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Optimization possibilities of decentration aberrations of an optical system with small decentration

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The conditions for optimization of the wave-aberrations of decentration, due to decentration of a chosen surface in an optical system are formulated for the separate second-order decentration aberrations. The possibilities of self-compensation of the decentration coma for a chosen surface in the system under consideration are discussed in detail. The indications of the optimized configuration of the optical system (with the elements which can be intensitive to decentration from the point of view of decentration coma) are analyzed.

1. Introduction

In my last works [1], [2] I have shown that the wave aberration of the optical system with small decentration can be described in the form of vector, with the aberration coefficients of the centered system introduced by Hopkins [3]. A useful design parameter for complex assessment of the image quality of a system with decentration – the variance of the aberration – was also given under Marechal approximation. The possibility of minimizing decentration sensitivity of the elements of the system was also discussed on the example of the coma of decentration [2]. The problem of balancing the aberrations of selected elements of a system from the point of view of the decentration sensitivity can be treated more generally for other decentration aberrations, as well. This aspect will be discussed in this paper.

2. Analysis of the optimization conditions

Equation (3) from [2] describes the wave aberration of the system (under the approximation of the primary aberrations and the first order decentration) with decentered k-th element. It can be rearranged to show the influence of the separate decentration aberrations more clearly in the form

$$\begin{aligned}\Phi(\bar{\varkappa}=0) = \sum_{i=1}^{n} \big[& w_{20i}\cdot\bar{\varrho}^2\cdot\bar{a}^2 + w_{40i}\cdot\bar{\varrho}^4 + (w_{11i}\cdot\bar{a}^2 + w_{31i}\cdot\bar{\varrho}^2)(\bar{\varrho}\cdot\bar{a}) \\ & + w_{22i}\cdot(\bar{\varrho}\cdot\bar{a})^2\big],\end{aligned} \tag{1.1}$$

and

$$\Phi(\varkappa \neq 0) = \Phi_c + \Phi_a + \Phi_{im} + \Phi_{d1} + \Phi_{d2} + \Phi_s \tag{1.2}$$

where the right hand side of Eq. (1.2) is given by the expressions:

a) coma of decentration Φ_c

$$\Phi_c = (\bar{\varrho} \cdot \bar{\varkappa}) \cdot \bar{\varrho}^2 [4 \cdot b_{40} \cdot p + b_{31} \cdot t + 4 \cdot d_{40} \cdot q + d_{31} \cdot t], \tag{2.1}$$

b) astigmatism of decentration Φ_a

$$\Phi_a = (\bar{\varrho} \cdot \bar{\varkappa}) \cdot (\bar{\varrho} \cdot \bar{a}) 2 [b_{31} \cdot p + b_{22} \cdot t + d_{31} \cdot q + d_{22} \cdot t], \tag{2.2}$$

c) image inclination Φ_{im}

$$\Phi_{im} = (\bar{a} \cdot \bar{\varkappa}) \cdot \bar{\varrho}^2 [b_{31} \cdot p + 2 \cdot b_{20} \cdot t + d_{31} \cdot q + 2 \cdot d_{20} \cdot t], \tag{2.3}$$

d) decentration distortion of the first kind Φ_{d1}

$$\Phi_{d1} = (\bar{\varrho} \cdot \bar{\varkappa}) \cdot \bar{a}^2 [2 \cdot b_{20} \cdot p + b_{31} \cdot t + 2 \cdot d_{20} \cdot q + d_{31} \cdot t], \tag{2.4}$$

e) decentration distortion of the second kind Φ_{d2}

$$\Phi_{d2} = (\bar{a} \cdot \bar{\varkappa}) \cdot (\bar{\varrho} \cdot \bar{a}) \cdot 2 [b_{22} \cdot p + b_{11} \cdot t + d_{22} \cdot q + d_{11} \cdot t], \tag{2.5}$$

f) effect of the transversal shift of the image point due to aberrations Φ_s

$$\Phi_s = (\bar{a} \cdot \bar{\varkappa}) \cdot \bar{a}^2 [b_{11} \cdot p + d_{11} \cdot q] \tag{2.6}$$

where:

$c_{mn} = \sum_{l=1}^{N} w_{mni}$ – sum of the coefficients of the primary aberration over the whole system,

$b_{mn} = w_{mnk}$ – coefficient of the aberration of the k-th element of the system,

$d_{mn} = \sum_{l=1}^{k-1} w_{mni}$ – sum of the aberration coefficients of the preceding part of the system (for the wavefront incident on k-th element of the system),

$\bar{\varrho}, \bar{a}$ – localization vectors of the points in the aperture and image plane, respectively (normalized to $\varrho_{max} = a_{max} = 1$),

$\bar{\varkappa}$ – inclination angle of the decentred, k-th surface of the system. It is given in the form of a vector for pointing out the possible azimuthal changes of his direction.

All normalization factors for the aperture and image heights and for the aperture and field magnifications, introduced by the k-th element of the system, are now included in the parameters p, q, t, for simplicity. They have different forms for spheres and planes. For a sphere with radius R they are expressed as follows:

$$p = -n \cdot \beta \cdot R \cdot \frac{1}{n' \cdot U' \cdot (h \cdot \beta' - y \cdot \alpha')}, \tag{3.1}$$

$$q = (n' \cdot \beta' - n \cdot \beta) \cdot R \cdot \frac{1}{n' \cdot U' \cdot (h \cdot \beta' - y \cdot \alpha')}, \tag{3.2}$$

$$t = (n' \cdot \alpha' - n \cdot \alpha) \cdot R \cdot \frac{1}{n' \cdot W' \cdot (h \cdot \beta' - y \cdot \alpha')}, \tag{3.3}$$

For a plane surface ($R = \alpha$) they become:

$$p = q = (n-n')\cdot y\cdot\frac{1}{n'\cdot U'\cdot(h\cdot\beta'-y\cdot\alpha')}, \tag{4.1}$$

$$t = (n-n')\cdot h\cdot\frac{1}{n'\cdot W\cdot(h\cdot\beta'-y\cdot\alpha')}, \tag{4.2}$$

where:

n, n' – refraction indexes before and after the surface under consideration,

R – radius of the surface under consideration,

h, α, α', y, β, β' – parameters of the paraxial and paraxial principal ray, respectively (on the k-th surface),

U' – aperture angle in the image space of the whole system,

W – object field angle of the whole system.

The index k is omitted in Eqs. (2) and (3) for simplicity.

It can be seen from Eqs. (2.1)–(2.5) that each aberration of decentration the of k-th surface is induced by two aberrations of the centered system. This is valid for both parts of each equation (2.1)–(2.5): the one containing the sum of the coefficients (with parameters given in Eq. (3) or (4)) of the wave aberration caused by the aberrations of the decentered surface itself (coeff. b_{mn}) and the one describing the aberration of the wavefront incident on the surface under consideration (coeff. d_{mn}). When the expressions in the square brackets are equal to 0 then the relevant aberration of decentration of the k-th surface is absent. The simple optimization conditions for reducing the sensitivity for decentration of the surface discussed can be formulated for each decentration aberration as follows:

a) compensation of the coma of decentration

$$b_{31}\cdot t = -4\cdot p\cdot b_{40}, \quad d_{31}\cdot t = -4\cdot q\cdot d_{40}; \tag{5.1}$$

b) compensation of the astigmatism of decentration

$$b_{22}\cdot t = -p\cdot b_{31}, \quad d_{22}\cdot t = -q\cdot d_{31}; \tag{5.2}$$

c) compensation of the image inclination

$$b_{20}\cdot t = -0.5\cdot p\cdot b_{31}, \quad d_{20}\cdot t = -0.5\cdot q\cdot d_{31}; \tag{5.3}$$

d) compensation of the first kind distortion of decentration

$$b_{11}\cdot t = -2\cdot p\cdot b_{20}, \quad d_{11}\cdot t = -2\cdot q\cdot d_{20}; \tag{5.4}$$

e) compensation of the second kind distortion of decentration

$$b_{11}\cdot t = -p\cdot b_{22}, \quad d_{11}\cdot t = -q\cdot d_{22}. \tag{5.5}$$

It can be noted that in order to have the conditions (5.2) and (5.3), as well as (5.4) and (5.5), fulfilled, it is necessary (but not sufficient, of course) that $b_{20} = 0.5\cdot b_{22}$ and $d_{20} = 0.5\cdot d_{22}$.

If the conditions (5.1)–(5.5) are fulfilled, the variance of the aberration of the optical system with decentration described by Eq. (6) of [2] is equal to 0, and the image quality cannot be influenced by any decentration of the k-th element discussed. This surface is also intensitive to decentration (in the approximation of primary aberrations and the first order of decentration). In any case the minimization of the values in the square brackets on the right hand side of Eqs. (2.1)–(2.5) leads to the reduction of the sensitivity of the surface under consideration to decentration from the point of view of separate decentration aberrations, respectively. The influence of the decentration of the k-th element on the final image quality deterioration can be estimated at any stage of the optimization process with the help of the relations qiven in [2]. As was mentioned in Chapter 3 of [2], the partial balancing for the selected elements of the system seems possible in practice. For the manufacturing the special importance is given to the minimization of the coma of decentration. More detailed analysis of this case will be given in the next Section.

3. Optimization of the coma of decentration

Let us analyse the conditions of Eqs. (5.1) for the compensation of the decentration coma, induced by inclination of the k-th surface of the system. Discussion of the first condition, describing the compensation of decentration coma, induced by the self-aberrations of the decentred k-th surface of the system, seems to be relatively simple. The wave aberration coefficiens b_{40} and b_{31} can be expressed by means of the Seidel sums S_{I} and S_{II} as

$$b_{40} = 1/8 \cdot S_{\mathrm{I}} \cdot U'^4, \qquad b_{31} = -1/2 \cdot S_{\mathrm{II}} \cdot U'^3 \cdot W \tag{6}$$

where:

- U' – maximal aperture angle of the whole optical system in the image space,
- W – maximal field angle in the object space of the whole system,
- S_{I}, S_{II} – Seidel sums for the spherical aberration and coma of the centered system, which can be expressed (see [4], for example) as

$$S_{\mathrm{I}} = h \cdot P, \qquad S_{\mathrm{II}} = h \cdot P \cdot \frac{\beta' - \beta}{\alpha' - \alpha}, \tag{7}$$

with

$$P = \left(\frac{\alpha' - \alpha}{\dfrac{1}{n'} - \dfrac{1}{n}}\right) \cdot \left(\frac{\alpha'}{n'} - \frac{\alpha}{n}\right).$$

For h, α, β, α', β' see comments to Eqs. (3) and (4).

Taking into account Eqs. (7), (3) and (4) we can write the first condition from (5.1) in the form

$$\frac{-(\beta' - \beta)}{n \cdot \beta} = \frac{\alpha' - \alpha}{n' \cdot \alpha' - n \cdot \alpha} \tag{8}$$

for the sphere. It can be rearranged taking into account that the object magnification M_o of the decentered k-th surface of the system

$$M_o = \frac{n \cdot \alpha}{n' \cdot \alpha'},$$

and the pupil magnification M_A

$$M_A = \frac{n \cdot \beta}{n' \cdot \beta'}.$$

We now obtain from Eq. (8)

$$M_o = M_A \cdot \frac{n-n'}{n} + 1. \tag{9}$$

If we want to find the position of the entrance pupil which can fulfil the condition (8) for a given radius of curvature R and a given distance s of the object point in the object space of the surface under consideration, we can write after some simple transformations

$$z = \frac{R}{n'-n} \cdot \left(n' - 2 \cdot n + n \cdot \frac{R}{s} \right). \tag{10}$$

In such a configuration, for the surface of radius R, the object distance s and entrance pupil distance z given by Eq. (10), the decentration coma, generated by the primary spherical aberration and coma of this surface, is self-compensated.

For the plane surface, taking into account Eq. (4) instead of Eq. (3), we can write Eq. (8) in the form

$$\frac{\beta' - \beta}{\alpha' - \alpha} = \frac{y}{h}. \tag{11}$$

It can be readily proved that the condition Eq. (11) cannot be fulfilled in a real system. Since, for the plane surface

$$\alpha' = \frac{n}{n'} \cdot \alpha \quad \text{and} \quad \beta' = \frac{n}{n'} \cdot \beta$$

we get

$$z = \frac{y}{\beta} = \frac{h}{\alpha} = s \tag{12}$$

where s and z are the respective object and pupil distances, in the object space of the plane surface discussed. It is evident that the configuration described by Eq. (12) is not possible in a real system.

More troublesome seems to be the analysis of the second expression of Eq. (5.1), which describes the condition for compensation of the decentration coma generated by the aberrations of the wavefront incident on the surface under consideration. As the sums over the preceeding part of the system are very complicated, the expressions for d_{40} and d_{31} are difficult to treat analytically. We can write

$$\frac{d_{40}}{d_{31}} = -4\cdot\frac{W}{U'}\cdot\frac{\sum S_{\mathrm{II}}}{\sum S_{\mathrm{I}}},$$

and, from Eq. (5.1),

$$-4\cdot\frac{q}{t} = 4\frac{W\cdot y_k}{U'\cdot h_k}$$

where h_k, y_k are the heights of the paraxial and paraxial principal ray on the k-th surface under consideration. The second condition from Eq. (5.1) can now be expressed as

$$\frac{y_k}{h_k} = \frac{\sum S_{\mathrm{II}}}{\sum S_{\mathrm{I}}}. \tag{13}$$

The self compensation of the decentration coma generated by the spherical aberration and coma of the wavefront incident on the k-th surface, can also be reached by the appropriate choice of the position of the surface discussed along the optical axis of the system. This is independent of the radius of curvature of this surface but can be specified by knowing the primary aberrations of the wavefront incident on the surface under consideration.

4. Summary

The conditions for optimization (self-compensation) of the decentration aberrations generated by the primary aberrations of the centered system are formulated for all the second order decentration aberrations. The possibilities of self-compensation of the decentration coma for a chosen surface in the system under consideration are discussed in detail. The analysis show the possibility of designing the optical elements which can be intensitive to decentration in the aberrational sense for the axial object point under the approximation of primary aberrations and the first order of decentration.

References

[1] RAFAŁOWSKI M., Opt. Appl. **17** (1987), 3–12.
[2] RAFAŁOWSKI M., Opt. Appl. **17** (1987), 333–339.
[3] HOPKINS H., *Wave Theory of Aberrations*, Clarendon Press, Oxford 1950.
[4] SLUSAREV G. G., *Metody raschota opticheskikh sistem*, Ed. Mashinostr. 1969, pp. 87—90.

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Возможности оптимизации аберрации децентрировки в оптической системе с малой децентрировкой

Представлены условия оптимизации волновой аберрации децентрировки для одного компонента оптической системы. Они формулированы отдельно для одиночных аберраций децентрировки, внесенных аберрациями третьего порядка центрированной оптической системы. Подробно проанализированы условия для компенсации комы децентрировки. Указаны возможности оптимальной конструкции оптической системы (кривизны и размещения оптических поверхностей), в которой избранные поверхности компенсированы с точки зрения комы децентрировки.