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Letter to the Editor

Measuring of birefringence with the use of the method of phase modulation. Double sensitivity achieved by the half-shadow technique *

W. Urbańczyk

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

The way of doubling the measurement sensitivity of the phase shift introduced by the birefringent objects has been described. This has been achieved by employing a half-shadow element in the form of a retardation plate of 180° phase shift.

The phase modulation technique is exploited commonly in the interference measurements (see e.g., [1]–[3]) and also in the birefringence measurements and ellipsometry (e.g., [4]–[10]). The essence of the method consists in the fact that besides the measured phase shift φ_S, additional phase shift changing in time is introduced between the interfering waves

$$\varphi(t) = \Omega t \tag{1}$$

where it has been assumed that the intensities of both interfering beams are equal to interfering beams changes in time in accordance with the formula

$$I_S(t) = I_0[1+\cos(\Omega t+\varphi_S)] \tag{2}$$

where it has been assumed that the intensities of both interfering beams are equal to each other and equal to $I_0/2$. Note that the phase shift $\Delta\Phi$ of the signal $I_S(t)$ with respect to the reference signal

$$I_R(t) = I_0[1+\cos\Omega t] \tag{3}$$

is equal to the measured phase shift $\Delta\Phi = \varphi_S$. In the ellipsometric measurements the amplitude of modulated component of the signal is additionally taken into account. The reference signal is generated usually by the modulator or is taken at the point in the interference field for which it can be assumed that $\varphi_S = 0$.

We shall show that the sensitivity of the birefringence measurement may be doubled by employing the half-shadow element. The idea is to split the modulated

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beam passing through the examined birefringent object into two parts. For one part of the beam the phase difference φ_S introduced by the examined birefringent object evokes a phase shift of the final beam intensity (2) by the value of $\Delta\Phi_1 = \varphi_S$. By employing the half-shadow element the phase shift of intensity of the other part of the beam becomes of opposite sign $\Delta\Phi_2 = -\varphi_S$. By measuring the relative phase shift of the signals corresponding to each of the two parts of the beam the sensitivity of the measurement becomes twice as much as it is in the case of the phase shift measurement with respect to the steady reference signal, i.e.

$$\Delta\Phi = \Delta\Phi_1 - \Delta\Phi_2 = 2\varphi_S \tag{4}$$

as compared to

$$\Delta\Phi = \varphi_S. \tag{5}$$

The schemes of the exemplifying measuring setups have been shown in Fig. 1. For the configuration of the inverse Senarmonte compensator (Fig. 1a) the function of

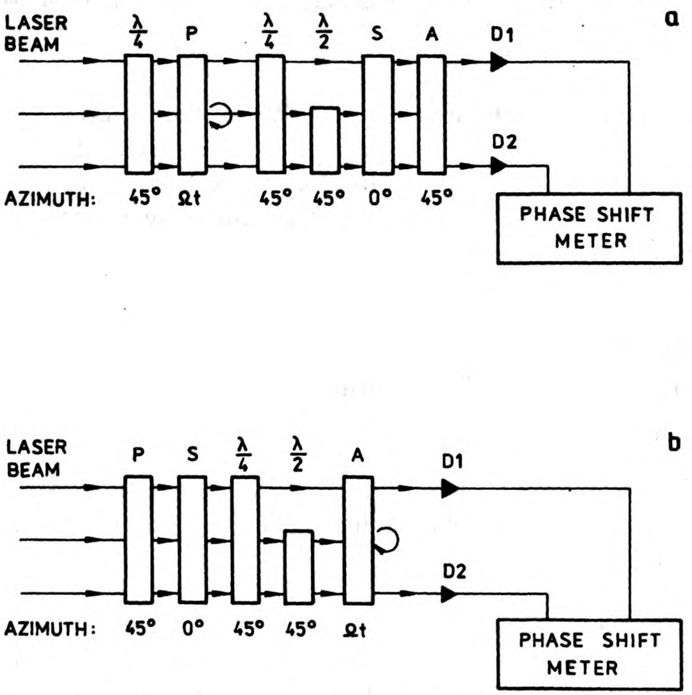

Fig. 1. Schemes of the measuring systems for birefringence determination with the use of the technique of the phase modulation with application of the half-shadow element. **a** — configuration of the Senarmonte inverse compensator, **b** — configuration of the Senarmonte direct compensator

particular elements of the system is as follows: the quarter-wave plate of the 45° azimuth in relation to the horizon changes the vertical polarization of the laser beam into a circular one. The polarizer P rotating (around the optical axis of the system) with a constant frequency plays the part of a modulator causing the spinning of the polarization azimuth. Next, the light falls onto the quarter-wave plate of 45° azimuth which transforms the spinning of the polarization azimuth into a linearly increasing change of the polarization state ellipticity for its steady azimuth being equal to 45°. Thus, the turning of the polarizer introduces the linear increment of the phase between the vertically and horizontally polarized components of the beam in the space behind the quarter-wave plate.

One part of the beam passes then through the half-shadow element which is a half-wave plate $\lambda/2$ of 45° azimuth with respect to the horizon while the other one falls immediately onto the examined birefringent sample S. The half-shadow element changes the handedness of the polarization ellipse into the opposite one with respect to that part of the beam which falls immediately on the examined sample. After having passed through the examined birefringent sample S of 0° azimuth an additional phase shift φ_S appears between the respective components of amplitude polarized vertically and horizontally, respectively. Next, both parts of the beam pass through the analyser A of 45° azimuth due to which an interference occurs between the vertically and horizontally polarized components of amplitude. Each of the beams falls subsequently on the corresponding intensity detectors D_1 and D_2.

Now, we are going to calculate how the intensities of both beams change in time. For this purpose the matrix description of Jones type [11] may be used while at the beginning the amplitudes of the beams falling on the detectors D_1 and D_2 will be determined

$$\hat{E}_1(t) = \hat{A}\hat{S}\hat{Q}\hat{P}(t)\hat{Q}\hat{E}_0, \tag{6a}$$

$$\hat{E}_2(t) = \hat{A}\hat{S}\hat{H}\hat{Q}\hat{P}(t)\hat{Q}\hat{E}_0 \tag{6b}$$

where the matrices $\hat{A}$, $\hat{S}$, $\hat{Q}$, $\hat{H}$, $\hat{P}(t)$ represent the particular elements of the measuring system such as analyser, sample under test, quarter-wave plate, half-wave plate and the rotating polarizer:

$$\hat{A} = \frac{1}{2}\begin{bmatrix} 1, & 1 \\ 1, & 1 \end{bmatrix}, \quad \hat{S} = \begin{bmatrix} e^{i\varphi_S/2}, & 0 \\ 0, & e^{-i\varphi_S/2} \end{bmatrix},$$

$$\hat{Q} = \frac{1}{2}\begin{bmatrix} 1, & i \\ i, & 1 \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} 0, & 1 \\ 1, & 0 \end{bmatrix}, \tag{7}$$

$$\hat{P}(t) = \begin{bmatrix} \cos^2 \Omega t, & \cos \Omega t \sin \Omega t \\ \cos \Omega t \sin t, & \sin^2 \Omega t \end{bmatrix},$$

and vector $\hat{E}$ represents the vertically polarized light beam entering the system

$$\hat{E}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{8}$$

After substituting (7) into the formulae (6) and performing the necessary calculations we obtain:

$$E_1(t) = \begin{bmatrix} ie^{i\varphi_S/2} + e^{-i(2\Omega t + \varphi_S/2)} \\ ie^{i\varphi_S/2} + e^{-i(2\Omega t + \varphi_S/2)} \end{bmatrix}, \tag{9a}$$

$$E_2(t) = \begin{bmatrix} ie^{-i\varphi_S/2} + e^{-i(2\Omega t - \varphi_S/2)} \\ ie^{-i\varphi_S/2} + e^{-i(2\Omega t - \varphi_S/2)} \end{bmatrix}. \tag{9b}$$

Knowing the amplitudes of both beams the light intensity incident on the first and the second photodetector may be easily calculated:

$$I_1(t) = I_0[1 - \sin(2\Omega t + \varphi_S)], \tag{10a}$$

$$I_2(t) = I_0[1 - \sin(2\Omega t - \varphi_S)]. \tag{10b}$$

Thus, the phase delay φ_S introduced by the examined birefringence sample causes the phase shift of the signals $I_1(t)$ and $I_2(t)$ by the same value but in opposite directions $\Delta\Phi_1 = \varphi_S$ and $\Delta\Phi_2 = -\varphi_S$. If the relative phase shift between the signals $I_1(t)$ and $I_2(t)$ is measured we obtain $\Delta\Phi = 2\varphi_S$, which means that the sensitivity of the measurement is twice as high as that in traditional measurement with respect to the constant reference signal. It may be easily shown that the same effect of sensitivity

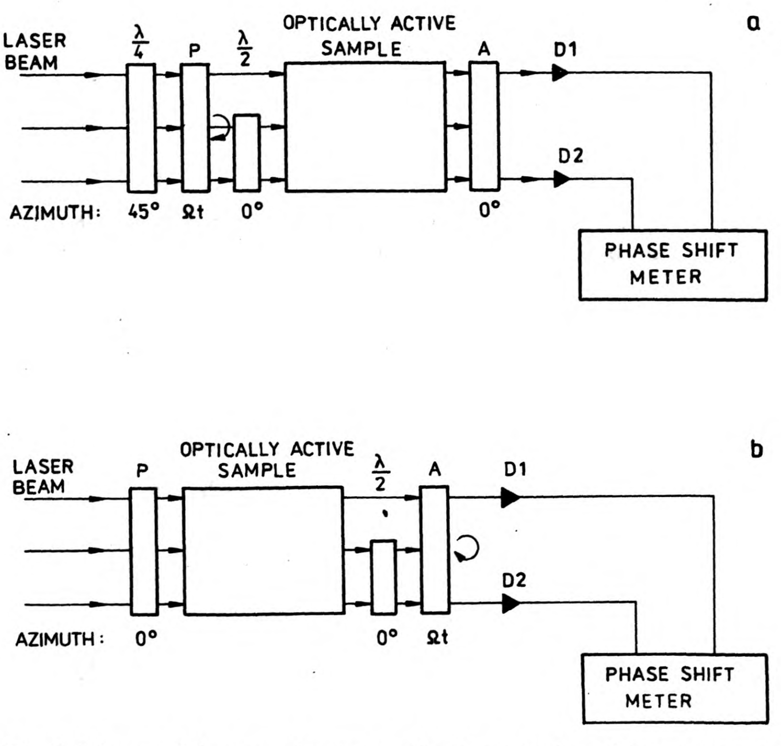

Fig. 2. Schemes of the measuring systems for determination of the rotation angle of polarization azimuth with the use of the technique of phase modulation with a half-shadow element. **a** – rotating polarizer as the modulating element, **b** – rotating analyser as the modulating element

enhancement may be achieved also in the case of the measuring system shown in Fig. 1**b**. The proposed modification may be also applied to increase the measurement sensitivity of the rotation angle of polarization azimuth in the optically active media. Exemplified systems are shown in Fig. 2. These are almost identical and the only difference is that in one case the modulating element is rotating polarizer (Fig. 2**a**) while in the other – an analyser (Fig. 2**b**).

The described way of doubling the measurement sensitivity suffers (as all half-shadow methods do) from some essential restriction of applicability. Namely, it requires the uniformity of the examined birefringent sample S, i.e., it is necessary that the phase shifts in these places of the sample, which are transilluminated by the one and the other parts of the beam, be identical $\varphi_{S_1} = \varphi_{S_2}$. If the uniformity conditions are not fulfilled, the phase shift measured by the proposed method will be equal to $(\varphi_{S_1}+\varphi_{S_2})/2$. The above restriction does not appear at all for many classes of objects like the fluids, for instance. In the remainder cases, the influence of the nonuniformity may be reduced by transilluminating the sample with a convergent beam (of small aperture angle) to make the beam cross-section on the surface of the examined sample as small as possible.

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Измерение двойного лучепреломления методом фазовой модуляции – двукратное повышение чувствительности благодаря применению полтеневого метода

Представлено способ двукратного повышения чувствительности измерения фазового сдвига внедряемого анизотропными средами. Повышение чувствительности было достигнуто благодаря применению полтеневого элемента в виде фазосдвигающей пластинки с фазовым сдвигом 180°.