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Cell-connecting in detour phase computer-generated hologram*

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Cell-connecting approach to detour phase methods of CGH encoding is described as a way of eliminating vertically diffracted false images. Practical realization and experimental examination of this approach to two-aperture Lohmann method is presented. Other promising properties of cell-connecting are stated.

1. Introduction

Detour phase methods of encoding data into CGH, first described by Brown and Lohmann in their fundamental work [1], are simple in use and possess high diffraction efficiency; hence they are commonly used in optical laboratories. There are different advantages and disadvantages of the various detour phase methods, but one disadvantage is common to all of them: there are many different images in observation plane, but only one of them is true and all the others, more or less differing from the true one, are false. This is caused by a finite size of elementary cells in hologram plane and cannot be avoided for the X-direction images, because of the main detour phase encoding principle, while for Y-direction images the limitations are not so serious. In this work we describe a simple modification of one of the Lohmann's data encoding methods consisting in vertical connecting of neighbouring cells. This modification implies practically a total disappearance of the vertical false images and possesses further advantages over the other methods.

2. Detour phase methods and cell connecting

Brown and Lohmann proposed three methods of data encoding into CGH; shown schematically in Fig. 1. In the first method, most often used in practice, the amplitude A is encoded by the aperture height (Fig. 1**a**), hence the vertically neighbouring cells cannot be connected. The other two methods enable vertical cell-connecting, since

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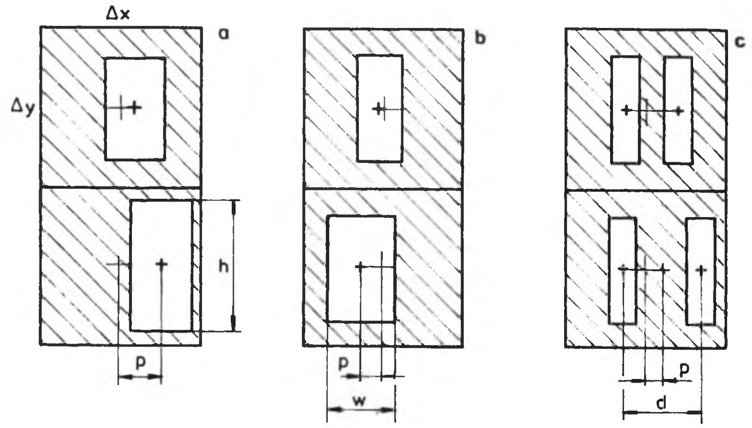


Fig. 1. Lohmann's detour phase methods of CGH encoding: **a** — $h = (A/A_{max})\Delta x$, $p = (\text{phase}/2\pi)\Delta y$ (classical Lohmann method), **b** — $w = \sin[(A/A_{max})(\pi/2)]\Delta y/2$, $p = (\text{phase}/2\pi)\Delta y$, **c** — $d = \cos[(A/A_{max}) \times(\pi/2)]\Delta y/2$, $p = (\text{phase}/2\pi)\Delta y$ (two-aperture Lohmann method)

the amplitude is encoded either by the aperture width or by the distance between two apertures in one cell (Figs. 1**b**, **c**).

Other principles of detour phase methods [2], [3] of data encoding do not allow cell-connecting, for nearly the same reason as in the classical Lohmann method.

3. Application of cell-connecting in two-aperture Lohmann method

There are some ways to obtain continuous lines rather than separate apertures in Y-direction on CGH. The most obvious one is to decrease cell height and increase the number of sampling points in Y-direction (Fig. 2**a**), however, it would very largely increase the computation time (N times more points $\rightarrow$ $N \times \log N$ times increase of computation time of Fourier hologram) and the size of graphical data. Another way of connecting cells is to compute intermediate points using any kind of interpolation, but this method also largely increases graphical data size and computation time, the latter, however, not to such an extent as the previous method.

In our opinion, the most promising way of cell-connecting is to use a plotter (or other plotting device) as interpolating machine (Fig. 2**b**). The computed centres of apertures are vertically connected by straight lines, so the graphical data size is smaller than in all three Lohmann's methods. This "plotter interpolation" seems to be a normal linear interpolation, but it is not, for the distance between the apertures is not a linear function of amplitude encoded. Only phase interpolation is linear. They way in which the amplitude is interpolated can be shown as follows:

$$A_{\text{between}}(y) = \cos\left(\frac{\delta x_{\text{down}} + y\cdot(\delta x_{\text{up}} - \delta x_{\text{down}})}{\Delta y}\right) \tag{1}$$

where: $x_{\text{up, down}} = \dfrac{\Delta x}{\pi}\arccos(A_{\text{up, down}})$,

$A_{\text{up, down}}$ — amplitudes computed for upper and lower cells.

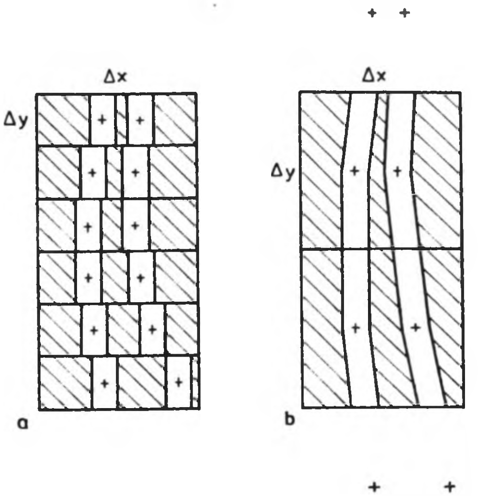


Fig. 2. Ways of cell-connecting: **a** – decrease of cell height and increase of number of cells, **b** – "plotter interpolation"

This manner of interpolation is difficult as far as an exact analysis is concerned, but in the case of small differences between A_{up} and A_{down} (slowly changing function describing wavefront) its properties resemble linear interpolation and the whole wavefront encoded into CGH can be considered as being sampled in X- and continuous (!) in Y-directions. Theoretical and practical importance of this fact might be significant, but it needs further studies.

4. Experimental

4.1. Theory

In this chapter we compare cell-connecting modification of two-aperture Lohmann method (Fig. 1**c**) described in the previous chapter with the classical Lohmann method (Fig. 1**a**), the simplest and the one most often used for data encoding into CGH produced by computer driven plotter. Both methods were applied to produce holographical differentiating spatial filters [4]. The wavefront encoded on these filters is represented by a complex function

$$Z(u, v) = \mathrm{const}\,(u+iv). \tag{2}$$

The amplitude of this function varies linearly from 0 in the centre to 1 on the filter edge, and the phase varies also linearly from 0 to 2π around the filter, thus it is suitable for practical testing of encoding methods. The accuracy of work of such a filter can be easily checked. A radially symmetrical object with transmittance

$$T(r) = a(b+\cos(cr)) \tag{3}$$

was prepared for examination of the influence of anisotropic way of encoding on filter operation (Fig. 3). The output intensity should be the square module of the first

Fig. 3. Test object

derivative of function T

$$I_{\mathrm{out}}(x, y) = \left|\frac{\partial}{\partial x} T(x, y) + i\frac{\partial}{\partial y} T(x, y)\right|^2 \tag{4}$$

and in the case of radial symmetry of $T(x, y)$

$$I_{\mathrm{out}}(x, y) = \left|\frac{\partial}{\partial r} T(r)\right|^2 \tag{5}$$

(this is the benefit of using this filter wavefront!). With our object (3) we should obtain

$$I_{\mathrm{out}}(r) = (ac)^2 \sin^2(cr). \tag{6}$$

4.2. Practice

The filters were produced by a plotter Digigraf A1008 in the way ensuring the shortest plotting time with the best use of the cell surface (Figs. 4**a**, **b**). The filters 60×60 cm large containing 100×100 square cells were reproduced photographically on Agfa 10E56 holoplates to 4×4 mm size. All the filters were of two kinds: black-and-white and bleached. The filters with connected cells were made in two versions: a normal one and with intercepted centre Fig. 5**c**) for reducing the noise caused by diffraction of the part of incident light undisturbed by the object and focused on the centre of the filter. All filters are shown in Fig. 5.

Because of the form (2) of filter transmission $Z(u, v)$ a certain weakness of classical Lohmann method (limitation of the minimal amplitude $A_{\min}$ encoded by plotter) is clearly shown.

$$A_{\min} = \frac{\text{pen width}}{\text{cell size}}. \tag{7}$$

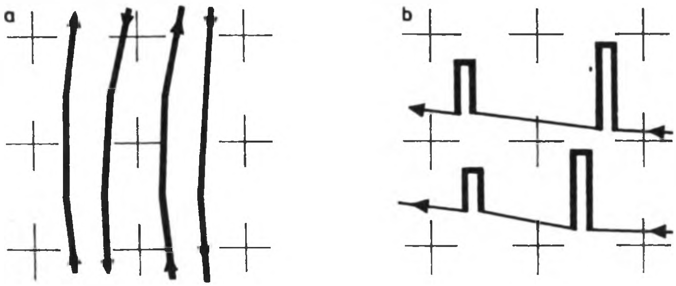


Fig. 4. Ways of pen up (thin line) and down (thick line) in producing hologram encoded by: **a** – two-aperture Lohmann method with cell-connecting, **b** – classical Lohmann method

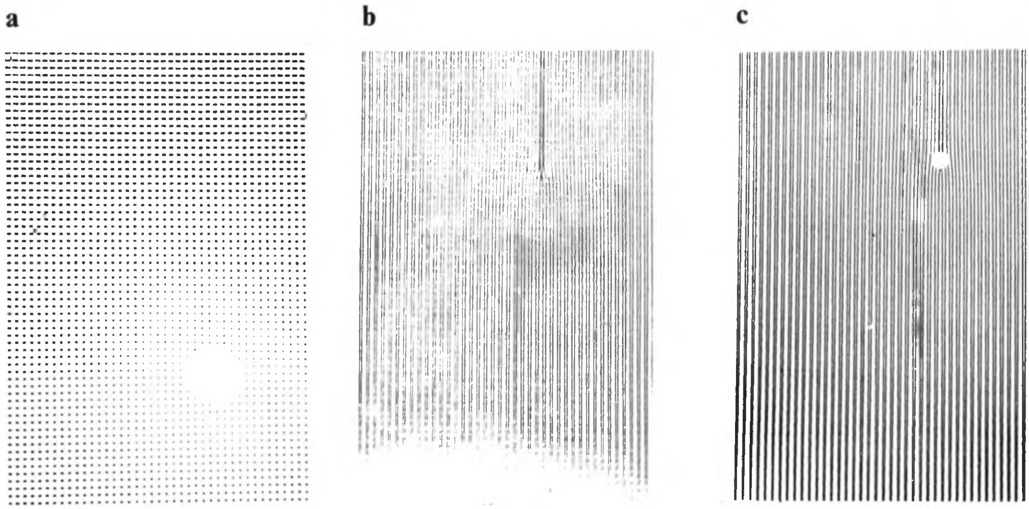


Fig. 5. Filters' maps: **a** – classical Lohmann method, **b** – two-aperture Lohmann method with cell connecting, **c** – as above with intercepted centre

The empty central part of the filter (Fig. 5**a**) corresponds to the complex numbers of amplitudes too small to be encoded. This limitation does not occur in two-aperture method, in which, however, the limitation concerns the maximum amplitude $A_{\max}$, causing a decrease of diffraction efficiency

$$A_{\max} = \cos\left(\pi \frac{\text{pen width}}{\text{cell size}}\right). \tag{8}$$

Another shortcoming of classical Lohmann method, not present in cell-connecting modification, is an error of encoding caused by irregular shape of the aperture at the site where the pen touches the paper.

A simple spatial filtering system (Fig. 6) was used for examination of filters. The results are shown in Figs. 7 and 8. They concern black-and-white filters, but the only

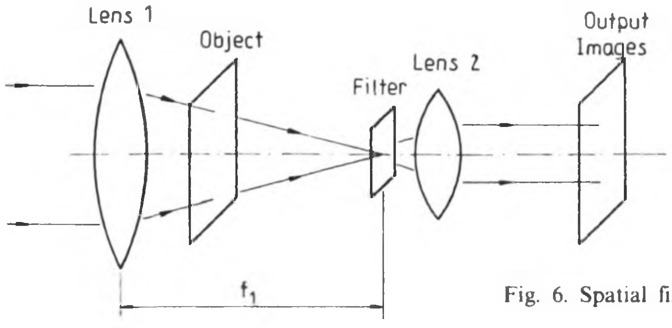
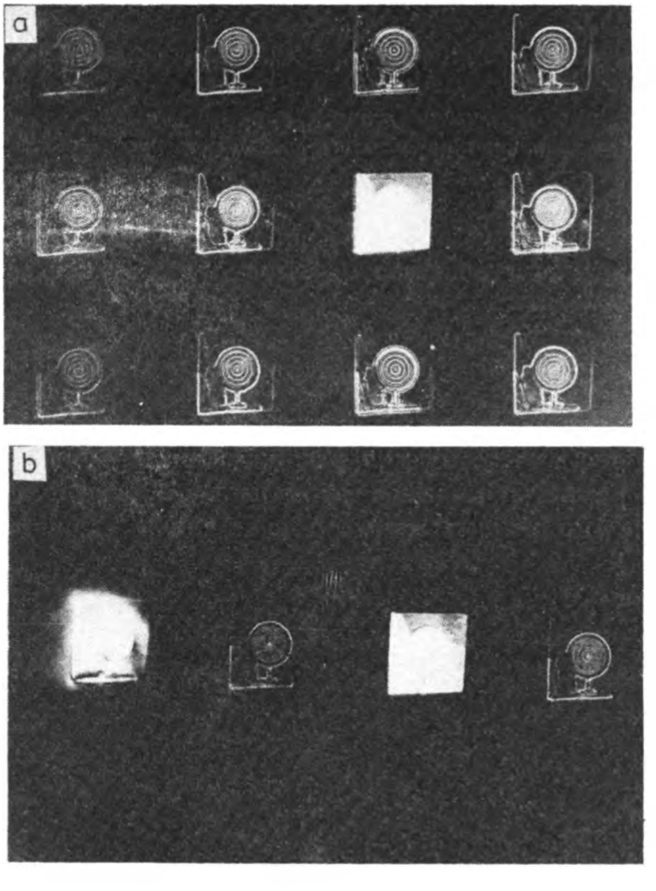


Fig. 6. Spatial filtering system used in experiments

difference observed in the work of bleached filters is a much higher diffraction efficiency. The whole observation planes are presented in Fig. 7, and the true images are shown in Fig. 8. In the black-and-white version the diffraction efficiency of classical Lohmann filter is about 4 times higher than that of cell-connecting, but in the bleached version the diffraction efficiencies are almost equal. Differential



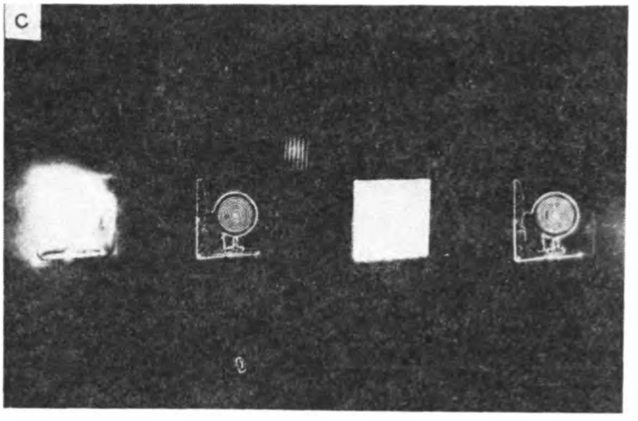
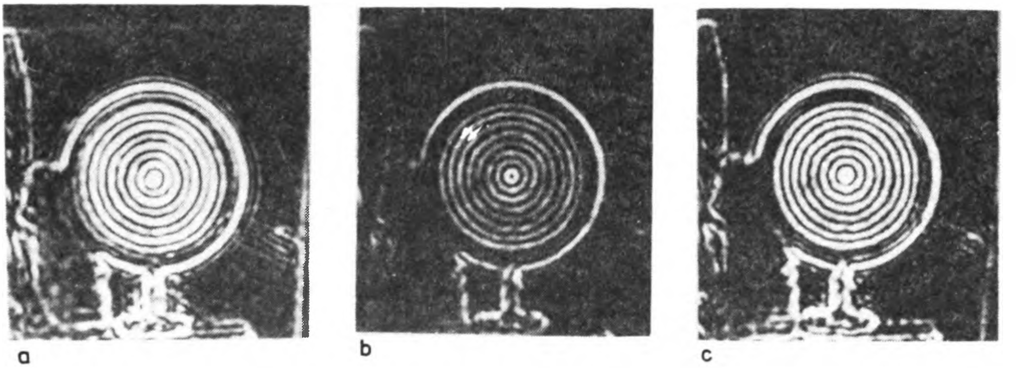


Fig. 7. Whole observation planes when using filters encoded by: **a** – classical Lohmann method, **b** – two-aperture Lohmann method with cell-connecting, **c** – as above with intercepted centre



a b c

Fig. 8. True output images when using filters encoded by: **a** – classical Lohmann method, **b** – two-aperture Lohmann method with cell-connecting, **c** – as above with intercepted centre

properties of all filters are similar, a more exact examination of negatives shows the most smooth shape of derivative for the cell-connected filter with intercepted centre. The quantitative researches on differential quality are in course.

5. Conclusions

The properties of Lohmann two-aperture method with cell-connecting when compared with the classical Lohmann method are as follows:

Advantages

– no vertical false images,
– no limitation of minimal amplitude encoded,
– fine shape of apertures (no lowering and uppering pen).

Shortcomings

- small diffraction efficiency in black-and-white version,
- limitation on maximum amplitude encoded,
- large minimum cell width.

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Соединение ячеек в детоурфазовых синтетических голограммах

Описан подход соединения ячеек в синтетических голограммах как метод элиминации вертикально отклоненных фальшивых изображений. Показана практическая реализация и экспериментальное исследование этого подхода в двуапертурном методе Ломана. Упомянуты другие многообещающие свойства соединения ячеек.