

which is consistent with the principle of summation after taking account of $T_i = 6\sigma(Z_i)$ and $T_j = 6\sigma(x_j)$.

Now, consider the case of arbitrary distribution. As an example consider the uniform distribution of the components of the sum for which the density function is described by the relation $f(x_j) = 1/T_j$ and the variance $\sigma^2(x_j) = T_j^2/12$. Following the procedure applied for the master case the field of tolerances for the i-th aberration amounts to

$$T_i = Z_{i\max} - Z_{i\min} = \sqrt{3}\sqrt{\Sigma a_{ij}^2 T_j^2} = 1.73\sqrt{\Sigma a_{ij}^2 T_j^2}. \tag{2}$$

The comparison of the relations (1) and (2) indicates that they differ by the factor appearing in front of the square root only. This is the sought coefficient $\lambda = 1.73$ of the uniform distribution. The values of the coefficient for other distributions may be found in the papers [5], [6]. Thus, in general, it may be written that for the arbitrary distribution

$$T_i = \sqrt{\Sigma a_{ij}^2 \lambda_j^2 T_j^2}. \tag{3}$$

There are no intuitive objections to the symmetric distributions, i.e., for the distributions the expected value of which lies in the centre of tolerance field while the deviations are symmetrically distributed around the mean value. As to the case of asymmetric distributions exemplified by triangle distribution the application of Eq. (3) meets the difficulty in interpreting both the field of tolerances and its position with respect to the mean value.

In such a case the expected value of the parameter x_j may be presented in general as

$$E(x_j) = x_{j0} + \varepsilon_j T_j$$

where x_{j0} — fixed value of the parameter x_j corresponding to the centre of the interval T_j, ε_j — coefficient characterizing the shift of $E(x_j)$ from the value of x_{j0}. For the right angle triangle distribution $\varepsilon_j = 2/3 - 1/2 = 1/6$. The mean square deviation from the mean value is also the function of the tolerance field and amounts to

$$\sigma(x_j) = \lambda_j T_j$$

where λ_j is a coefficient indicating the relation between the standard deviation of the real distribution and the deviation in the master case. This is thus the same coefficient which appears in Eq. (3). Hence, in accordance with the elementary relation for independent random variables we have

$$E(Z_i) = \varphi(x_{j0} + \varepsilon_j T_j). \tag{4}$$

After having taken $T_j = 6\sigma(x_j)$ into account, the standard deviation becomes

$$\sigma(Z_i) = \frac{1}{6}\sqrt{\Sigma a_{ij}^2 \lambda_j^2 T_j^2},$$

and after further transformations, analogical to those used in the master case, we obtain

$$T_i = \sqrt{\Sigma a_{ij}^2 \lambda_j^2 T_j^2}. \tag{5}$$

This form is identical with that of (3), and hence it may be concluded that the nonsymmetry of the distribution has no influence on the value of the tolerance field (it continues to a_{ij} and λ_j only). This field, as it follows from (4), is positioned nonsymmetrically with relation to the centre of the interval, in a way characteristic to a given type of the distribution with respect to the expected value.

3. Concluding remarks

The determination of the working tolerances by using the modified Fialovszky method leads to a relatively small change consisting merely in additional consideration of the distribution coefficient λ_i only. However, a much greater problem appears due to the reference of these tolerances to the expected values of the design parameters and to the so-called probable aberrations connected with the latter. In the discussed case of nonsymmetric distributions the mean values lie beyond the centre of the tolerance field, (Eq. (4)), and this means that the most probable parameters of the optical system will differ from their nominal values. Consequently, the aberrations of the system will change too. These are the above mentioned probable aberrations. In order to illustrate this fact, the graph of the longitudinal spherical aberration for a typical optical system has been shown in Fig. 3. The broken line denotes the curve of this aberration for nominal values of the design parameters of the optical system, i.e., for the centre of tolerance field. The continuous line is used to mark the same aberration but obtained from the expected values (most probable) of the design parameters. To illustrate this fact better the statistical spread of the aberration values is also shown in this figure. As it can be seen in the figure, it is impossible to obtain a system with aberrations which would meet the nominal values of the statistical parameters (their values exceed the six-standard tolerance field, where the probability of occurrence amounts to $P < 0.0027$). In this specific case an advantageous change of aberrations occurred but in general case some worsening of the state of system correction took place. This altered state of system aberrations, caused exclusively by nonsymmetric distributions of design parameters, must be corrected so that the optical system could work properly. Therefore an additional calculational procedure (taking account of the additional correction of the aberrations) is necessary at the stage of tolerance determination. Its task is to change the nominal values of the design parameters in such a way that the probable aberrations could represent the

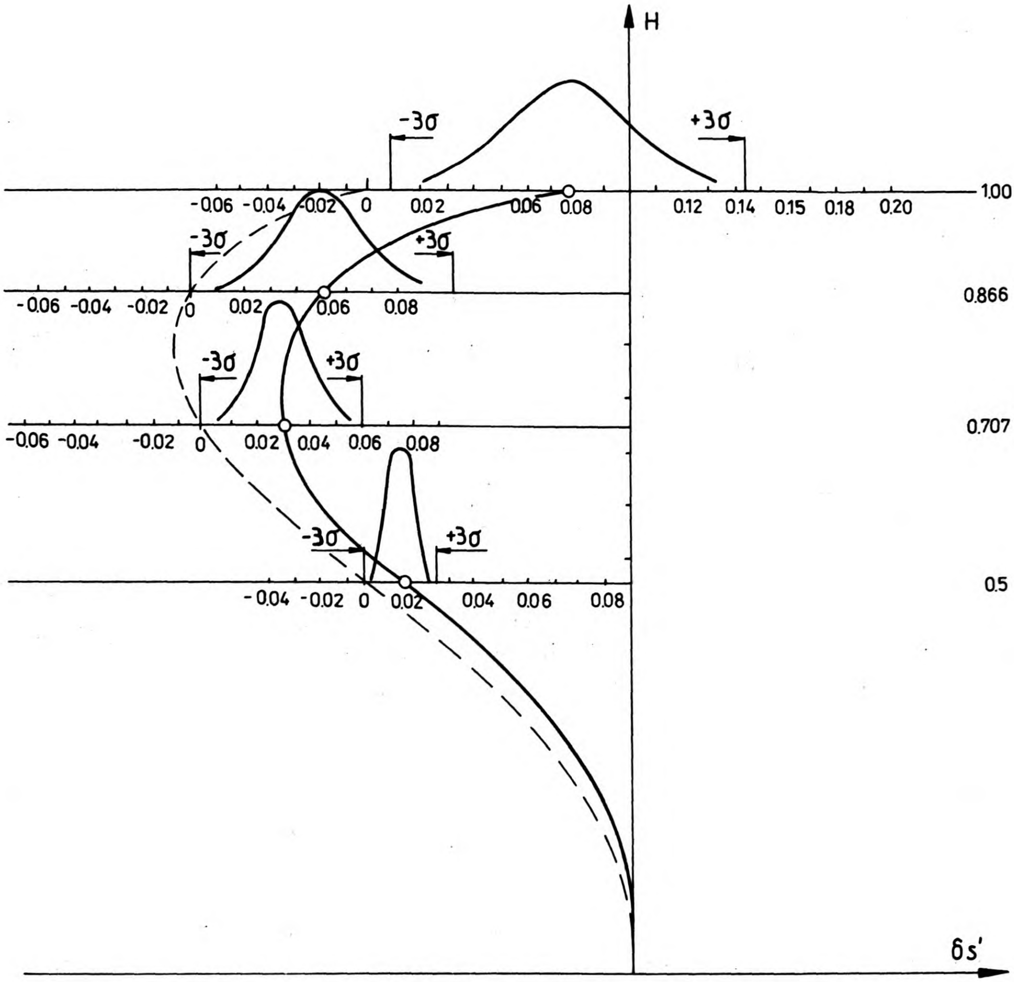

Fig. 3. Graph of the longitudinal spherical aberration for typical optical system. The continuous line marks the probable aberration together with its statistical spread for particular zones of the aperture, while the broken line denotes the same aberration determined on the basis of nominal values of the design parameters of the system (H – radius of the pupil, $\delta s'$ – longitudinal spherical aberration)

suitable state of the system correction. The fulfilment of this postulate is easy only with respect to the spacings (thickness) where any change of the nominal value is not problematic. However, for the curvatures there appear significant difficulties due to the necessity of using the radii recommended by the industrial standards [7] (for instance, Polish Standards) or the radii consistent with the actual set of used gauges.

There are practically two ways of solving this problem. The first one is to use the so-called deviation gauges suggested by the author in paper [8], where the change of the nominal radius is done by execution of the "deviation" replica of the working gauge, serving to control a given technological part only. This way requires formally no additional calculation stage because of application of the deviation gauges, and

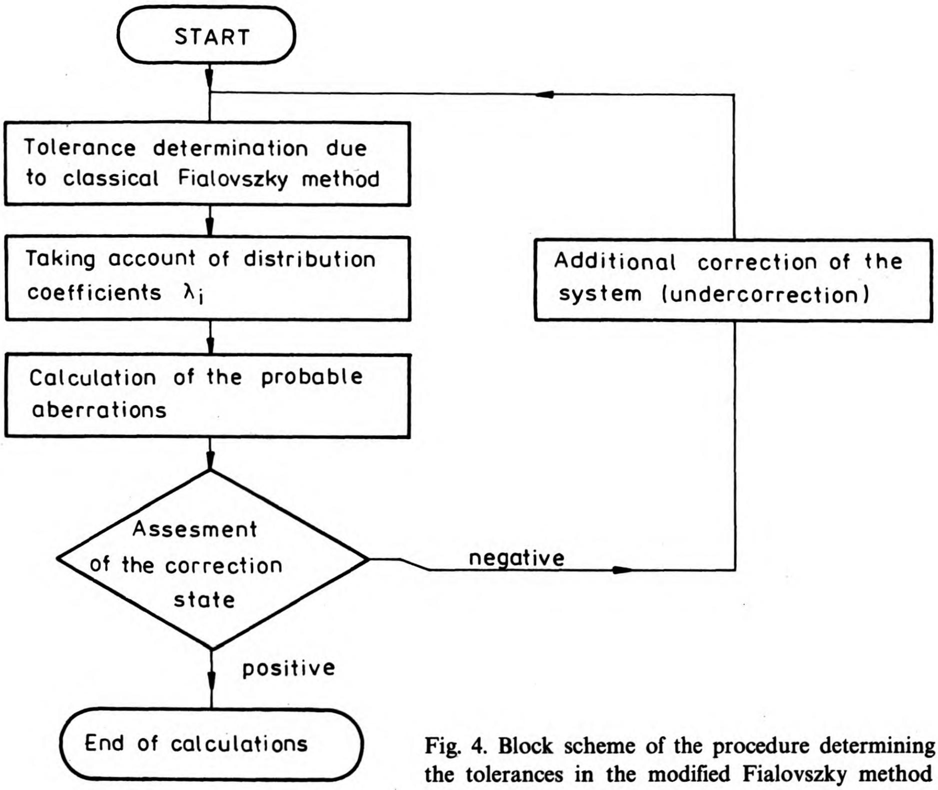

Fig. 4. Block scheme of the procedure determining the tolerances in the modified Fialovszky method

for this reason is recommendable. However, this introduces a kind of danger of the mistakable use of the gauges (a given deviation gauge may be applied to that technological part for which it was produced). The other way is to change the nominal values of the parameters of the optical system by its undercorrecting (see Fig. 3), in such a way that the probable aberrations become identical with those for the nominal variant, i.e., with the aberrations of the starting system for tolerance calculations. The block scheme of such a procedure is shown in Fig. 4.

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Received December 3, 1986
in revised form February 23, 1987

Модификация метода Фиялковского для несимметрических распределений допусков

В статье разработана проблема определения допусков на оптические детали с учётом несимметрических распределений.