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Variance of the wave-aberration of the optical system with small decentration

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The variance of the wave-aberration of the composed optical system with small decentration of single element is derived using the vector form of wave-aberration equation of phase distorter with decentration. The possibilities of optimization of aberrations of centred system from the standpoint of minimization of the decentration sensitivity of the selected single element are discussed. The conditions for minimization of coma of decentration and their practical aspects are presented.

1. Introduction

In the previous article [1] it was shown that the wave-aberration of the optical system with small decentration can be described in the vector form with the aberration coefficients of centred system introduced by Hopkins [2]. It makes possible to introduce the phase distorter description of decentration of any lens in the optical system. Under limitations of the Fresnel approximation and in the isoplanatic region of the object the distorter of any type can be transferred through focusing elements to the object space of the system without altering its influence on the imaging process [3]. It is also possible to determine the perturbation of the wavefront in the exit pupil. Moreover, the methods of image assessment based on the diffraction theory of imaging can be applied. They enable, among others, the variance calculation for the wavefront in the exit pupil plane (under Marechal approximation). This can be done conveniently by numerical methods. In simple cases the analytical approach is possible as well. As an example the variance for the system with spherical aberration and defocusing was discussed in [1]. In the present paper the variance of the wave aberration for the system with all primary aberrations and single decentration will be presented. The possibility of minimization of the decentration sensitivity of the selected element by optimization of aberrations of centred system will be discussed, and the conditions for minimization of coma of decentration will be formulated.

2. Analysis

It was stated in [1] that the optical system with decentration can be expressed as a set of:

i) Purely quadratic phase correctors that describe focusing properties of all elements.

ii) Distorters due to aberrations of the centred system, which can be described as

$$\Phi(\overline{\Delta\varrho}=0)=\sum_{i=1}^{N}[w_{20i}\bar{\varrho}^2\bar{a}^2+w_{40i}\bar{\varrho}^4+(w_{11}\bar{a}^2+w_{31}\bar{\varrho}^2)(\bar{\varrho}\cdot\bar{a})+w_{22}(\bar{\varrho}\cdot\bar{a})^2]. \quad (1)$$

iii) Distorters due to the decentrations of the first order. They can be expressed as

$$\Phi(\overline{\Delta\varrho}\neq 0)=\sum_{i=1}^{N}\Big\{w_{20i}[2(\bar{\varrho}\cdot\overline{\Delta\varrho_i})\bar{a}^2+2(\bar{a}\cdot\overline{\Delta a_i})\bar{\varrho}^2]$$
$$+w_{40i}4(\bar{\varrho}\cdot\overline{\Delta\varrho_i})\bar{\varrho}^2$$
$$+w_{11i}[(\bar{a}\cdot\overline{\Delta\varrho_i})\bar{a}^2+(\bar{\varrho}\cdot\overline{\Delta a_i})\bar{a}^2+2(\bar{a}\cdot\overline{\Delta a_i})(\bar{\varrho}\cdot\bar{a})]$$
$$+w_{31i}[(\bar{a}\cdot\overline{\Delta\varrho_i})\bar{\varrho}^2+2(\bar{\varrho}\cdot\overline{\Delta\varrho_i})(\bar{\varrho}\cdot\bar{a})+(\bar{\varrho}\cdot\overline{\Delta a_i})\bar{\varrho}^2]$$
$$+w_{22i}[2(\bar{a}\cdot\overline{\Delta\varrho_i})(\bar{\varrho}\cdot\bar{a})+2(\bar{\varrho}\cdot\overline{\Delta a_i})(\bar{\varrho}\cdot\bar{a})]\Big\} \quad (2)$$

where: w_{mni} – coefficients of primary aberrations of i-th element of the system,
$\bar{\varrho}$ – localization vector of a point in the aperture plane,
$\bar{a}$ – localization vector of the point in the image plane,
$\overline{\Delta\varrho_i}$ – the shift of the abcrrational function in the exit pupil of i-th element,
$\overline{\Delta a_i}$ – the shift of the image point in the image plane of i-th element.
$\bar{\varrho}$, $\bar{a}$, $\overline{\Delta\varrho_i}$, $\overline{\Delta a_i}$ are normalized to the maximal pupil and image heights respectively in the image space of i-th element. Equation (2) should be taken into account twice. Firstly, for describing the effect of own aberrations of i-th element and, secondly, for describing the effect of aberrations of the wavefront incident on the i-th element of the system.

For the case of a system of N elements with the single decentration of k-th element Eqs. (1) and (2) can be rewritten as

$$\Phi=\Phi(\overline{\Delta c}=0)+\Phi(\overline{\Delta c}\neq 0) \quad (3)$$

where:

$$\Phi(\overline{\Delta c}=0)=c_{20}\bar{\varrho}^2\bar{a}^2+c_{40}\bar{\varrho}^4+(c_{11}\bar{a}^2+c_{31}\bar{\varrho}^2)(\bar{\varrho}\cdot\bar{a})+c_{22}(\bar{\varrho}\cdot\bar{a})^2, \quad (3.1)$$

$$\Phi(\overline{\Delta c}\neq 0)=2b_{20}[p\bar{a}^2(\bar{\varrho}\cdot\overline{\Delta c})+t\bar{\varrho}^2(\bar{a}\cdot\overline{\Delta c})]$$
$$+4b_{40}p\bar{\varrho}^2(\bar{\varrho}\cdot\overline{\Delta c})$$
$$+b_{11}[p(\bar{a}\cdot\overline{\Delta c})\bar{a}^2$$
$$+t(\bar{\varrho}\cdot\overline{\Delta c})\bar{a}^2+2t(\bar{a}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})]$$
$$+b_{31}[p(\bar{a}\cdot\overline{\Delta c})\bar{\varrho}^2+2p(\bar{\varrho}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})$$
$$+t(\bar{\varrho}\cdot\overline{\Delta c})\bar{\varrho}^2]+2b_{22}[p(\bar{a}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})+t(\bar{\varrho}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})]$$

$$+2d_{20}(q\bar{a}^2(\bar{\varrho}\cdot\overline{\Delta c})+t\bar{\varrho}^2(\bar{a}\cdot\overline{\Delta c})]$$
$$+4d_{40}q\bar{\varrho}^2(\bar{\varrho}\cdot\overline{\Delta c})$$
$$+d_{11}[q(\bar{a}\cdot\overline{\Delta c})\bar{a}^2$$
$$+t(\bar{\varrho}\cdot\overline{\Delta c})\bar{a}^2+2t(\bar{a}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})]$$
$$+d_{31}[q(\bar{a}\cdot\overline{\Delta c})\bar{\varrho}^2+2q(\bar{\varrho}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})$$
$$+t(\bar{\varrho}\cdot\overline{\Delta c})\bar{\varrho}^2]$$
$$+2d_{22}[q(\bar{a}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})+t(\bar{\varrho}\cdot\overline{\Delta c})(\bar{\varrho}\cdot\bar{a})] \tag{3.2}$$

where: $\overline{\Delta c}$ – vector of lateral shift of centre of curvature of decentred k-th surface (or shift of focus of the thin lens); the index k is omitted for simplicity,

$c_{mn} = \sum_{i=1}^{N} w_{mni}$ – sum of the coefficients of primary aberration for the whole system,

$b_{mn} = w_{mnk}$ – coefficient of aberration of k-th surface or thin lens of the system,

$d_{mn} = \sum_{i=1}^{k-1} w_{mni}$ – sum of aberration coefficients of preceeding part of the system for the wavefront incident on k-th element of the system,

$p = -M_A$, $q = 1-M_A$, $t = 1-M_o$, M_A, M_o – pupil and image magnifications of k-th element of the system under con siderations.

In the considerations only a small decentration due to the production tolerances of fairly well-corrected centered system are taken into account. In this case it is useful to employ the variance E of the wavefront as a desing parameter. MARECHAL [4] showed that for small aberration, i.e., $SC \geqslant 0.8$ we have

$$SC \geqslant [1-1/2\,k^2 E]^2. \tag{4}$$

Obviously, the variance minimization maximizes the Strehl criterion. Under Marechal approximation, for vectors described in the polar-coordinate system, the variance of the wavefront is given for a circular aperture by

$$E = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \Phi^2 \varrho d\varrho d\alpha - \frac{1}{\pi^2}\Big[\int_0^{2\pi}\int_0^1 \Phi\varrho\, d\varrho\, d\alpha\Big]^2. \tag{5}$$

If the aberration function is given on the polar-coordinate system the integrations are simple. Because of the extent of analysis only the final result is quoted in Eqs. (6). We assume the vectors $\bar{\varrho}(\varrho\cos\alpha, \varrho\sin\alpha)$, $\bar{a}(a\cos\gamma, a\sin\gamma)$, $\overline{\Delta c}(\Delta c\cos\beta, \Delta c\sin\beta)$ and, for a simplification of Eqs. (6), we introduce the parameter $A = \cos\beta\cos\gamma+\sin\beta$

$\sin\gamma = \cos(\beta-\gamma)$ – cosine of the angle between the azimuth of a chosen image point and the decentration azimuth of k-th element. From Eqs. (5) and (3) we have

$$E = E_1+E_2+E_3+E_4+E_5+E_6 \tag{6}$$

where

$$E_1 = c_{20}a^2\left[\frac{1}{12}a^2c_{20}+\frac{1}{6}c_{40}+\frac{1}{12}a^2c_{22}\right]+c_{40}\left[\frac{4}{45}c_{40}+\frac{1}{12}a^2c_{22}\right]$$
$$+c_{11}a^4\left[\frac{1}{4}a^2c_{11}+\frac{1}{3}c_{31}\right]+c_{31}^2\frac{1}{8}a^2+c_{22}^2\frac{1}{16}a^4, \tag{6.1}$$

$$E_2 = \Delta caA\Bigg\{(c_{20}a^2+c_{40})\left[\frac{1}{3}t\,b_{20}+\frac{1}{3}p\,b_{31}+\frac{1}{6}t\,b_{22}\right]$$
$$+c_{11}a^2\left[a^2\,pb_{20}+\frac{4}{3}pb_{40}+\frac{3}{2}a^2\,tb_{11}+\frac{1}{3}tb_{31}+a^2\,pb_{22}\right]$$
$$+c_{31}\left[\frac{2}{3}a^2\,pb_{20}+pb_{40}+a^2\,tb_{11}+\frac{1}{4}tb_{31}+\frac{2}{3}a^2\,pb_{22}\right]$$
$$+c_{22}a^2\left[\frac{1}{6}tb_{20}+\frac{1}{3}pb_{31}+\frac{1}{4}tb_{22}\right]\Bigg\}, \tag{6.2}$$

$$E_3 = \Delta caA\Bigg\{(c_{20}a^2+c_{40})\left[\frac{1}{3}td_{20}+\frac{1}{3}qd_{31}+\frac{1}{6}td_{22}\right]$$
$$+c_{11}a^2\left[a^2qd_{20}+\frac{4}{3}qd_{40}+\frac{3}{2}a^2td_{11}+\frac{1}{3}td_{31}+a^2\,qd_{22}\right]$$
$$+c_{31}\left[\frac{3}{2}a^2\,qd_{20}+qd_{40}+a^2\,td_{11}+\frac{1}{4}td_{31}+\frac{2}{3}a^2\,qd_{22}\right]$$
$$+c_{22}a^2\left[\frac{1}{6}td_{20}+\frac{1}{3}qd_{31}+\frac{1}{4}td_{22}\right]\Bigg\}, \tag{6.3}$$

$$E_4 = (\Delta c)^2\Bigg\{a^2b_{20}\Bigg[\left(p^2a^2+\frac{1}{3}t^2A^2\right)b_{20}+\frac{8}{3}p^2b_{40}+(1+2A^2)a^2\,ptb_{11}$$
$$+\frac{2}{3}(1+A^2)\,ptb_{31}+\left(2a^2p^2+\frac{1}{3}t^2\right)A^2b_{22}\Bigg]$$
$$+b_{40}\left[2p^2b_{40}+\frac{4}{3}(1+2A^2)a^2\,ptb_{11}+ptb_{31}+\frac{8}{3}a^2A^2p^2b_{22}\right]$$
$$+a^2b_{11}\left[\frac{1}{4}(1+8A^2)a^2t^2b_{11}+\frac{1}{3}(1+2A^2)t^2b_{31}+3a^2A^2\,ptb_{22}\right]$$

$$+b_{31}\left[\frac{1}{6}(1+2A^2)a^2p^2b_{31}+\frac{1}{8}t^2b_{31}+\frac{1}{3}(1+3A^2)a^2\,ptb_{22}\right]$$

$$+a^2b_{22}^2\left[p^2a^2+\frac{1}{12}(2+A^2)\,t^2\right]\Bigg\}, \tag{6.4}$$

$$E_5=(\Delta c)^2\Bigg\{a^2d_{20}\Bigg[\left(q^2a^2+\frac{1}{3}t^2A^2\right)d_{20}+\frac{8}{3}q^2d_{40}+(1+2A^2)a^2\,qtd_{11}+\frac{2}{3}(1+A^2)qtd_{31}$$

$$+\left(2a^2q^2+\frac{1}{3}t^2\right)A^2d_{22}\Bigg]+d_{40}\left[2q^2d_{40}+\frac{4}{3}(1+2A^2)a^2\,qtd_{11}+qtd_{31}+\frac{8}{3}a^2A^2q^2d_{22}\right]$$

$$+a^2d_{11}\left[\frac{1}{4}(1+8A^2)a^2t^2d_{11}+\frac{1}{3}(1+2A^2)t^2d_{31}+3a^2A^2qtd_{22}\right]$$

$$+d_{31}\left[\frac{1}{6}(1+2A^2)a^2q^2d_{31}+\frac{1}{8}t^2d_{31}+\frac{1}{3}(1+3A^2)a^2\,qtd_{22}\right]$$

$$+a^2d_{22}^2\left[q^2a^2+\frac{1}{12}(2+A^2)t^2\right]\Bigg\}, \tag{6.5}$$

$$E_6=2(\Delta c)^2\left\{a^2b_{20}\,d_{20}\left(pqa^2+\frac{1}{3}t^2A^2\right)+\frac{4}{3}a^2pq\,(b_{20}\,d_{40}\,b_{40}\,d_{20})+a^4\left(\frac{1}{2}+A^2\right)\right.$$

$$\times(p\,t\,b_{20}\,d_{11}+qtb_{11}\,d_{20})+\frac{1}{3}a^2\,[(p+qA^2)\,tb_{20}\,d_{31}+(q+pA^2)\,t\,b_{31}\,d_{20}]$$

$$+a^2A^2\left(pqa^2+\frac{1}{6}t^2\right)(b_{20}\,d_{22}+b_{22}\,d_{22})+2\,pqb_{40}\,d_{40}+\frac{2}{3}a^2\,(1+2A^2)\,(ptb_{40}\,d_{11}$$

$$+qtb_{11}\,d_{40})+\frac{1}{2}(ptb_{40}\,d_{31}+qtb_{31}\,d_{40})+\frac{4}{3}a^2pq\,(b_{40}\,d_{22}+b_{22}\,d_{40})$$

$$+\frac{1}{4}a^4\,(1+8A^2)t^2b_{11}\,d_{11}+\frac{1}{6}a^2\,(1+2A^2)t^2(b_{11}\,d_{31}+b_{31}\,d_{11})+\frac{3}{2}a^4A^2\,(qtb_{11}\,d_{22}$$

$$+ptb_{22}\,d_{11})+\left[\frac{1}{6}a^2\,(1+2A^2)pq+\frac{1}{8}t^2\right]b_{31}\,d_{31}+\frac{1}{6}a^2\,[(A^2+1)\,(ptb_{31}\,d_{22}$$

$$+qtb_{22}\,d_{31})+2A^2\,(qtb_{31}\,d_{22}+ptb_{22}\,d_{31})]$$

$$+a^2\left[p\,q\,a^2A^2+\frac{1}{12}t^2\,(A^2+2)\right]b_{22}\,d_{22}. \tag{6.6}$$

E_1 given by Eq. (6.1) is the variance due to the aberrations of the whole centered system. The part of the variance given by Eqs. (6.2) and (6.3) contains the first order terms of decentration. They vary with image height and the image azimuth. E_4, E_5, E_6 combine quadratic terms of Δc. E_2 and E_4 are the functions of aberration coefficients of k-th element only, and E_3, E_5 of aberrations of the wavefront incident on k-th element of the system. E_6 is a combined term.

3. Discussion

The variance expression given enables to determine the influence of decentration on the image quality with reduced amount of data. For primary aberrations only five coefficients, paraxial aperture and image magnifications for every optical element are required. Since the terms E_2, E_3 and E_4, E_5, E_6 are mathematically similar (E_3 reduced to E_2, E_5 to E_4 and E_6 to $2E_4$ for $p = q$, $b_{mn} = d_{mn}$) the numerical algorythm can be very simple.

At this point we can minimize the variance or minimize the decentration sensitivity of k-th element of the system by appropriate choices of the selected aberration coefficients. As pointed out by BARAKAT and HOUSTON [5] this procedure is not generally possible in practice. Even when we solve the mathematical problem and obtain the derived functional relations between the aberration coefficients, it is rarely possible to achieve these relations exactly in the system design. The practical balancing problem reduces to achieving the partial balancing. Secondly, the variance terms $E_2 - E_6$ are functions of A (angle between azimuths of the decentration and image point). The image quality changes not only with the image height but also with the azimuth. This is not true for $a = 0$. In such a case the variance is influenced only by spherical aberration of the whole system and coma of decentration. Equation (6) simplifies to

$$E = \frac{4}{45}c_{40} + (\Delta c)^2\left[2p^2b_{40}^2 + ptb_{31}b_{40} + \frac{1}{8}t^2b_{31}^2\right] + (\Delta c)^2\left[2q^2d_{40}^2 + qtd_{31}d_{40} + \frac{1}{8}t^2d_{31}^2\right] + 2(\Delta c)^2\left[2pqb_{40}d_{40} + \frac{1}{2}(ptb_{40}d_{31} + qtb_{31}d_{40}) + \frac{1}{8}t^2b_{31}d_{31}\right]. \quad (7)$$

Coma of decentration is also caused by spherical aberration and coma of the decentred element and the same aberrations of the wavefront incident of this element only. The balancing problem seems very simple in this case. For

$$\frac{\partial E}{\partial b_{40}} = 0, \quad \frac{\partial E}{\partial d_{40}} = 0,$$

we have the conditions

$$b_{40} = -\frac{1}{4}\frac{t}{p}b_{31}, \quad (8a)$$

$$d_{40} = -\frac{1}{4}\frac{t}{q}d_{31}. \quad (8b)$$

If these conditions are fulfilled the decentration of the element under consideration cannot influence the value of the variance for the whole system. As mentioned above, it is not always possible to fulfil these conditions exactly. However, they can be used as optimization criteria for minimizing the coma of decentration sensitivity for a selected element of the optical system.

The coma of decentration is the one of all decentration aberrations which is observed in a simple way in the alignment process in production. This aberration deteriorates the image quality uniformly in the whole field of view of the optical systems. It is discussed very often when analyzing the decentration effects on the image quality. The possibility of minimization of this aberration (by optimization of wavefront aberrations for the selected element inside the optical system) in the desing process seems to be a very interesting one. It can be used for the desing of the elements of the optical system, which are difficult for practical adjustment and assembly as well as for the desing of these elements of the system which are used for compensation for the coma of decentration. Microscopic objectives can serve as an example. The sensitivity to decentration of these elements can be optimized during the desing from the point of view of requirements of the adjustment.

The more detailed analysis of the possibility of implementation of the conditions given by Eqs. (8a) and (8b), and of the optimization of other decentration aberrations will be given in the following work.

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Варианция волновой аберрации оптической системы с малой децентрировкой

Выведена формула для варианции волновой аберрации сложной оптической системы с децентрировкой одного элемента на базе векторного уравнения волновой аберрации интерпретируемой как фазовый дистортер. Рассмотрены возможности оптимизации аберрации центральной оптической системы для минимализации чувствительности выбранного элемента системы на децентрировку. Представлены условия для минимализации комы децентрировки и практические возможности их использоания.