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RISK CAPITAL IN UNIT-LINKED INSURANCE POLICIES

KAPITAŁ PODWYŻSZONEGO RYZYKA W UBEZPIECZENIACH UFK

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Abstract: Unit-linked insurance policies (ULIP) are endowment policies with shares in selected investment funds held by financial institutions, which combine insurance coverage with investment. Therefore, the risk related to this type of insurance is analysed as: subject of insurance risk and financial risk which determine the value of ULIP. To secure its liquidity, the insurer should take this enhanced risk. Thus, the aim of this text is to determine the appropriate capital levels which should offset the additional risk to which the insurer is exposed to in order to ensure the financial security of the insured persons and protect the insurer's solvency.

Keywords: unit-linked insurance, Solvency II, risk capital, Monte-Carlo.

Streszczenie: Polisy ubezpieczeniowe z ubezpieczeniowym funduszem kapitałowym (ULIP) to polisy z udziałami w wybranych funduszach inwestycyjnych będących w posiadaniu instytucji finansowych, które łączą ochronę ubezpieczeniową z inwestycyjną. Dlatego też ryzyko związane z tym typem ubezpieczeń analizowane jest jako przedmiot ryzyka ubezpieczeniowego i finansowego determinującego wartość ULIP. Aby zabezpieczyć swoją płynność, ubezpieczyciel powinien podjąć takie zwiększone ryzyko. Celem niniejszego dokumentu jest zatem określenie odpowiednich poziomów kapitału, które powinny zrównoważyć dodatkowe ryzyko, na które narażony jest ubezpieczyciel, dla zapewnienia bezpieczeństwa finansowego ubezpieczonych i ochrony wypłacalności ubezpieczyciela.

Słowa kluczowe: ubezpieczenia UFK (unit-linked), kapitał podwyższonego ryzyka, Solvency II, metoda Monte Carlo.

1. Introduction

Insurance companies have to meet numerous requirements ensuring the security of their insurance operations. The key aspect of the regulatory framework concerns the need to determine the appropriate capital level. For this purpose, in order to protect itself against unforeseeable losses, the insurer should, on the basis of cash flow valuation, determine the appropriate capital level to offset the insurer's risk, in order to ensure the financial safety of the insured persons. The financing requirements adjusted to actual risk to which the insurance companies are exposed are specified in Solvency II [Directive 2009/138/EC]. The underlying idea of Solvency II is to ensure stricter correlation between the capital level and the level of risk undertaken by the insurance companies. In other words, the capital level must be sufficient to cover the actual risk.

Valuation should be conducted according to the current disposal value, i.e. the value should correspond to the current amount that the insurance company would have to pay in the case of immediate transfer of its contractual rights and obligations to another insurance company. In order to achieve this goal, the insurers' assets must be valuated according to the actual market value, i.e. the so-called best estimate. Because the Unit-Linked policies are endowment policies combined with shares in selected investment funds, they are exposed to enhanced risk related not only to the subject of insurance, but also to the financial risk. The benefits under this kind of policies are directly related to the realisation of the reference portfolio and thus, their stochastic nature must be provided for in the valuation. Consequently, the valuation of policies prepared according to best estimate should be based on the market value and provide for all relevant risks. An insurer offering ULIP should take this additional financial risk into account and take the combined actuarial and financial approach to determining the required capital level [Graf et al. 2011]. This is related to the need to determine the so-called risk capital.

2. Unit-linked insurance concept and the related pay-out

The financial institutions offering unit-linked insurance policies are life insurance companies, however, ULIP are not separate products, but are supplementary to basic products, i.e. traditional life insurance policies. They are endowment insurance contracts between the insured person and the insurer, under which the insured person pays the relevant premiums and, in exchange, the insurance company provides the benefit in the amount of the larger of the following values:

- the sum assured (G_{Π}),
- the sum arising from the reference portfolio value (FV_t).

The ULIP is thus combined saving and investment product. In simple terms, it allows the insured person to invest a portion of their premium in the investment funds offered by the insurance company, with various risk levels. They combine, in a uniquely flexible way, the insurance element and the saving element, creating numerous opportunities for investing the saved capital. The investment funds differ in terms of risk levels and investment policies, and since the ULIP have an open and transparent nature, the insured persons are able to decide on the composition of their portfolio during the insurance period. In Poland, ULIP are available mainly as part of individual life coverage in the following forms:

- permanent life policies,
- term life and term endowment policies,
- typical savings policies.

Regardless of the form, ULIP are either life or endowment insurance contracts between the insured person and the insurer, under which the insured person pays the relevant premiums and, in exchange, the insurance company provides them with appropriate benefits. Unlike the traditional life policy, where the cost of insurance (expressed in the premium) remains constant throughout the insurance period and does not arise from the risk level in the given year, but from the average risk for the entire insurance period, in ULIP, the cost depends on the age of the insured person and varies according to payments, interest rates, administrative costs and risk of death or disability, but also the financial risk depending on the fund units' price.

Concerning the benefit, in traditional life insurance, its value is specified in the insurance contract as the sum assured, while in ULIP the benefit value arises from the value of the reference portfolio and depends on the prices of the given fund units, which in turn result from the applied investment policy. Therefore, the pay--out from a ULIP is the relevant function of the accumulated investment, depending on the price of fund units, and equals [Moller 2007]:

$$f(FV_t) = FV_t \text{ or } f(FV_t) = \max\{G_\Pi, FV_t\},$$

where G_Π – the sum assured depending on the insurance premium,

FV_t – the value of the insurance (reference) portfolio at t.

In the first case, the pay-out depends only on the value of the reference portfolio, while in the second case, the insurer guarantees a minimum sum assured. Then, if an event under insurance coverage occurs, the insurer pays the insured person the higher of the two values, i.e. the minimum sum assured (often considered to be the insurance protection element) and the market value of the insurance portfolio. The majority of insurance companies in Poland minimise the protection coverage, focusing on the saving aspect. As a result, contracts of this type are not guaranteed, which means that the payment due is calculated according to the first of the above formulas.

Providing for the changes of the value of money over time, the market value as of the benefit payment will be:

$$Z_t(FV_T) = \frac{\upsilon(T)}{\upsilon(t)} f(FV_T) = \frac{\upsilon(T)}{\upsilon(t)}\left(G_\pi + \left(X_T - G_\pi\right)^+\right).$$

The value of the insurance portfolio related to coverage of this type X_T is random and depends on: the fund unit price, the fund unit prices in the past (purchased with the premiums), and the volume of investment for the ULIP portion of the premium π_t. Assuming that the insured person invests in the selected assets at a price determined as a process S_t, providing for the changes of the value of money over time, the market value of the reference portfolio at the moment of policy lapse will be equal to:

$$Z_t(FV_T)=\frac{\upsilon(T)}{\upsilon(t)}\cdot G_\pi+\frac{\upsilon(T)}{\upsilon(t)}\cdot\left(S_T\sum_{u=0}^{\tilde{n}_T-1}\pi_u\cdot S_u^{-1}-\cdot G_\pi\right)^+ .$$

3. Risk capital for unit-linked policies

The new solvency assessment system, conforming to SOLVENCY II is to match the actual risks to which insurance companies are exposed to. In the case of insurance institutions, potential risks are specific to the types of insurance contracts. For insurance policies related to financial risk, the stochastic variability of economic variables is the key factor determining the future value of liabilities. Therefore, in the case of a unit-linked insurance, the cash flow should be valuated in consideration of the total risk to which the insurer is exposed, i.e. including the filtration providing the comprehensive information available at t on the process of mortality and price fluctuations determined by the financial market. Consequently, the history (filtration) can be divided into three groups to be considered during valuation:

- $\mathcal{L}_t$ is the knowledge of the contract options,
- $\mathcal{G}_t$ is the knowledge of the process of prices S_t and value of cash units B_t,
- $\mathcal{H}_t$ is the knowledge of the mortality process.

Filtration $\mathcal{L}_t$, interpreted as the contract options risk, determines the type of cash flows arising from the underwritten insurance policy. Furthermore, it assumes that the financial market's nature is perfect and every player has the access to the same knowledge of it, and information is inferred solely from the observation of the process of prices S_t and the process of cash unit value B_t. Then, considering the $\mathcal{G}_t$, interpreted as the knowledge acquired until t, we assume that: $\mathcal{G}_t=\mathcal{G}_t^{S\wedge B}$.

On the other hand, $\mathcal{H}_t$ is the knowledge acquired until t on the mortality process, and the information on the process is determined by the insured person's future life span, therefore it is assumed that: $\mathcal{H}_t=\mathcal{H}_t^T$. Thus, filtration $\mathcal{F}_t$ determines the comprehensive information available at t on the mortality process and price fluctuations, and takes the following form [Homa 2015]:

$$\begin{aligned}\mathcal{F}_t &= \mathcal{L}_t \wedge \mathcal{G}_t \wedge \mathcal{H}_t \\ &= \mathcal{L}_t \wedge \sigma\{(B_t, S_t), 0 \le t \le T\} \wedge \sigma\{\mathrm{I}(T_i \le t), 0 \le t \le T, i = 1, \ldots, l_x\},\end{aligned}$$

where: T_i – the future life span of the i-th insured person,

l_x – the number of persons in the insurance portfolio.

Considering this filtration and consequently, the enhanced actuarial risk, the capital required under SOLVENCY II should be determined as the probability-weighted average of the future cash flows:

$$V_t^{ULIP} = E\left(\left(Z_t\left(FV_T\right) - Z\Pi_t\right) \mid \mathcal{L}_t \wedge \mathcal{G}_t^{S\wedge B} \wedge \mathcal{H}_t^T\right).$$

Assuming the independence of the insurance and financial market which generate the process history, we further obtain:

$$\begin{aligned}V_t^{ULIP} &= E\left(\frac{\upsilon(T)}{\upsilon(t)} Z_t\left(FV_T\right) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot E\left(\mathrm{I}_{\{T>t\}} \mid \{T > t\}\right) + \\ &+ \int_t^{T\vee n} E\left(\frac{\upsilon(\tau)}{\upsilon(t)} \cdot Z_t\left(FV_\tau\right) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot E\left(d\mathrm{I}\{\tau \le T\} \mid \{T > t\}\right) \\ &+ \int_t^{T\vee n} E\left(\frac{\upsilon(\tau)}{\upsilon(t)} \cdot \pi(\tau) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot E\left(d\mathrm{I}\{\tau > T\} \mid \{T > t\}\right).\end{aligned}$$

Assuming a fixed interest rate account and continuous compounding on the market, the cash unit value process is:

$$B_t = e^{\delta t} = e^{t\ln(1+r)}$$

and ${}_t p_x$ is the probability of survival, then the formula above takes the following form:

$$\begin{aligned}V_t^{ULIP} &= {}_{T-t}p_{x+t} \cdot E\left(e^{-\delta(T-t)}\left(G_\pi + \left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+\right) \mid \mathcal{G}_t^{S\wedge B}\right) + \\ &+ \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(e^{-\delta(\tau-t)} \cdot \left(G_\pi + \left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+\right) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau \\ &- \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot E\left(e^{-\delta(\tau-t)} \cdot \pi(\tau) \mid \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau =\end{aligned}$$

$$= e^{-\delta(T-t)} \cdot {}_{T-t}p_{x+t} \cdot E\left(G_\pi \,|\, \mathcal{G}_t^{S\wedge B}\right) + \int_t^{T\vee n} e^{-\delta(\tau-t)} \cdot {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(G_\pi \,|\, \mathcal{G}_t^{S\wedge B}\right) d\tau +$$

$$- \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot E\left(e^{-\delta(\tau-t)} \cdot \pi_{ubez}(\tau) \,|\, \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau$$

$$+ {}_{T-t}p_{x+t} \cdot e^{-\delta(T-t)} E\left(\left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+ \,|\, \mathcal{G}_t^{S\wedge B}\right) +$$

$$+ \int_t^{T\vee n} e^{-\delta(\tau-t)} \cdot {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(\left(S_T \sum_{u=0}^{\tilde{n}_T-1} \pi_u \cdot S_u^{-1} - \cdot G_\pi\right)^+ \,|\, \mathcal{G}_t^{S\wedge B}\right) d\tau$$

$$- \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot E\left(e^{-\delta(\tau-t)} \cdot \pi_{inw}(\tau) \,|\, \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau.$$

It should be noted that the conditional expected value, provided in the aforementioned formula is, by definition, the derivative arbitrage price [Wüthrich et al. 2007]. Therefore, assuming the designation:

$$E\left[e^{-\delta(T-t)} \cdot \left(X_T - G_\pi\right)^+ \,|\, \mathcal{G}_t\right] = C_t\left(X_T, G_\Pi\right)$$

and following further conversions, we obtain:

$$\begin{aligned} V_t^{ULIP} &= e^{-\delta(T-t)} \cdot {}_{T-t}p_{x+t} \cdot E\left(G_\pi \,|\, \mathcal{G}_t^{S\wedge B}\right) + \\ &+ \int_t^{T\vee n} e^{-\delta(\tau-t)} \cdot {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot E\left(G_\pi \,|\, \mathcal{G}_t^{S\wedge B}\right) d\tau + \\ &- \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot E\left(e^{-\delta(\tau-t)} \cdot \pi(\tau) \,|\, \mathcal{G}_t^{S\wedge B}\right) \cdot d\tau + \\ &+ {}_{T-t}p_{x+t} \cdot C_t\left(X_T, G_\Pi\right) + \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot C_t\left(X_\tau, G_\Pi\right) d\tau. \end{aligned}$$

In the formula provided above, the first three components are the capital that should be secured by an insurance company offering traditional endowment policies with sum assured G_Π and premiums in the amount of $\pi(t)$, the final form of the formula for the risk capital covering the reference portfolio risk is:

$$KR_t^{ULIP} = {}_{T-t}p_{x+t} \cdot C_t\left(X_T, G_\Pi\right) + \int_t^{T\vee n} {}_{\tau-t}p_{x+t} \cdot \mu(x+\tau) \cdot C_t\left(X_\tau, G_\Pi\right) d\tau .$$

According to the aforementioned formula, in order to determine the value of the risk capital, the actuarial approach should be combined with the financial approach in terms of the tools used to valuate the European or American call option, depending on the type of endowment coverage.

4. Simulation results

As an example, a ULIP term endowment contract of a 30-year-old male was analysed. Under the contract, the insurer undertakes to pay the maturity benefit (MB), as well as the death benefit (DB). The insurer shall pay to the insured person the sum assured of 1,000 cash units plus bonus arising from the reference portfolio value as of the pay-out moment. It was assumed that the insured person paid premiums in the amount of $\pi(t_k)$ at time t_k, furthermore, also continuous compounding and 5% risk-free interest rate was included. In order to determine the survivability and mortality, mortality tables based on Gompertz-Makeham law on mortality were used [Dahl 2004]. On the basis of the life expectancy for males, function approximation was performed and the following maximum likelihood estimation of its parameters was obtained:

$$A = 0.0004;\ B = 0.0000034674;\ c = 100.06.$$

The insured person invests in the best capital funds offered on the Polish market, in four main fund groups:

- Stock Portfolio,
- Balanced Portfolio,
- Stable Growth Portfolio,
- Debt Securities Portfolio.

The insurance portfolios listed above, developed on the basis of the aforementioned funds, differ in terms of investment policy and hence the financial risk, which makes it possible to assess the impact of the financial risk on the capital level. For this purpose, a simple risk assessment of the funds in individual groups was conducted, taking into consideration the basic measures such as standard deviation and Sharpe Ratio. On that basis, one fund was selected in each group with the highest rank and the results are provided in Table 1.

Table 1. Information about the best funds indices in groups

Group	S	Sharpe ratio	Benchmark
Stock Portfolio	3.99	0.12	100% WSE Index
Balanced Portfolio	2.05	0.05	50% WSE Index + 50% Bond Index
Stable Growth Portfolio	1.27	0.06	20% WSE Index + 80% Bond Index
Debt Securities Portfolio	0.76	0.02	100% EFFAS Bond Indices Poland

Source: own study.

According to the last presented formula for the determination of the risk capital level, it is essential to correctly valuate the call option $C_t(X_T, G_\Pi)$. Depending on the insurance version, it is either the European or the American call option. The European call option may be exercised only upon the lapse of T (applicable to MB policies), while the American option may be exercised at any time (applicable to DB policies), which is why this study is focusing on simulation methods. In the classical approach, i.e. the Black-Scholes model, the price of the European call option is expressed by an analytical formula; the valuation of the American call option is more complicated, which is why the Monte Carlo (MC) method was applied. As the financial market model, the simplest price evolution model was adopted, i.e. the geometric Brownian motion. Therefore, the price of the stock fund unit S_t is described by the geometric Brownian motion, with appropriate drift, which can be done using the Euler method [Perna, Sibillo 2008]. Applying this mathematical apparatus, we obtain a formula simulating the future value of the base instrument:

$$S_{t_k}^i = S_{t_{k-1}}^i \exp\left[\left(r - \frac{\sigma^2}{2}\right)(t_k - t_{k-1}) + \sigma\sqrt{t_k - t_{k-1}}\,\varepsilon_k^i\right],$$

where ε_k^i are the independent values generated from the normal distribution, r is the risk-free interest rate, while S_{t_k} determines the instrument price variability. The stock price process is simulated in a finite number of points at time t_k.

Using the MC method, the risk capital value for unit-linked insurance contracts MB and DB, as presented by Figures 1 and 2.

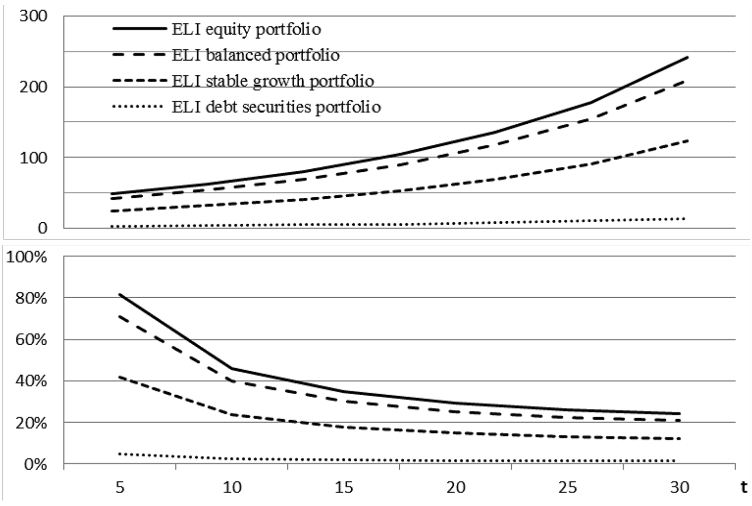

Fig. 1. Risk capital and percentage increase arising from the insurance capital fund during the insurance period in ULIP (MB)

Source: own study.

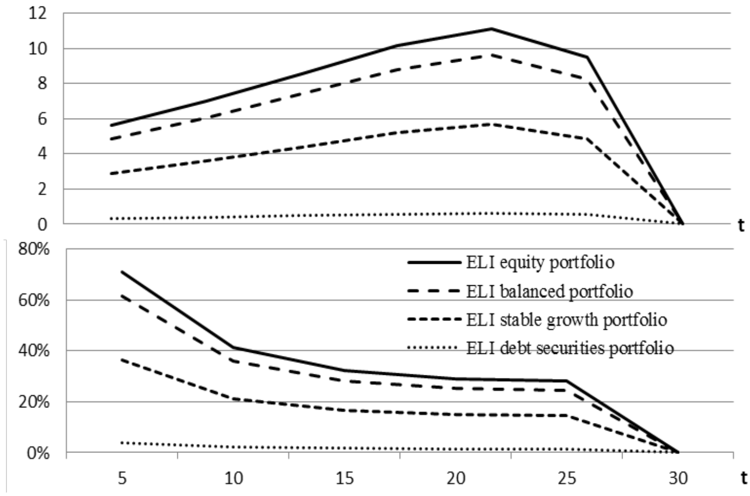

Fig. 2. Risk capital and percentage increase arising from the insurance capital fund during the insurance period in ULIP (DB)

Source: own study.

On the basis of the charts above, it can be concluded that the financial risk of the ULIP reference portfolio does not affect the functional form of the risk capital, nor does it change its structure during the insurance period. On the other hand, the reference portfolio risk has significant impact on its level. In the case of the debt securities fund (the lowest-risk ULIP portfolio), the ULIP risk capital value is close to zero, which means that the capital that the insurer should gather in this case, is similar to traditional insurance products, such as MB and DB. Therefore, in actuarial calculations, the risk margin of this portfolio is negligible. As evident from the simulations, the increase of the reference portfolio risk is followed by the increase of capital levels, which means that the insurer should not neglect this risk in its calculations and valuations. Consequently, taking an additional financial risk of the ULIP related to investments in all funds except for the debt securities fund, requires gathering additional capital. It should also be pointed out that in the first years of the policy, the risk capital amounts to nearly 80% of the required reserve. This is confirmed by the diagrams of insurance capital fund financial risk margins for individual portfolios, and the percentage increase of mathematical reserves.

5. Conclusion

The obtained results for ULIP with various strategies confirm that the standard approach used in traditional endowment insurance policies does not correctly reflect the actual risk related to ULIP insurance products and may thus lead to shortages of reserve funds. It was shown that only in the case of the debt securities fund, which is exposed to the lowest risk, the issue of increased capital is negligible. When valuating the cash flows of a ULIP with other investment strategies, the risk capital level must be determined and the formula proposed in the paper should be used. This solution is a key aspect of the regulatory framework according to Solvency II.

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