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# Focusing and imaging of annular beams of laser radiation*

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A simple model of annular laser beam transformations in optical systems has been described in terms of ABCD matrices. This model generalizes the Gaussian beam model as it comprises both the beams of annular intensity distribution and the case of partial coherence. The focusing properties of the annular beams have been analysed for different relative apertures. The influence of the beam geometry on the intensity distribution in the vicinity of caustics has been demonstrated. A symmetry of distribution for very small relative apertures and the appearance of an image of the ring in the plane conjugate to the exit plane of the system have been observed. The applicability of the model to describe the diffraction-free beams has been also shown.

## 1. Introduction

Optical systems of annular apertures have been known since many years [1]–[3] to mention astronomic Cassegrain telescopes, for instance. Such systems have been employed also in radar and microwave techniques. For an aplanatic system of annular aperture the field distribution in the focal plane or the optical transfer function may be calculated analytically [4], [5].

Since the discovery of the laser the interest in such systems has grown significantly. For instance, in the high power laser, mirrors of annular aperture were applied in order to lead the energy out of the resonator which results in the annular intensity distribution in the beam generated therefrom [6]. The whole series of elements transforming the annular beam into a Gaussian one and vice versa have been elaborated. They include reflecting and refracting axicones, waxicones, and so on, [7], [8]. The waxicones of definite curvature of the reflecting surfaces [9] transform the annular beam into a Gaussian one without any significant increase of

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the intensity on the optical axis, which was difficult to achieve when using the axicones. The laser setups working with an annular beam are exploited in the laser processing of the various materials [10], in microsurgery [11], and so on.

In the course of the last 20 years many works devoted to the annular beams and their caustics have been published [12]–[14], most of them being based on the vector diffraction theory and the Richards-Wolf method given in [15], [16].

## 2. Method of calculation

In many cases, especially for rather dark systems of *f*-number higher than 5, the scalar theory approximation appears to be sufficient. Such systems may be well corrected without greater problems involved by reducing the wave aberration down to tenth fraction of the wavelength $\lambda_0$. In the optical systems focusing the laser beams the vigneting of the light beam by any optical elements is usually also negligible. Thus, the application of the generalized Fresnel transform [17], describing the transformation of the field $E_1$ in the input plane of the optical system, described by the ABCD transition matrix, into a field $E_2$, defined in the output plane of the system (see Fig. 1), seems to be justified. This transformation written in the cylindric

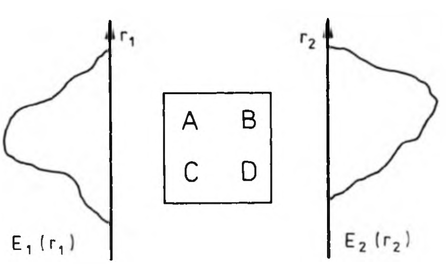


Fig. 1. Transformation of the field in an optical system described by the ABCD matrix

coordinate system takes the following form:

$$E_2(r_2, \Phi_2) = \frac{i}{\lambda_0 \mathrm{B}} \int_0^{\infty} \int_0^{2\pi} E_1(r_1, \Phi_1) \exp[-ik\varphi(r_1, r_2, \Phi_1, \Phi_2)] r_1 dr_1 d\Phi_1 \tag{1}$$

where:

$$\varphi(r_1, r_2, \Phi_1, \Phi_2) = L_{\mathrm{opt}} + \frac{1}{2\mathrm{B}}[Ar_1^2 - 2r_1 r_2 \cos(\Phi_1 - \Phi_2) + \mathrm{D}r_2^2],$$

$L_{\mathrm{opt}}$ – optical path along the $z$-axis, A, B, C, D – parameters of the transition matrix of the optical system.

In this case the field distribution is usually represented in terms of the eigenfunctions of this transformation, i.e., the Laguerre-Gauss functions.

In our opinion, another set of functions [18]–[20] may be used to analyse the annular fields which does not fulfil the orthogonality relations but offer some other advantageous properties. Let us consider the propagation in the optical system, described by the ABCD matrix, of an axial symmetrical laser beam given by the following formula:

$$G_{k,1}(t_1) = t_1^{k-1}\exp(-t_1) \tag{2}$$

where:

$$t_1 = \alpha r_1^2, \ \alpha = 1/w_1^2 \tag{3}$$

($w_1$ – radius of the Gaussian beam). It may be shown that the field in the output plane of the optical system described by the ABCD matrix is given by the formula

$$G_{k,2}(t_2) = \frac{H_0}{(1-ih)^k} L_{k-1}(t_2)\exp(-t_2) \tag{4}$$

where:

$$\begin{aligned} t_2 &= \frac{p}{1-ih}, \ h = AN_{\mathrm{F}}, \ p = \left[\frac{N_{\mathrm{F}} r_2}{w_1}\right]^2, \\ H_0 &= i^{-1} N_{\mathrm{F}} \exp[iN_{\mathrm{F}} \mathrm{D} r_2^2/w_1^2], \ N_{\mathrm{F}} = \frac{\pi w_1^2}{\lambda_0 \mathrm{B}}, \end{aligned} \tag{5}$$

$L_k$ is the Laguerre polynomial described by the following expression:

$$\begin{aligned} L_{k+1}(t) &= [2k+1-t]L_k(t) - kL_{k-1}(t), \\ L_0 &= 1, \ L_1 = 1-t. \end{aligned} \tag{6}$$

For $k = 1$, the formulae for Gaussian beam transformation are similar to the case of the L–G function of the zero order. The function $G_{k,1}$ in the input plane represents the annular distribution of intensity which changes along the propagation path, in contrast to the L–G function which preserves the same intensity distribution across the whole region while its phase changes. The intensity of the beam $I_k$ in the entrance plane of the optical system is given by the following formula:

$$I_k = |G_{k,1}|^2 = t^{2k-2}\exp(-2t) \tag{7}$$

where: $t = (r_1/w_k)$.

For $t = k-1$, the first derivative of intensity $dI_k/dt = 0$. If the parameter $w_k$ for each function $G_{k,1}$ is equal to

$$w_k = \frac{w_1}{\sqrt{k-1}}, \ \text{for} \ k > 1, \tag{8}$$

the intensity maximum for each function $G_{k,1}$ occurs at $r = w_1$. The function $G_{1,1}$ describes the usual Gaussian beam, while the function $G_{k,1}$ describes the annular beams of the same diameter of maximum $d_{\max} = 2w_1$ and different thicknesses of the ring, as shown in Fig. 2. From the disappearance condition for the

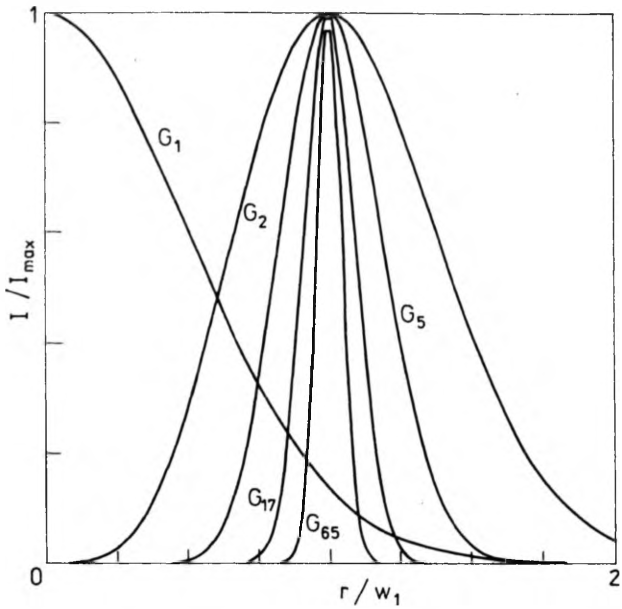


Fig. 2. Intensity distribution of the beams $G_k$ in the input plane of the optical system

second derivative of intensity we may determine the thickness of the ring. If

$$d^2 I_k/dt^2 = 0, \tag{9}$$

$t$ fulfils the following equation:

$$2t^2 - 4t(k-1) + (k-1)(2k-3) = 0. \tag{10}$$

then

$$\Delta t = t_1 - t_2 = \sqrt{2(k-1)}. \tag{11}$$

Let us introduce the quantity $g_k$, defined as a ratio of the radius $w_1$, to the thickness of the ring

$$g_k = \frac{w_1}{\Delta r} = \sqrt{2(k-1)}. \tag{12}$$

As it follows from the above analysis, the thickness of the ring is inversely proportional to $(k-1)^{1/2}$. The functions $G_{k,1}$ may well approximate the annular beams of different diameters and thicknesses of the ring.

The relations transforming the mutual coherence function in an optical system may be derived in an analogical way [19], [20]. In this paper, we restrict our interest to the examination of the properties of the totally coherent annular beams.

## 3. Focusing of annular beams

The $f$-number $F^{\#} = f/2w_1$ is decisive so far as the qualitative description of caustic is concerned. However, as it follows from the calculations, for different $F^{\#}$ not greater than 20–30, the field distributions in the caustic are similar, if a new system of dimensionless coordinates is introduced. The centre of the coordinate system is located in the geometrical focus, while the new dimensionless coordinates $z^{\#}$ and $r^{\#}$ are given by the following expressions:

$$z^{\#} = \frac{z}{\lambda F^{\#2}}, \quad r^{\#} = \frac{r}{\lambda F^{\#}}. \tag{13}$$

The waist of the entrance beam is located in the principal of lens plane of the focal length $f$. In the coordinate system $(z^{\#}, r^{\#})$, the image of the caustics for the given input function $G_{k,1}$ is symmetric with respect to the focal plane and identical for different $F^{\#}$. Thus, it suffices to analyse the field distribution in the caustic only within the rectangular

$$z^{\#} \in (0, z^{\#}_{\max}), \quad r^{\#} \in (0, r^{\#}_{\max}). \tag{14}$$

With the increase of the parameter $g_k$ the caustic becomes longer and the contribution of the power propagating in higher orders of diffraction increases. This is consistent with the intuition. The narrower is the ring, the smaller is the part of the power propagating in the zero order of diffraction and the richer the diffraction image in the focal plane. The results of calculations of intensity distribution along the optical axis and in the focal plane for few selected functions $G_k$ are shown in Figs. 3 and 4.

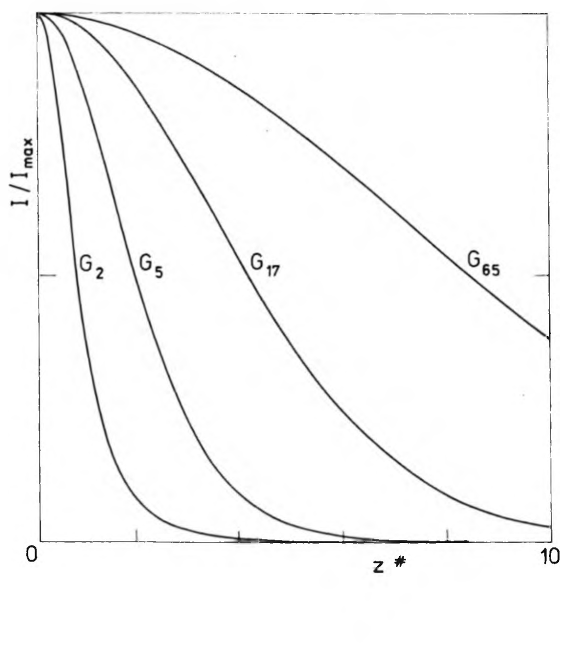


Fig. 3. Intensity distribution along the optical axis for different beams $G_k$ and $F^{\#} \leqslant 25$

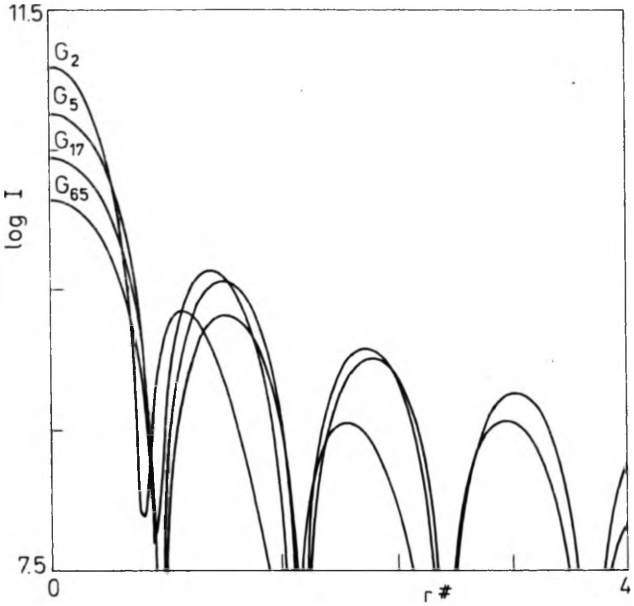


Fig. 4. Intensity distribution in the logarithmic scale in the focal plane for various $G_k$

The maximum intensity of the focus diminishes with the increase of $g_k$. The narrowest and shortest caustic is produced by the Gaussian beam $G_1$ of the diameter in the input plane equal to that of the ring $2w_1$. Hence, from the view point of both the maximization of the intensity in the focus and the minimization of the caustic dimensions, a well known conclusion that the most advantageous field distribution in the input plane of the optical system is that produced by the Gaussian beam.

## 4. Focusing and imaging of the annular beams for large *f*-number

For large *f*-numbers the field distributions in the caustics cease to be symmetric with respect to the geometric focus. For the laser beams, this phenomenon has been known and examined since many years [21]–[23].

In the case when the waist of the annular beam is distant from the principal plane of the lens (see Fig. 5), the image of the beam from the waist plane appears additionally in the image plane. Due to superposition of the two effects the field distributions becomes very complicated which offers some interesting possibilities of modelling the caustic. In the case of not too great $F^{\#}$ although the plane of the beam waist is remote from the lens by the distance $R_{\mathrm{inp}}$, the field distribution in caustic is close to that of the case discussed in Sect. 3, i.e., for $R_{\mathrm{inp}} = 0$. For great $F^{\#}$, however, the situation is quite different.

As it is shown in Figures 6 and 7, the field distributions along the optical axis loose their symmetry; besides a distinct minimum is observed at the points

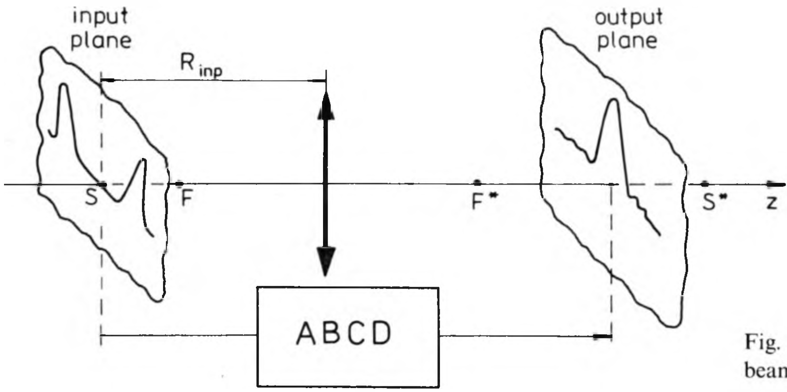


Fig. 5. Scheme of the laser beam imaging

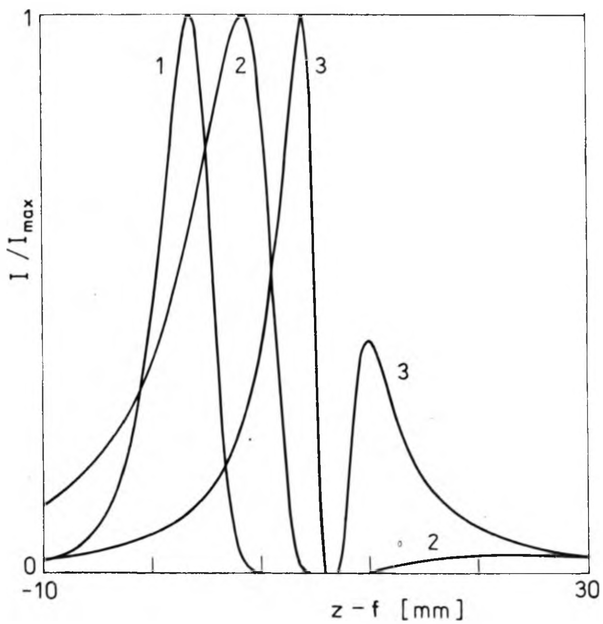


Fig. 6. Intensity distribution of the $G_5$ beam on the optical axis for different $f$-numbers and the waist distant from the lens by $R_{inp} = 1$ m and the focal length $f = 100$ mm (1 – $F^{\#} = 50$, 2 – $F^{\#} = 100$, 3 – $F^{\#} = 200$)

corresponding to the position of the image. With the increase of $g_k$ the caustic becomes more localized around the image point. A local maximum is observed also behind the image point. For the large $k$ numbers an absolute maximum of intensity may be found behind the image point.

The intensity distributions in the vicinity of the image plane are shown in Figs. 8 and 9.

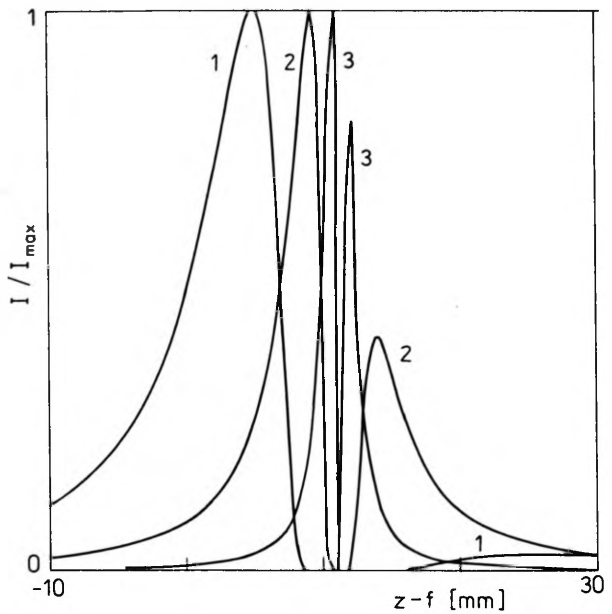


Fig. 7. The same as in Fig. 6, but for $G_{65}$ beam

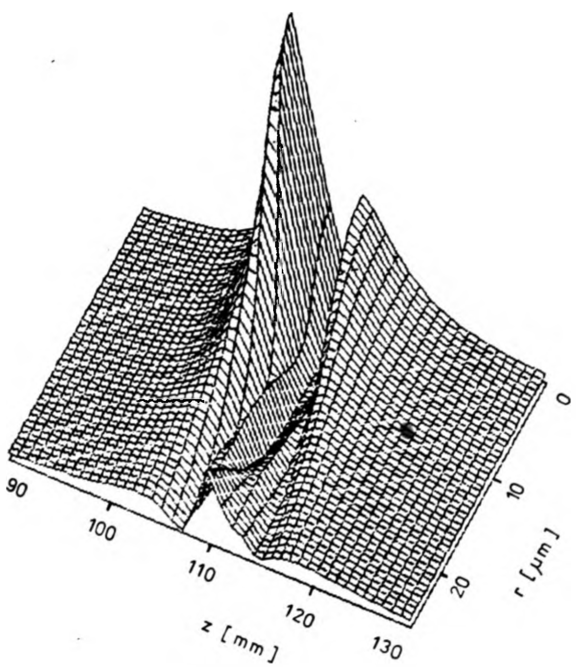


Fig. 8. Intensity distribution of the $G_5$ beam in the caustics for $F^{\#} = 200$, $R_{inp} = 1$ m, $f = 100$ mm, $\lambda_0 = 0.5$ μm, $z \in (90, 130)$ mm, $r \in (0, 25)$ μm

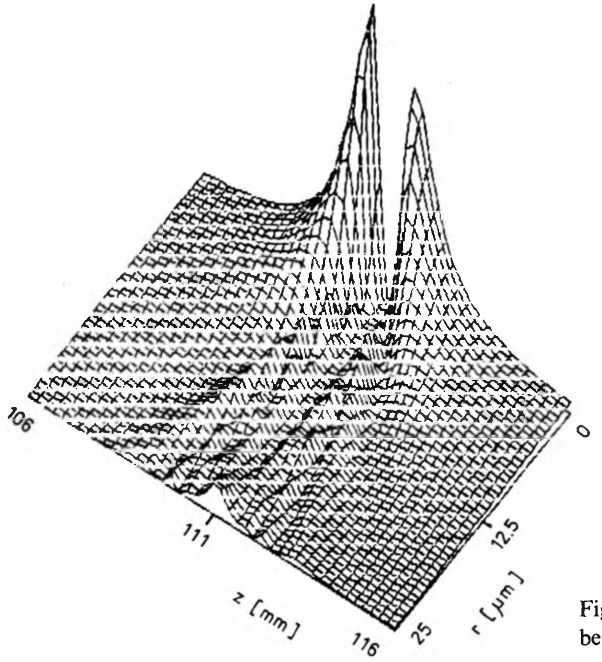


Fig. 9. The same as in Fig. 8, but for $G_{65}$ beam ($z \in (106, 116)$mm)

## 5. "Diffraction-free" beams

In the case when $R_{inp} = f$, i.e., when the waist of the laser beam is positioned in the front focal plane of the lens, the beam becomes collimated after having passed through the lens. When propagating in the free space, the Gaussian beam suffers from divergence $\Delta\theta_{dif} = \lambda_0/\pi w_0$. It appears that there exist such field distributions which propagate in the free space almost without diffraction. These are, as it was shown by DURNIN [24], the fields of amplitude distribution described by the Bessel function $J_0$. The Bessel function has no convergent norm, thus it represents a nonphysical case since such a field would carry an unlimited power. GORI and GUATARI [25] have proposed another field described by the product of Bessel and Gauss functions. Such fields, named by them the Bessel-Gauss functions, propagate in the free space with a significantly less convergence than the ordinary Gaussian beams, while in the far zone they show an annular distribution of intensity. DURNIN et al. [26] examined the imaging of the annular beams of waist located at the focal length of the lens. It turned out that such beams, in the case of high $f$-numbers and very low thickness of the ring, propagate also almost without suffering from diffraction.

Thus, it may be expected that the set of functions $G_k$ may also well approximate the diffraction-free beams. Let us examine the field distributions behind the lens for

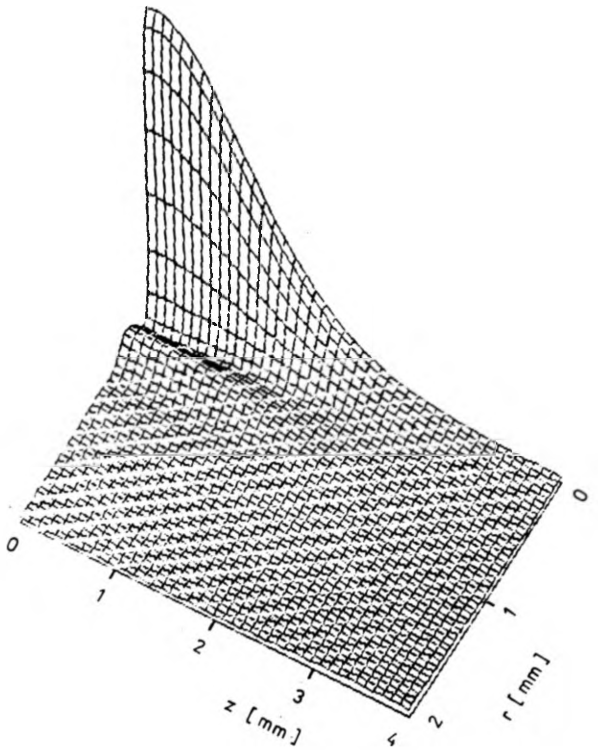


Fig. 10. Intensity distribution of the $G_5$ beam behind the lens for a system characterized by: $f = 100$ mm, $\lambda_0 = 0.5\,\mu$m, $R_{inp} = 100$ mm, $w_1 = 50$ µm, $r \in (0, 2)$ mm, $z \in (0, 4000)$ mm

different annular beams, the waists of which are located in the focal plane. In accordance with the expectations, it turns out that for very high $f$-numbers of order of 1000 the zero order of diffraction propagates almost without diffraction as $g_k$ increases. In Figures 10–12, the field distributions behind the lens are shown for different beam $G_k$.

As it is seen in the above figures, the intensity on the optical axis diminishes slower and slower as $g_k$ increases, while the width of the zero order remains practically unchanged. With the increase of $z$ the contribution of the power irradiated into the higher diffraction orders grows up. The field distribution in the cross-section, but as it is well known, it is described by the Laguerre polynomial multiplied by the Gauss function. The results of calculations indicate a high usefulness of the set of functions $G_k$ to describe the different phenomena connected with the annular beam propagation.

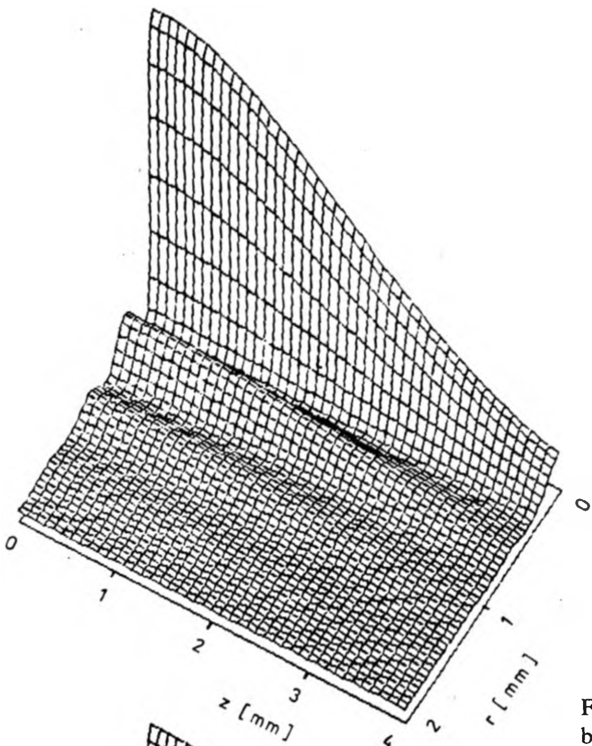


Fig. 11. The same as in Fig. 10, but for $G_{17}$ beam

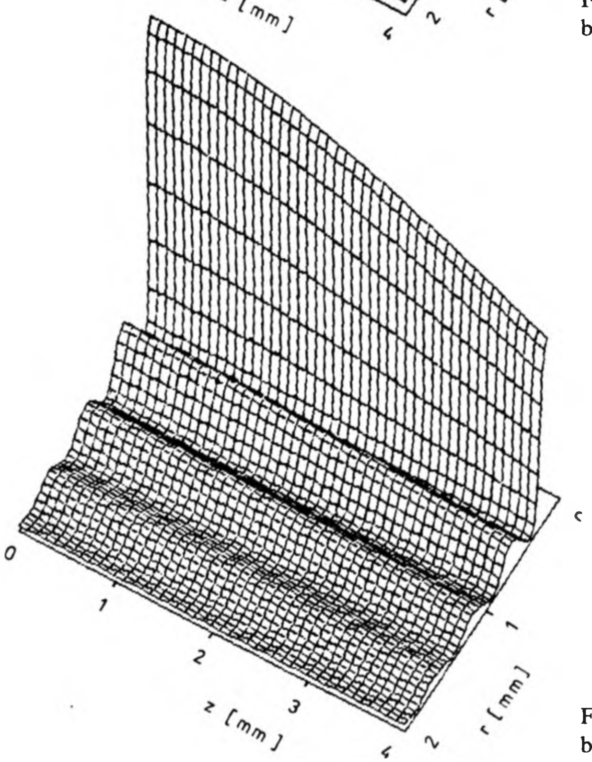


Fig. 12. The same as in Fig. 10, but for $G_{65}$ beam

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### Фокусировка и изображение кольцевых пучков лазерного излучения

Разработана простая модель трансформации кольцевых лазерных пучков в оптических системах, описанных матрицей ABCD, которая является расширением модели гауссового пучка. Проанализирована фокусировка кольцевых пучков для разных относительных зрачков. Представлено влияние геометрии пучка на распределение интенсивности в окрестности фокуса. Наблюдались асимметрия распределения для очень маленьких относительных зрачков и восстановление кольцевого распределения в плоскости изображения. Кроме того показана возможность применения модели для случая „бездифракционных" пучков.