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# Shearing interferometry approach for producing shear strain maps

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Principles of the shearing interferometry and spatial filtering method for generating whole field contour maps of shear strains $\gamma_{xy}$ are presented. Cross-derivatives of in-plane displacements are obtained by two beam lateral shear interferometry. The interferograms with high spatial frequency carrier fringes added are overlapped to form a cross type structure. Spatial filtering of its appropriate diffraction orders leads to the interference pattern with the information about $\gamma_{xy}$.

## 1. Introduction

Moiré fringe technique is a well established method in strain analysis [1]. Line deformations of the specimen grating fixed to the object under test correspond to the object in-plane displacements. Their subsequent differentiation or differentiation of moiré fringes provides the contours of normal strains $\varepsilon_x$ and $\varepsilon_y$. The list of several differentiation methods can be found, for example, in paper [2].

Recently, some optical or quasi-optical methods have been proposed to determine additional parameter characterizing the strain field, i.e., the shear strain $\gamma_{xy}$ [3]–[6]. They utilize a combination of well known subsidiary techniques: mechanical differentiation [7], coherent optical filtering and additive moiré [7]. In the methods reported the cross-derivatives $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$, where $u(x, y)$ and $v(x, y)$ represent the in-plane displacements in $x$ and $y$ directions, respectively, are generated by mechanical differentiation. Two identical copies of the deformed specimen grating are mutually shifted in the direction parallel to grating lines. This direction is perpendicular to the registered displacement field and must be very precisely determined. A slight departure results in the erroneous strain value.

An alternative approach to obtain the maps of the cross-derivatives $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$ is the shearing interferometry method. The direction of the derivative corresponds to the lateral shear direction. The latter one is defined by angular setting of the shearing optical element which can be done with high accuracy. The same remark concerns the amount of shear. A considerable advantage of the optical differentiation approach is the possibility of producing the maps of cross-derivatives $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$ in the form of deformed high frequency fringes. Therefore, new methods of generating the shear strain contours can be developed. The mechanical differentiation gives the derivative information under the uniform field display mode, i.e., without the carrier fringes.

The aim of this paper is to propose new optical methods for mapping the shear strain $\gamma_{xy}$. Their common feature is the application of lateral shear interferometry to the diffraction orders of a deformed specimen grating. Spatial filtering of properly overlapped cross-derivative interferograms in the coherent optical processor leads to final two beam interferograms giving the map of shear strain $\gamma_{xy}$.

## 2. Analysis

The discussion presented below is valid for the cross-type as well as linear amplitude gratings fixed to the specimen under study. As in the methods described by KATO et al. [3]–[5] we have to photograph the deformed specimen grating to have its copies for further interferometric investigations.

The next stage is to produce high density fringe interferograms carrying the information about $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$. For this purpose the most simple seem to be the solutions utilizing the low frequency linear diffraction grating or the Wollaston prism [8]. The configuration with former type beam-splitting and shearing element will be discussed in the following.

Figure 1 shows schematic representation of the optical system. The copy of the deformed specimen grating is inserted in the input plane of the coherent optical processor L1–L2. Plane wave front illumination is provided. If the illuminating beam

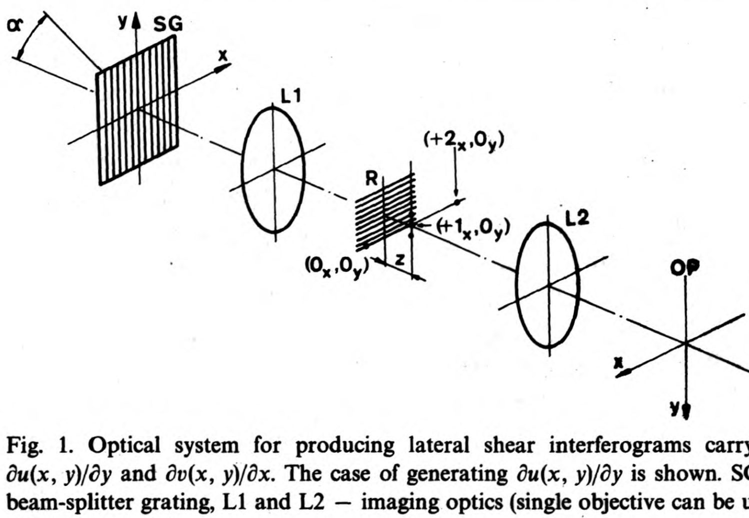


Fig. 1. Optical system for producing lateral shear interferograms carrying the information about $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$. The case of generating $\partial u(x, y)/\partial y$ is shown. SG – specimen grating, R – beam-splitter grating, L1 and L2 – imaging optics (single objective can be used for this purpose as well), OP – observation plane

impinges at an angle $\alpha$ equal to the first order diffraction angle of the specimen grating (incidence plane can be in the planes $xz$ or $yz$), then the appropriate first diffraction order propagates along the grating normal coinciding with the optical axis. When the specimen grating of spatial frequency 40 lines/mm and the transforming objective L1 of focal length 600 mm is used, the distance between the diffraction spots in the frequency plane equals 15.2 mm for $\lambda = 0.633$ μm.

Let us assume a linear specimen grating SG with the lines perpendicular to $x$ axis. Its amplitude transmittance can be represented by a Fourier series

$$T_{SG}(x, y) = \sum_n b_n \exp\left\{in\frac{2\pi}{d}[x+u(x, y)]\right\} \tag{1}$$

where $b_n$ is the amplitude coefficient, $d$ is the grating period, and $u(x, y)$ designates the in-plane displacement function, i.e., the local departures of grating lines from straightness. The complex amplitude of the first diffraction order propagating along the grating normal (Fig. 1) is $b_1 \exp\{i2\pi u(x, y)/d\}$. Now a linear diffraction grating R with lines parallel to $x$ axis is introduced near the back focal plane of L1 where the spatial frequency spectrum is displayed. It is more convenient to place R in front of this plane to have the possibility to filter out the proper diffraction beams of R. This is important from the point of view of the contrast of produced lateral shear interferogram. Spatial filtering is not possible for the beam-splitting grating R placed to the right of the focal plane.

When the filter transmits the beams $(+1_x, 0_y)$ and $(+1_x, +1_y)$, where the first and second number in the parenthesis designate the order number at SG and R, respectively, the amplitude in the observation plane OP can be described by

$$E(x, y) = b_1 a_0 \exp\left\{i\frac{2\pi}{d}u(x, y)\right\} + b_1 a_1 \exp\left\{i\left[-k\Theta_y y + \frac{2\pi}{d}u(x, y-\Delta y)\right]\right\} \tag{2}$$

where $a_0$ and $a_1$ denote the amplitude transmittance coefficients of the zero and first diffraction orders of R, respectively, $k = 2\pi/\lambda$, $\Theta_y$ is the incidence angle of the first order of R onto the observation plane OP and equal to

$$\Theta_y = \left(\frac{z}{f_{L2}}\right)\tan\left(\frac{\lambda}{d_R}\right) \tag{3}$$

where $z$ is the distance of the grating R from the focal plane, $f_{L2}$ is the focal length of L2, and $d_R$ is the spatial period of R. In Eq. (2) $\Delta y$ denotes the lateral displacement of the two interfering beams in the observation plane, i.e., the lateral shear value along the $y$ direction given by

$$\Delta y = f_{L2}\tan(\lambda/d_R). \tag{4}$$

Exact derivations of Eqs. (2), (3) and (4) were presented by the present author in [9]. The intensity distribution is given by

$$I(x, y) = a_0^2 b_1^2 + a_1^2 b_1^2 + 2a_0 a_1 b_1^2 \cos\left\{k\Theta_y y + \Delta y\frac{2\pi}{d}\frac{\partial u(x, y)}{\partial y}\right\}. \tag{5}$$

Equation (5) describes a lateral shear type interferogram with carrier (tilt) fringes of spatial period $p = (f_{L2}/z)d_R$ deformed proportionally to the derivative $\partial u(x, y)/\partial y$. The fringe period can be easily chosen by changing the axial localization $z$ of the beam-splitter grating R. For $z = 0$ the uniform field display mode of $\partial u(x, y)/\partial y$ is encountered. The shear amount can be controlled by using gratings R of various period $d_R$. They are commercially available or can be produced interferometrically.

The interferogram with maximum contrast is obtained when the spatial filter transmits the beams $(+1_x, +1_y)$ and $(+1_x, -1_y)$. The intensity distribution in this case is

$$I(x, y) = 2a_1^2 b_1^2 \left\{1+\cos 2\left[k\Theta_y y+\Delta y\frac{2\pi}{d}\frac{\partial u(x, y)}{\partial y}\right]\right\}. \tag{6}$$

With comparison to Eq. (5) it is readily noticed that the shear amount and the carrier fringe frequency are doubled. If necessary the fringe frequency can be reduced by axially displacing grating R.

In a similar way the interferogram giving the information about $\partial v(x, y)/\partial x$ can be obtained. In the input plane a linear specimen grating carrying the information about $v(x, y)$ is inserted and the lines of R are properly oriented. In general, the lines of R must be perpendicular to the lines of the analysed grating. The method can be readily extended to the case of cross-type specimen grating by providing proper spatial filtering and utilizing the beam-splitter grating R of sinusoidal amplitude transmittance. Because of the limited space we will not describe this problem in detail here.

Having the maps of $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$ without the carrier fringes, i.e., $z = 0$, we can employ the method of finding $\gamma_{xy}$ described in [3], [6]. The two cross-derivative fringe patterns are superimposed and the additive moiré fringes corresponding to diagonal curves are traced by hand. This method, although being relatively simple, is time consuming and requires the knowledge of boundary load conditions when numbering the fringe orders of component cross-derivative patterns.

Additional possibilities arise due to an easy availability of the cross-derivative interferograms with dense carrier fringes. In the following they will be called gratings of derivatives $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$. It is necessary to note that the lines of these two gratings are mutually perpendicular. Several methods using these gratings can be proposed now to generate the map of shear strain given by

$$\gamma_{xy} = \frac{\partial u(x, y)}{\partial y}+\frac{\partial v(x, y)}{\partial x}. \tag{7}$$

Taking into consideration the conclusions following from the comparison of the methods proposed by KATO et al. [3]–[5] we will not consider the configurations based on the Mach–Zehnder interferometer. Much simpler systems seem to be the ones using coherent optical processors with spatial filtering.

The first solution is shown schematically in Fig. 2. The two previously described derivative gratings $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$ are superimposed and form the cross-type structure. The product or sum type superimpositions are possible. The first one corresponds to overlapping of two separately recorded interferograms and the second one to double exposure overpal. Let us consider the first case. The lines of the derivative grating $\partial v(x, y)/\partial x$ are vertical and the lines of the grating $\partial u(x, y)/\partial y$ are horizontal. If the single aperture spatial filter is placed at the location $(+1_x, +1_y)$

(Fig. 2), then the information carried by the diffraction beam in the observation plane can be described as

$$E_{+1_x,+1_y}(x, y) \propto \exp\left\{i\frac{2\pi}{p}\left[x+y+\Delta\frac{\partial v(x, y)}{\partial x}+\Delta\frac{\partial u(x, y)}{\partial y}\right]\right\} \tag{8}$$

where $\Delta = \Delta x = \Delta y$ and $p$, as before, designate the period of the derivative gratings. It is seen that the phase of the beam is proportional to $\gamma_{xy}$. Therefore, the intensity map of $\gamma_{xy}$ can be generated by interference with the plane wave front beam. The latter one is obtained by introducing a small aperture on the axis in the plane SF (Fig. 2) transmitting the zero order beam. To equalize the amplitudes of both

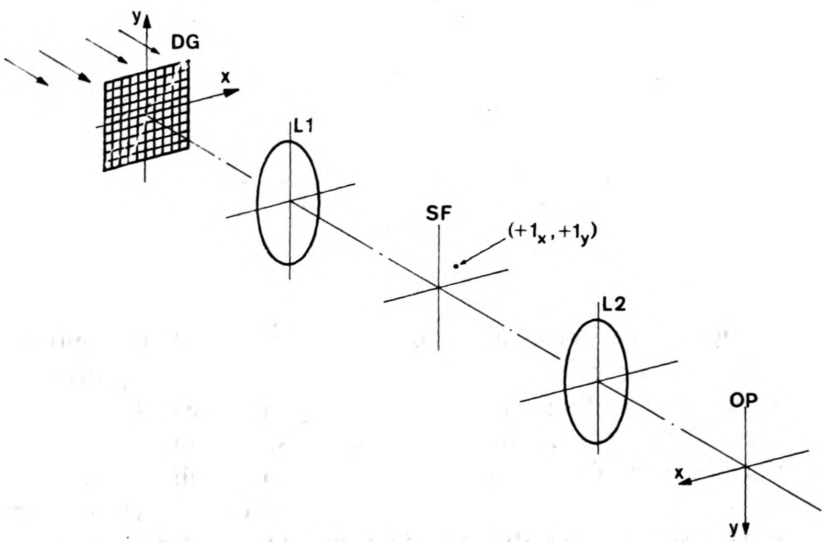


Fig. 2. Schematic layout of the spatial filtering system for producing the map of $\gamma_{xy}$ from two cross-derivative $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$ gratings with lines mutually perpendicular. DG – cross-type derivative grating, SF – spatial filter plane. Other symbols mean as before

interfering beams the zero order should be attenuated. The intensity distribution of the resulting interference pattern is

$$I(x, y) \propto DC+\cos\frac{2\pi}{p}\left\{x+y+\Delta\left[\frac{\partial u(x, y)}{\partial y}+\frac{\partial v(x, y)}{\partial x}\right]\right\} \tag{9}$$

where $DC$ denotes the bias term. To obtain the contour map of $\gamma_{xy}$ it is necessary to reduce the spatial frequency of the above interferogram. For this purpose we can introduce a linear master grating of spatial period $p/\sqrt{2}$ with lines set parallel to the lines of the interferogram of Eq. (9). The moiré fringes will give the uniform field display mode of $\gamma_{xy}$. The master grating can be produced in the optical system shown in Fig. 1 by lateral shearing of the zero diffraction beam of the gratings $u(x, y)$ or $v(x, y)$.

It is worthwhile to note that in Fig. 2 the spatial filter can be in the form of two openings located at the coordinates $(+1_x, 0)$ and $(-1_y, 0)$. The same interference pattern as the one described by Eq. (9) is obtained. The multiplicative or additive superimposition of component cross-derivative interferograms can be used.

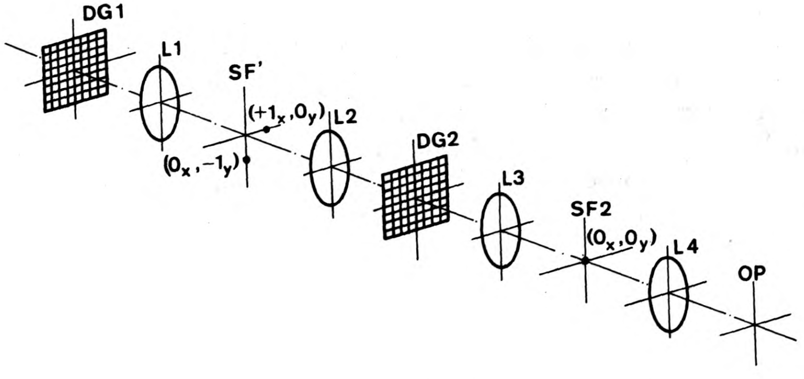


Fig. 3. Double optical processor for generating the map of $\gamma_{xy}$. DG — cross-type grating of derivatives $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$, MG — master cross-type grating, SF1 and SF2 — spatial filters, L1–L2 and L3–L4 — imaging systems

Another more sophisticated solution is shown in Fig. 3. It utilizes double optical processor and spatial filtering. As before, in the input plane of the first processor L1–L2, the cross-type derivative grating is located. In the input plane of the second processor L3–L4 (image plane of L1–L2) we have master cross-type grating MG of the same spatial period as the derivative grating DG. Let us consider first the case of multiplicative superimposition of the gratings $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$. For example, the openings in the spatial filter SF1 can be located at $(-1_x, 0)$ and $(0, +1_y)$. A single opening of the second filter SF2 is located on the optical axis. The master grating serves the purpose of colinear propagation of diffraction orders carrying the information about $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$. The intensity distribution formed by two beam interference on the observation plane is

$$I(x, y) \propto \left| \exp\left[ -i\frac{2\pi}{p}\Delta\frac{\partial v(x, y)}{\partial x} \right] + \exp\left[ i\frac{2\pi}{p}\Delta\frac{\partial u(x, y)}{\partial y} \right] \right|^2$$
$$= 2\left\{ 1+\cos\frac{2\pi}{p}\Delta\left[ \frac{\partial u(x, y)}{\partial y} + \frac{\partial v(x, y)}{\partial x} \right] \right\}. \tag{10}$$

The same result is obtained with the openings of SF1 placed at $(+1_x, 0)$ and $(0, -1_y)$ and the filter SF2 unchanged. In both cases the contrast of the intensity pattern attains maximum.

The cross-type master grating can be produced in the optical system of Fig. 1 by shearing the zero diffraction order of SG. When the period of MG is slightly different from the period of DG the finite fringe display mode of $\gamma_{xy}$ is encountered.

If the cross-type grating DG of the derivatives $\partial u(x, y)/\partial y$ and $\partial v(x, y)/\partial x$ is produced by double exposure technique, Fig. 3, its amplitude transmittance is given by the sum (not product) of the component gratings. However, when the spatial filter SF1 has the openings with the coordinates, for example, $(-1_x, 0)$ and $(0, +1_y)$, the situation is the same as in the case of multiplicative superimposition and the equations derived above remain valid.

## 3. Conclusions

The use of lateral shear devices such as diffraction grating or a Wollaston prism permits an easy generation of the map of derivative of a phase function. Their application for obtaining the cross-derivatives of in-plane displacement encoded in the deformed lines of the specimen grating has been described. Since the carrier fringes are readily introduced into the interferograms the latter ones serve, subsequently, as the cross-derivative diffraction gratings. By overlapping two such structures and spatial filtering of the doubly diffracted beams in the coherent optical processor the intensity pattern carrying the information about shear $\gamma_{xy}$ is obtained. Its spatial frequency can be reduced by the moiré fringe technique or double spatial filtering operation.

Experimental verification of the principles proposed will follow in a separate paper.

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### Применение интерферометрии с поперечным смещением волнового фронта для изготовления карт деформаций при сдвиге

Представлен метод интерферометрии с поперечным смещением волнового фронта и фильтрации пространственных частот, служащей для получения карты угла сдвиговой деформации $\gamma_{xy}$. Производные перемещений в плоскости генерируются при помощи сдвиговой двухлучевой интерферометрии. Получаемые интеферограммы с интерференционными полосами с высокой пространственной частотой служат затем для изготовления крестовых сеток. При помощи фильтрации соответствующих порядков дифракции в когерентном оптическом процессоре изготовляется двухлучевая интерферограмма составляющая карту $\gamma_{xy}$.