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Analysis of light propagation in a *Y*-junction of diffused Ti:LiNbO$_3$ strip waveguides and in a guided-wave Mach–Zehnder electrooptic modulator

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Expressions for refractive index distribution in a diffused *Y*-junction have been obtained. The dependence of the power losses on the *Y*-junction half angle has been determined and the possibility of reduction of losses by dereasing refractive index in the junction centre has been proved. Calculated field distributions oscillate from side to side in the *Y*-junction and Mach-Zehnder interferometric modulator's monomode branches: by taking account of these oscillations the excessive losss can be avoided. The obtained results allow us to optimize the *Y*-junction and geometric dimensions of the modulator.

1. Introduction

Diffused waveguide *Y*-junctions have numerous applications in integrated-optic devices, e.g. in a waveguide Mach–Zehnder interferometric modulator [1], shown in Fig. 1. The half-angle α is an essential parameter of *Y*-junction. To its optimization, two opposite requirements should be taken into consideration: when this anfle is greater the optical circuit may be shorter at the same time, however, the power losses increase. The refractive index distribution in the *Y*-junction is of an essential importance in the light propagation; so far, however, it has not been described sufficiently.

In this paper we present an analytic expression for the refractive-index distribution, obtained by means of anisotropic diffusion theory (Sect. 2), and the results of two-dimensional propagating beam analysis [2] of light propagation in a *Y*-junction (Sect. 3), and Mach–Zehnder interferometric modulator (Sect. 4). The conclusions are outlined in Sect. 5. In the Appendix, a simple and efficient way of computer calculations of guided field parameters from the discrete Fourier transform of the field is developed. We assume that the coordinate system shown in Fig. 1 coincides with the crystallographic axes of the LiNbO$_3$ crystal. We also assume that the

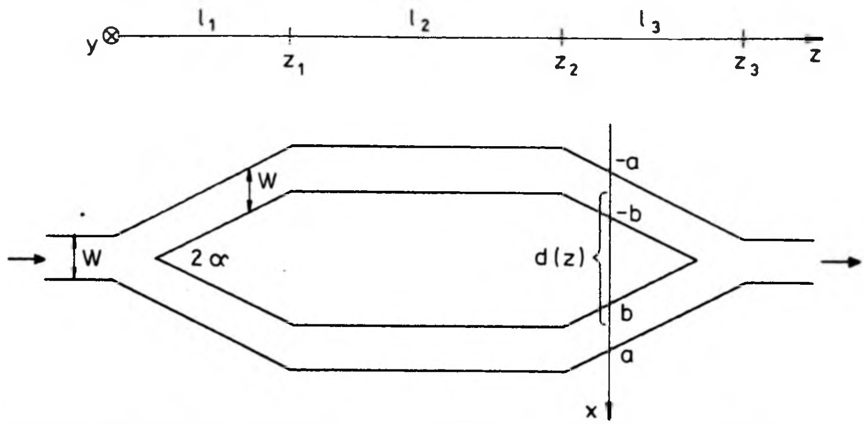

Fig. 1. Diffused Y-junction in a Mach-Zehnder interferometric waveguide modulator

branches separation parameter $d(z)$ only slightly depends on the z-coordinate. As it will be shown in Sect. 5, the half-angle α should not exceed 1° for keeping the power losses at an acceptable level.

2. Refractive-index distribution in a diffused Y-junction

The Ti-doped $LiNbO_3$ channel waveguides are usually formed by the Ti indiffusion from a titanium (or titanium-dioxide) strip of initial thickness h and width W [1], as

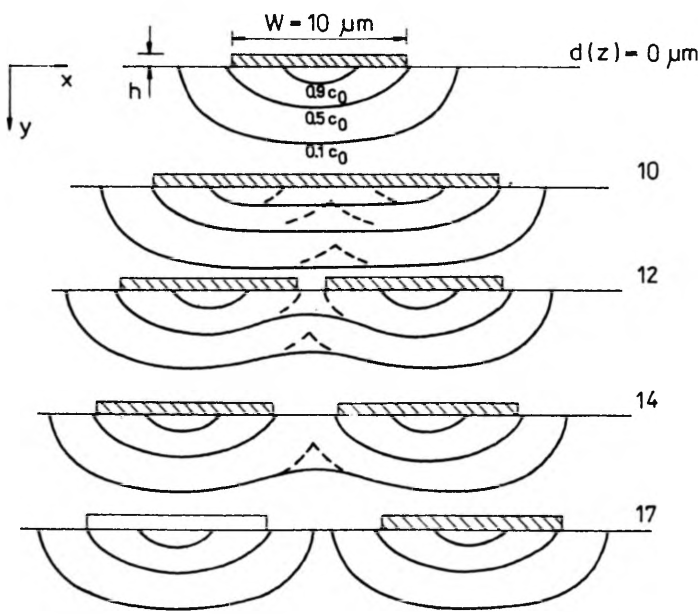

Fig. 2. Distribution of titanium concentration in a Y-junction for several values of branches separation parameter. Solid curves represent actual concentration distribution obtained after diffusion from titanium strip (dashed area). Dashed curves correspond to a non-actual profile which exhibits smaller power losses. Other parameters are as follows: $D_x = D_y = 3$ μm, $W = 10$ μm

shown inFig. 2. The increase of ordinary δn_o and extraordinary δn_e lithium niobate indices of refraction are determined by the titanium concentration c [3]:

$$\delta n_e = E c, \tag{1a}$$

$$\delta n_o = (F c)^\gamma \tag{1b}$$

where: $E = 1.2 \times 10^{-23}$ cm^3, $F = 0.13 \times 10^{-24}$ cm^3, and $\gamma = 0.55$ at $\lambda = 633$ nm.

The titanium concentration is a solution of the 3-D Fick's differential diffusion equation

$$\mathscr{D}_x \frac{\partial^2 c}{\partial x^2} + \mathscr{D}_y \frac{\partial^2 c}{\partial y^2} + \mathscr{D}_z \frac{\partial^2 c}{\partial z^2} = \frac{\partial c}{\partial t}, \tag{2}$$

with $\mathscr{D}_x$, $\mathscr{D}_y$, $\mathscr{D}_z$ as the diffusion coefficients in the x, y, z directions. The initial boundary conditions are as follows:

$$c = 0, \quad t = 0, \quad y \geqslant 0, \quad \lim_{x,y \to \infty} c = 0,$$

Assumption of slight dependence of the branches separation parametr $d(z)$ on the z-coordinate makes it possible to approximate $\partial^2 c/\partial z^2 \approx 0$; thus the diffusion equation (2) takes its 2-D form

$$\mathscr{D}_x \frac{\partial^2 c}{\partial x^2} + \mathscr{D}_y \frac{\partial^2 c}{\partial y^2} = \frac{\partial c}{\partial t}, \tag{3}$$

for which the particular solution is

$$c = (1/t) \exp[-(x/D_x)^2 - (y/D_y)^2 \tag{4}$$

where the diffusion depth, $D_\beta = 2(\mathscr{D}_\beta t)^{1/2}$ ($\beta = x, y$). The D' s are of order of 2–3 μm for monomode waveguides, according to the optical wavelength. Physically, the particular solution (4) corresponds to the concentration profile obtained from a linear source existing at $x = y = t = 0$, $-\infty < z < +\infty$. Similarly, a linear source placed at $x = x'$, $y = y'$, $t = t'$ causes a concentration distribution of the shape

$$c = \frac{1}{t-t'} \exp\left[-\frac{(x-x')^2}{4\mathscr{D}_x(t-t')} - \frac{(y-y')^2}{4\mathscr{D}_y(t-t')} \right], \quad t > t'. \tag{5}$$

If the infinitesimal dimensions of such a source are dx', dy', dt', it causes the following titanium concentration increase:

$$dc = \frac{A_1}{t-t'} \exp\left[-\frac{(x-x')^2}{4\mathscr{D}_x(t-t')} - \frac{(y-y')^2}{4\mathscr{D}_y(t-t')} \right] dx'\, dy'\, dt' \tag{6}$$

where A_1 is a constant.

Now, and in the following we assume that time t is equal to the time t_d of the diffusion process, i.e., $t \equiv t_d$.

In the expression (6), the variables x and y are separated; thus the final Ti concentration for a given z value can be written as

$$c(x, y, z, t_d) = c_0(t_d) f(y, t_d) g(x, t_d). \tag{7}$$

Functions f and g are usually normalized:

$$f(y=0) = 1, \quad \text{and} \lim_{W \to \infty} g(x=0) = 1. \tag{8}$$

When determining the depth shape of the concentration profile $f(y, t_d)$ from the expression (6), two mathematical approaches are used. The one employed more frequently, consists in considering the initial titanium strip as the part of the $LiNbO_3$ crystal with its diffusion coefficients [4]; thus the Ti concentration results from the integration of the expression (6) with respect to x' and y', over the intervals defining titanium strip

$$c(x, y, z, t_d) = \frac{A_2}{t_d} \int_{y'=-h}^{0} \exp[-(y-y')^2/D_y^2]\, dy \int_{x'} \exp[-(x-x')^2/D_x^2]\, dx'. \tag{9}$$

The concentration dependence on the z-coordinate is within the limits of the second integral.

The second and more justified approach to the problem is to assume a constant strength of the titanium source [5]; in this cause the integration of (6) must be performed with respect to x' and t', over the appropriate intervals, this gives

$$c(x, y, z, t_d) = \int_{t'=0}^{\tau} \frac{A_1}{t_d - t'} \exp\frac{-y^2}{4\mathscr{D}_y(t_d - t')} \int_{x'} \exp\frac{-(x-x')^2}{4\mathscr{D}_x(t_d - t')}\, dx'\, dt', \quad t_d > \tau \tag{10}$$

where τ denotes the time necessary for the entire indiffusion of the titanium strip into the bulk.

It is generally accepted that the most efficient optical fiber to $Ti{:}LiNbO_3$ waveguide coupling occurs when the diffusion time t_d is much greater than the time required for the entire indiffusion of the titanium source, i.e., $t_d \gg \tau$ and $h \ll D_y$; under these assumptions the expressions (9) and (10) take forms more similar to each other:

i) by neglecting in (9) the dependence of the integrand on y'-coordinate within a small interval $(-h, 0)$ we obtain

$$c(x, y, z, t_d) = \frac{A_2}{t_d} \exp(-y^2/D_y^2) h \int_{x'} \exp[-(x-x')^2/D_x^2]\, dx'; \tag{9a}$$

ii) by neglecting in (10) the dependence of the integrand on t'-coordinate within a small time period (O, τ) we get

$$c(x, y, z, t_d) = \frac{A_1}{t_d} \exp(-y^2/D_y^2) \tau \int_{x'} \exp[-(x-x')^2/D_x^2]\, dx'. \tag{10a}$$

Obviously, (9a) and (10a) are identical if $A_1 \tau = A_2 h$. They both exhibit a Gaussian shape in the y direction

$$f(y, t_d) = \exp(-y^2/D_y^2). \tag{11}$$

The constants A_1 and A_2 are determined by Ti-atoms total number conservation condition.

The limits of the integrals in (9a) and (10a) depend on the relation between the value of branches separation parameter $d(z)$ and the titanium strip width W(see Fig. 1):

$d \leqslant W\colon -a < x' < a,$

$$g(x, y, z, t_d) = 0.5\,[\operatorname{erf} A - \operatorname{erf} B], \tag{12a}$$

$d > W\colon -a < x' < -b \cup b < x' < a,$

$$g(x, z, t_d) = 0.5\,[\operatorname{erf} A - \operatorname{erf} B - \operatorname{erf} C + \operatorname{erf} D], \tag{12b}$$

with $a = (d+W)/2$, $b = (d-W)/2$, $A = (a-x)/D_x$, $B = (-a-x)/D_x$, $C = (b-x)/D_x$, $D = (-b-x)/D_x$.

For the functions f and g of the forms (11) and (12), appearing in (7) the constant $c_0(t_d)$, when determined with the use of the Ti-atoms total number conservation condition, is

$$c_0 = \frac{2A_0 \varrho h}{D_y \sqrt{\pi}\,\mu} = 6.4 \times 10^{22}\,\mathrm{cm}^{-3}\,\frac{h}{D_y} \tag{13}$$

where $A_0 = 6.022 \times 10^{23}$/mol, ϱ is the titanium density and μ is its molar mass. It should be noted that the density of titanium layer deposited by evaporation can be lower than that of titanium metal by about 10% [6].

The expression (7) with f, g and c_0, given by (11) and (12) and (13), determines the titanium concentration; thereby the $LiNbO_3$ refractive indices increase δn_o and δn_e can be determined with the help of relations (1). The expressions (11) and (12) are normalized in the sense of (8).

Figure 2 shows the calculated titanium concentration distributions in a Y-junction or modulator's branches for several values of the branches separation parameter $d(z)$. The profile assumed in [2], with the reduced values of function $g(x)$ in the proximity of the junction's centre, which exhibits smaller power losses (as we have presented in Sect. 3), is also marked in this figure.

3. Analysis of light propagation in a *Y*-junction

The calculated light power distribution in a Y-junction is shown in Fig. 3. It has been assumed that the fundamental mode enters at the input of the structure. Although the junction branches are monomode, the field propagating in them oscillates in the transversal direction with spatial period of order of 1 mm, which indicates that the propagation is not only of the fundamental mode, but also of the radiation modes. This effect is analogical to that at corner bends of dielectric waveguides, as described, e.g., in [7]. The direction of these oscillations is strictly related to field divergence parameter k_d developed by the formula (A7) in the Appendix. Its values are given in Fig. 3.

The fundamental mode power P_0 in one of the Y-junction branches vs. the half-angle α is shown in Fig. 4 for both diffusion profiles given in Fig. 2. It is assumed that the input of the junction is excited by normalized to unity power fundamental

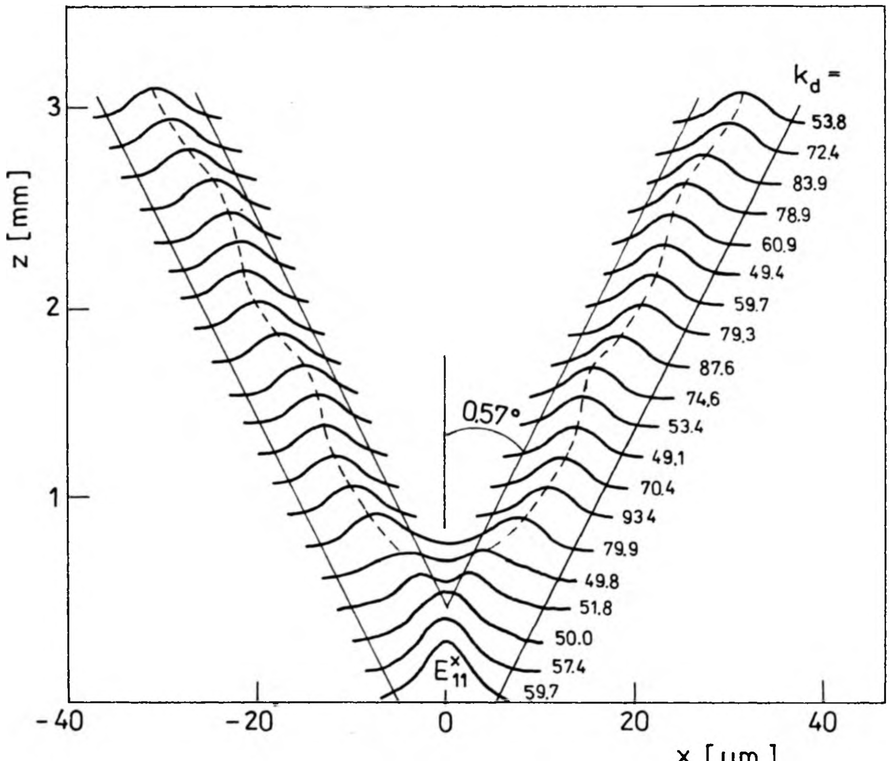

Fig. 3. Light power distribution $|E|^2$ in a Y-junction with fundamental mode E^x_{11} entering the input. Y-cut, X-propagating Ti:$LiNbO_3$ waveguide is assumed. The dashed curve indicates the intensity maxima. Values of field divergence parameter k_d are given in arbitrary units. The parameters are as follows: $W = 10$ μm, $D_x = D_y = 3$ μm, $\lambda = 1.3$ μm, the bulk refractive index 2.20, at the surface 2.21

mode. The values of P_0 were estimated by using the formula (A6) given in the Appendix.

The power losses increase rapidly for the values of half-angle α higher than 1°. For half-angles α smaller than 0.5° the power P_0 reaches the values of 46.5% in each

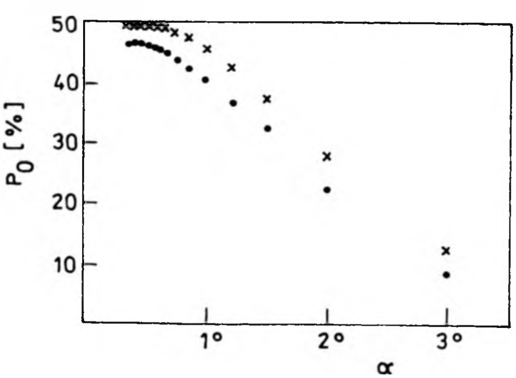

Fig. 4. Fundamental mode power P_0 in one of the Y-junction branches vs the junction half-angle for both diffusion profiles from Fig. 2: actual (points) and reduced at the junction's center (crosses). Fundamental mode carrying the power of unity is assumed at the junction input. Other parameters are as in Fig. 3

branch, i.e., 7% (0.32 dB) of the fundamental mode power is lost. For the profile which is reduced at the junction's center the power P_0 exceeds 49.5% for α smaller than 0.6°, therefore, the losses are lower than 1% (0.044 dB).

4. Analysis of light propagation in a Mach–Zehnder interferometric modulator

Similarly to the Y-junction branches, the field propagating in the parallel branches of the interferometer, shown in Fig. 1, exhibits also transversal oscillations. A skillful choice of lengths l_1, l_2, l_3, in order to make the change in the waveguides' direction adequate to the direction of field oscillations, will help us to avoid excessive power losses. The calculations accomplished with the same half-angle $\alpha = 0.57°$ and other parameters, as in Fig. 3, have shown:

i) In the most favourable case ($l_1 = l_3 = 1280$ μm, $l_2 = 720$ μm), the fundamental mode power P_0 at the output of the interferometer is as high as 93.5% of the fundamental mode power at the input, i.e., the loss is 0.29 dB.

ii) In the least favourable case ($l_1 = l_3 = 1692$ μm, $l_2 = 864$ μm), $P_0 = 37.5\%$, only, i.e., the loss is 4.26 dB.

The values of the field divergence parameter k_d in the bends z_1 and z_2 were (in arbitrary units) 48.0 and 49.5 in the case i), 87.0 and 64.3 – in the case ii).

The influence of modulating electric field on the guided field oscillations, thus on the magnitude of losses, will be the subject of our future investigations. It seems, however, that in the case of electrooptically induced phase difference not exceeding π radians the change in oscillations' behaviour will be negligible, their spatial period exceeding considerably light wavelength in the waveguide.

5. Conclusions

For keeping power losses at the satisfactory level, the Y-junction half-angle must not exceed 1°. A decrease of its value below 0.5° gives insignificant decreases of losses. Ar additional decrease of losses is possible due to the decrease of refractive index in the centre of the Y-junction – resulting from smaller titanium strip thickness before diffusion or from an additional magnesium indiffusion [8].

Design of the Mach–Zehnder modulator and similar integrated-optic devices' geometry should be preceded by a numerical analysis leading to consideration of power's transversal oscillations in order to minimize the power losses at corner waveguide bends.

Although we have approximated the 3-D diffusion equation to its 2-D form we do not expect it to cause considerable errors for halfangle α values not exceediing 1°. The expressions (12) can be easily generalized to describe asymmetric Y-junctions (i.e., with different values of half-angle α) by a proper adoption of the integral limits.

Appendix

The optical field transverse component distribution in a monomode waveguide when written as a superposition of its eigenmodes is

$$E(x) = \alpha u(x) + \Delta(x) \tag{A1}$$

where $u(x)$ is the normalized to unity power field of the fundamental mode

$$\int |u(x)|^2 dx = 1, \tag{A2}$$

α is a complex constant, and $\Delta(x)$ is the part of optical field composed of radiation modes. From (A1) it follows that the fundamental mode power in the waveguide is

$$P_0 = \int |\alpha u(x)|^2 dx. \tag{A3}$$

When the field $u(x)$ is normalized in Eq. (A2) sense, then from the waveguide eigenmodes orthogonality condition it follows that

$$P_0 = |\int E(x) u^*(x) dx|^2 \tag{A4}$$

where $*$ denotes a complex conjugation.

Let fields $E(x)$ and $u(x)$ are insignificantly small beyond the properly chosen interval (0, L), and their Fourier transforms $A(k_x)$ and $a(k_x)$ vanish for $|k_x| > N\pi/L$, where N is a natural number big enough; then the fields can be expanded into Fourier series as:

$$E(x) = \frac{1}{N} \sum_{k=-N/2}^{N/2} A_k \exp(2\pi jkx/L), \tag{A5a}$$

$$u(x) = \frac{1}{N} \sum_{k=-N/2}^{N/2} a_k \exp(2\pi jkx/L) \tag{A5b}$$

where A_k, a_k are discrete Fourier transform coefficients of the fields $E(x)$ and $u(x)$. Substituting (A5) into (A4) and reducing the integration limits to the interval (0, L), one obtains

$$P_0 = \left| \frac{L}{N^2} \sum_{k=-N/2}^{N/2} A_k a_k^* \right|^2 . \tag{A6}$$

Expression (A6) leads to a fast and precise evaluation of the integral (A4), i.e., of the fundamental mode power.

The guided field declination from the z-direction in the Y-junction and modulator's branches is an increasing function of the following field divergence parameter k_d

$$k_d = \sum_{k=-N/2}^{N/2} A_k A_k^* |k| 2\pi/L. \tag{A7}$$

The elements of the sum (4) represent relative intensities of plane waves, with A_k as the amplitudes and $k_x = 2\pi k/L$ as the wave-vector transversal components.

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Анализ распространения света в разветвлении типа *Y* диффузных полосных волноводов Ti:LiNbO$_3$ и в волноводном электрооптическом модуляторе Маха–Цендера

Получены выражения для распределения коэффициента преломления в области разветвления *У*. Определена зависимость потерь мощности от угла разветвления, а также было показано, что потери можно уменьшить путем понижения коэффициента преломления вблизи оси разветвления. Выяснено распределение поля в разветвлении одномодных волноводов. В плечах разветвления и модулятора поле обнаруживает колебания в поперечном направлении к оси волноводов; знакомство хода этих колебаний существенно при проектировании интерферометра с малыми потерями