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Correction procedures of the immersion mismatching in interferometric determination of refractive index profile. Part I. Wavefront correction*

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The suggested method of wavefront correction is a universal one since it may be applied to different types of interference (such as those of plane reference wave, radial shearing or transversal shearing, for example). The advantage of this method consists in a possibility of applying the algorithms valid for the case of perferct immersion matching in this method, as well.

1. Introduction

The methods of correction presented in this cycle of works have been inspired by those practical needs of measurement, when the results of measurements need not be very accurate but rendered in a possibly quick way. Such demands appear typically when controlling the profiles of refractive indices of preforms produced under the industrial conditions. Then the preforms are quickly exchanged in the measuring setup (an interferometer, for example). Usually, the methods suggested are very simple while the measurement is reduced to measuring the interference fringe deformation at one point on the coat of the measured object. This enables, in turn, the exploitation of the whole area of the CCD detector in the region of the core of the examined object, only. The denser is the sampling in this region the higher the fidelity of the examined profile reconstruction. The methods suggested in this cycle of works allow us to apply quick algorithms including those basing on the assumption that the refractive index of immersion and that of the coat are equal to each other. In the methods very accurate a detailed analysis is carried out on a large number of data, taken from the regions of core, coat and immersion, respectively [1]–[5]. The number of sampling in the core region decreases, if the same CCD detector is applied and the computer calculating the Abell transform, for instance, works respectively longer.

The present paper is the first of the cycle of works concerning nondestructive interference methods applicable to determine the refractive index distribution in both

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preforms and optical waveguides, under the condition of refractive index mismatching in the immersion liquid and the coat of the measured objects. In all the discussed interference methods, the object under test was located in the immersion liquid. In both the cases of perfect matching and mismatching, the refractive index in the object core is calculated on the base of the reconstructed wavefront appearing due to the passage of the plane wave through the examined object perpendicularly to its axis of symmetry. In the mathematical models, which are used in order to reconstruct the wavefront and calculate the refractive index of the core, it is assumed that the wavefront is a continuous function in the region considered, while both the object and the refractive index distribution are of cylindric symmetry, and the refractive indices of the immersion liquid and the object coat are usually equal to each other. The influence of the deviation from the assumptions concerning the examined object on the accuracy of the calculations of the refractive index distribution in the core was considered [3], [6], among others. In the present cycle of publications, the problem of calculation of refractive index in the core is discussed, for the case when the conditions of measurement do not allow us to match the refractive index of immersion to that of the core of the examined object. The perfect matching of the immersion liquid requires, as a rule, manipulations with more than one liquid, which may jeopardize the uniformity of the immersion as well as the steady construction of its components. Due to the above difficulties, a modification of the "old" mathematical model [4] used in the case of the perfect matching of immersion to the coat has been proposed. Since the modification is carried out immediately after wavefront reconstruction, it appears to be identical for all types of interference.

In the next works of this cycle we shall suggest another solution consisting of the correction of the input data, i.e., the orders of the interference fringes. This second way of correction does not require any introduction of the changes in "old" mathematical model, but is different in different versions of interference methods (i.e., in the method with plane reference wave, radial shearing, transversal shearing). The said problems have been solved to the zero order approximation, i.e., under the assumption that the light beam passes through the examined object without changing its direction. In Sects. 6 and 7, the same problem will be considered for the real run of the rays.

2. Principle of calculation of refractive index changes in the core of the phase object of cylindric symmetry

The phase objects considered below are preforms and light waveguides. The methods discussed here make it possible to determine the distribution of the difference of the refractive indices in the core and the coat of the examined object, respectively.

2.1. Case of matched refractive indices of both immersion and the object coat ($n_i = n_p$)

A plane wavefront passing through the object examined suffers from deformation both in the coat and core regions (Fig. 1). Within the core region ($0 \leqslant |x| < r$) it takes

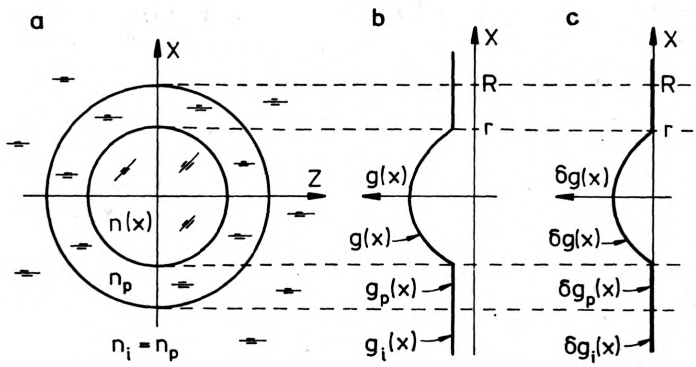

Fig. 1. Wavefront after its passage through the object examined ($n_i = n_p$): **a** – object, **b** – wavefront, **c** – wavefront referred to the object coat wavefront

the form

$$g(x) = 2\int_{0}^{z_r} n(x)dz + 2\int_{z_r}^{R} n_p dz, \tag{1}$$

while in the coat region ($r \leqslant |x| < R$) as well as outside the coat ($(|x|) \geqslant R$) it has the form

$$g_p(x) = 2\int_{0}^{R} n_p dz \tag{2}$$

where: r – core radius in the examined object,
R – external radius of the coat,

$$z_r = \sqrt{r^2 - x^2}, \quad z_R = \sqrt{R^2 - x^2}.$$

Since the changes in the core refractive index $\delta n(x)$ are referred to the refractive index of the core, the wavefront $g(x)$ in the region of the object core is also referred to the wavefront in the region of the object coat (δ is a difference operator). This relative wavefront is defined as follows:

$$\delta g(x) = 2\int_{0}^{z_r} \delta n(x)dz. \tag{3}$$

The dependence (3) is a basis to calculate the changes in the refractive index distribution $\delta n(x)$ in the object core. The information about $\delta g(x)$ is encoded immediately on the interferogram in the form of interference fringes. When analysing the distribution of fringes on the interferogram the wavefront $\delta g(x)$ may be reconstructed. The reconstruction method has been described in works [7], [8].

2.2. Case of mismatched refractive indices of the immersion liquid and the object coat ($n_i \neq n_p$), respectively

If the refractive indices of the immersion liquid and the coat of the measured object are different, the plane wavefront passing through this object is transformed (in the core region $0 \leqslant |x| < r$) into the wavefront $G(x)$ different from $g(x)$ which was generated in the case of $n_i = n_p$. This wavefront $(G(x))$ is a sum of $g(x)$ and some additional wavefront $G_k(x)$, which is called correcting wavefront $G(x) = g(x) + G_k(x)$ (Fig. 2). These wavefronts, as it was the case for $n_i = n_p$, when referred to the

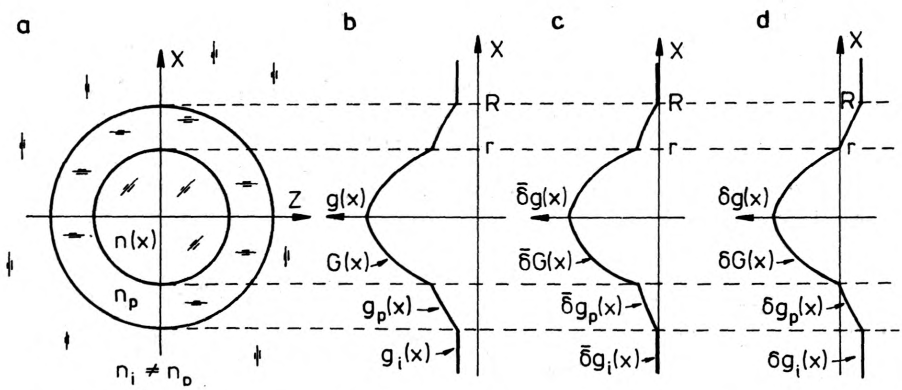

Fig. 2. Wavefront after its passage through the object examined ($n_i \neq n_p$): **a** – object, **b** – wavefront, **c** – wavefront referred to the object coat wavefront, **d** – wavefront referred to the immersion wavefront

wavefront of the object coat, resulted in the respective wavefronts $\delta G(x)$, $\delta g(x)$ and $\delta G_k(x)$ (Fig. 2c). The information of the wavefront $\delta G(x)$, which has been recorded immediately on an interferogram in the form of interference fringes, may be reconstructed on the base of data from the interferogram (as it was the case for the wavefront $\delta g(x)$ when $n_i = n_p$). In the case when $n_p \neq n_i$, the wavefront $\delta g(x)$ should be calculated by using the reconstructed wavefront $\delta G(x)$. The distribution of the refractive index changes $(\delta n(x))$ in the object core is calculated on the base of $\delta g(x)$ in the same way as in the case when $n_i = n_p$.

3. Calculation of the wavefront $\delta g(x)$ in the case of mismatching of the immersion refractive index to that of the coat of the phase object of cylindric symmetry ($n_i \neq n_p$)

The plane wavefront passing through the phase object suffers from deformation in the core region ($0 \leqslant |x| < r$) of refractive index $n(x)$, in the coat region ($r \leqslant |x| < R$) of refractive index $n_p = \text{const}$, and in the region of immersion liquid ($|x| \geqslant R$) of refractive index $n_i = \text{const}$ (Fig. 2). The wavefront in the core region has the form

$$G(x) = 2\int_0^{z_r} n(x)dz + 2n_p(\sqrt{R^2-x^2} - \sqrt{r^2-x^2}) + 2n_i(R - \sqrt{R^2-x^2}), \tag{4}$$

while in the coat region,

$$g_p(x) = 2n_p\sqrt{R^2-x^2} + 2n_i(R - \sqrt{R^2-x^2}), \tag{5}$$

in the immersion liquid region

$$g_i(x) = 2n_i R \tag{6}$$

where $z_r = \sqrt{r^2-x^2}$.

The respective relative wavefronts referred to the wavefront in the coat of the examined object (Fig. 2d) are defined as follows:

$$\begin{aligned}
\delta G(x) &= \delta g(x) + \delta G_k(x),\\
\delta g_p(x) &= 2\delta n_i(\sqrt{R^2-x^2} - \sqrt{R^2-r^2}),\\
\delta g_i(x) &= -2\delta n_i\sqrt{R^2-r^2}
\end{aligned} \tag{7}$$

where:

$$\begin{aligned}
\delta n(x) &= n(x) - n_p,\\
\delta g(x) &= 2\int_0^{z_r} \delta n(x)dz,\\
\delta G_k(x) &= 2\delta n_i(\sqrt{R^2-x^2} - \sqrt{R^2-r^2}),\\
\delta n_i &= n_p - n_i.
\end{aligned} \tag{7a}$$

As it may be seen from Eqs. (7) and (7a), the sought wavefront is of the form

$$\delta g(x) = \delta G(x) - 2\delta n_i(\sqrt{R^2-x^2} - \sqrt{R^2-r^2}). \tag{8}$$

The wavefront $\delta G(x)$ is enclosed immediately on the interferogram, and may be reconstructed by using one of the methods discussed in the works [6]–[10]. The distribution of the refractive index changes $\delta n(x)$ is calculated from the dependence (3).

The way of determining the refractive index difference δn_i is different for different types of interference, and will be described in the works of this cycle to follow. Since, for the sake of determining δn_i, we shall employ the wavefronts referred to the wavefront from the immersion region, the expression for these wavefronts is given below, and the accuracy of the wavefront determination will be discussed in the further part of this work. The relative wavefronts referred to the immersion in the core, coat and immersion regions (Fig. 2c) take respectively the following forms:

$$\bar{\delta}G(x) = \delta g(x) - \bar{\delta}G_k(x),$$

$$\bar{\delta}g_p(x) = 2\delta n_i\sqrt{R^2-x^2},$$

$$\bar{\delta}g_i(x) = 0 \tag{9}$$

where

$$\bar{\delta}G_k(x) = 2\delta n_i\sqrt{R^2-x^2}. \tag{9a}$$

As it is visible from Eqs. (7), (7a) and (9), (9a), the correcting wavefronts $\delta G_k(x)$ and $\bar{\delta}G_k(x)$ are the same forms as the respective wavefronts $\delta g_p(x)$ and $\bar{\delta}g_p(x)$ in the region of the examined object. The correcting wavefronts are the functions of both geometric parameters (r, R) of the examined object and the refractive index difference δn_i. In order to visualize the character of these changes, independently of the examined object and the measurement conditions, they have been normalized by a product $\delta n_i R$ (Fig. 3a). The values of the correcting wavefronts normalized in this

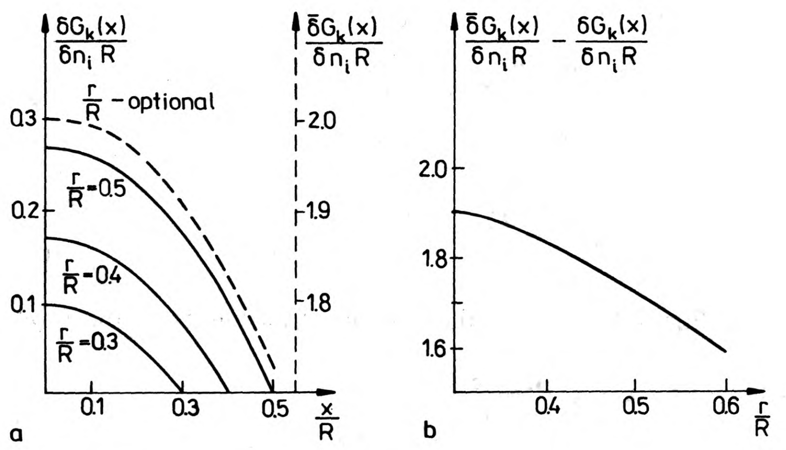

Fig. 3. Normalized correcting wavefronts (**a**) and the dependences among them (**b**)

way very along the radius of the examined object (x/R) and depend on the ratio of the dimensions of the core and coat (r/R) for the correcting wavefront $\delta G_k(x)$ (Fig. 3**a** – steady lines), while for the wavefront $\bar{\delta}G_k(x)$ they are the same for all r/R (Fig. 3**a** – broken lines). Both the normalized correcting wavefronts differ from each other only by a constant depending on r/R (Fig. 3**b**).

4. Accuracy of the correcting factor determination for the wavefront (zero-order approximation)

As it follows from Equations (7) and (9), the reconstruction accuracy of the wavefronts $\delta G(x)$ or $\bar{\delta}G(x)$ as well as the accuracy of correcting factor determination for the wavefront $\delta G_{\mathrm{k}}(x)$ or $\bar{\delta}G_{\mathrm{k}}(x)$ influence the accuracy of the wavefront determination $\delta g(x)$, the knowledge of the latter being necessary in order to calculate $\delta n(x)$. (This problem was considered in papers [7], [8]). The latest problem will be considered below. The relative errors for $\delta G_{\mathrm{k}}(x)$ and $\bar{\delta}G_{\mathrm{k}}(x)$ are defined as follows:

$$\frac{\Delta\delta G_{\mathrm{k}}(x)}{\delta G_{\mathrm{k}}(x)} = \frac{\Delta\delta n_{\mathrm{i}}}{\delta n_{\mathrm{i}}} + A\Delta R + B\Delta x + C\Delta r), \tag{10}$$

$$\frac{\Delta\bar{\delta}G_{\mathrm{k}}(x)}{\bar{\delta}G_{\mathrm{k}}(x)} = \frac{\Delta\delta n_{\mathrm{i}}}{\delta n_{\mathrm{i}}} + \bar{A}\Delta R + \bar{B}\Delta x, \tag{11}$$

where:

$$A = \left|\frac{R}{\sqrt{R^2-x^2}-\sqrt{R^2-r^2}}\left(\frac{1}{\sqrt{R^2-x^2}}-\frac{1}{\sqrt{R^2-r^2}}\right)\right|,$$

$$B = \left|\frac{1}{\sqrt{R^2-x^2}-\sqrt{R^2-r^2}}\,\frac{x}{\sqrt{R^2-x^2}}\right|,$$

$$C = \left|\frac{1}{\sqrt{R^2-x^2}-\sqrt{R^2-r^2}}\,\frac{r}{\sqrt{R^2-r^2}}\right|,$$

$$\bar{A} = \left|\frac{R}{R^2-x^2}\right|,$$

$$\bar{B} = \left|\frac{x}{R^2-x^2}\right|.$$

As may be seen, both the relative errors depend on: the measurement accuracy of the refractive index difference δn_{i}, the geometric parameters r, R of the examined object, and the accuracy of the determination of the coordinate x (Fig. 4). The relative errors presented in Fig. 4 are referred to the level $\Delta\delta n_{\mathrm{i}}/\delta n_{\mathrm{i}}$. It has been assumed that the accuracies of determination of both the coordinate x and the core radius r are the same ($\Delta x = \Delta r$). In the case of wavefront $\delta G_{\mathrm{k}}(x)$, the relative errors increase faster (with the increase of x/R) the less the value of r/R being the greatest at the core-coat border. In the case of the wavefront $\delta G_{\mathrm{k}}(x)$, the relative errors increase with the increasing of x/R (faster than it was the case above), and are the greater (in the whole considered range) the greater the value of the parameter r/R. The errors presented in Fig. 4 refer to the objects of various geometrical sizes (preforms, light waveguides). They are proportional to the errors $\Delta r/r$ and $\Delta R/R$ of the determination of both the core and coat of the object, respectively.

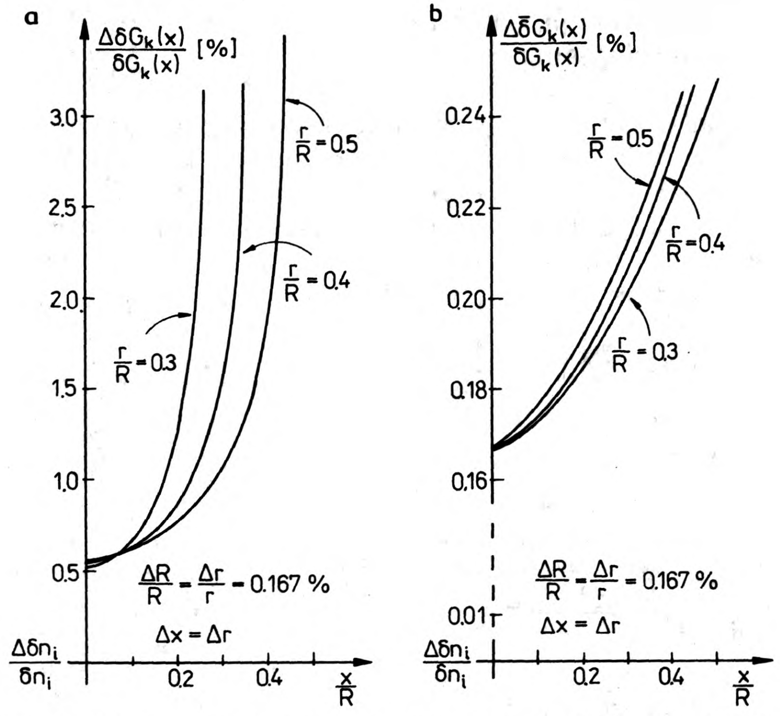

Fig. 4. Relative errors of the correcting wavefronts: **a** — for the wavefront $\delta g(x)$, **b** — for the wavefront $\bar{\delta} g(x)$

5. Errors introduced by the zero-order approximation

The range of applicability of the zero order approximation considered in Sect. 4 is restricted by the accuracy which is to be achieved while correcting the wavefront. The discussion of this problem, under the assumption that the light ray passing through the examined object does not change its direction (approximation of the zero order), leads to the results charged with errors. The relative error of this approximation, as compared to the exact solution (for a real run of the ray), depends on mismatching of the refractive indices of the immersion and the coat of the examined object and is the function of the normalized coordinate x/R (Fig. 5)

$$\frac{\delta G_{\mathbf{k}0} - \delta G_{\mathbf{kd}}}{\delta G_{\mathbf{k}0}} 100\% = \sum_{i=0}^{5} A_i (x/R)^i \tag{13}$$

where

$$A_i = \sum_{j=0}^{3} a_{ij} (\delta n_i)^j$$

(for a_{ij}, see Table). This additional error is superposed on the errors presented in Fig. 4. From Figures 4 and 5 the limits of the applicability of the zero order

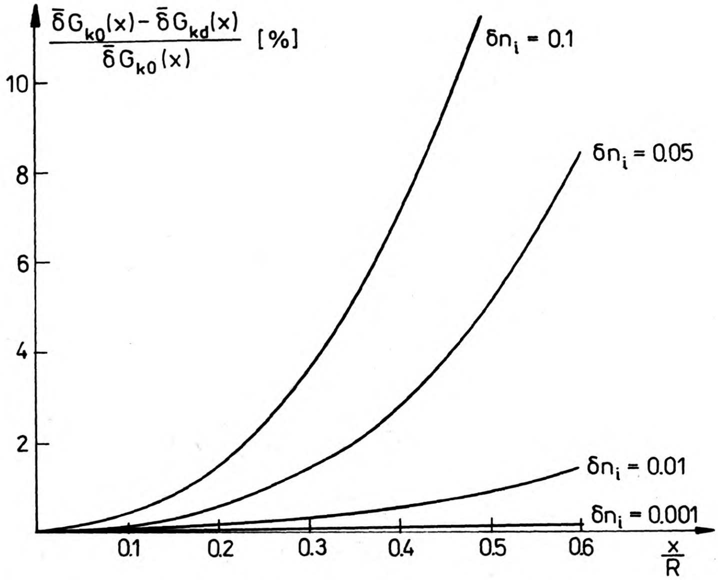

Fig. 5. Relative error of the zero-order approximation

Coefficients a_i for the zero-order approximation error

i	a_{i0}	a_{i1}	a_{i2}	a_{i3}
0	−0.000032	−0.008122	−0.425	3.3
1	0.026029	3.119000	214.660	−1552.8
2	−0.319526	166.200400	−2513.007	28031.0
3	1.454774	503.230400	17446.373	−122059.0
4	−2.716030	−1144.963800	−37091.244	259276.0
5	1.745153	1456.659900	29947.000	−174902.0

approximation may be determined for given values of parameters: $\Delta R/R$, $\Delta r/r$, $\Delta\delta n_i/\delta n_i$, δn_i. Thus, if the accuracy of correcting wavefront determination (as referred to the immersion, Fig. 4**b**) may be as high as 2% the accuracy of the geometrical sizes of the measured object must be as high as: $\Delta R/R = \Delta r/r = 0.6\%$, for $r/R = 0.4$, δn_i 0.01, and $\Delta\delta n_i/\delta n_i = 0.5\%$.

6. Calculation of the correcting wavefront for real run of the ray

The solutions of the correcting wavefront problem obtained in the previous sections are of limited applicability. In order to determine the latter the run of the ray through the preform will be considered, and thus the effect of refraction at the borders of the medium taken into account (Fig. 6).

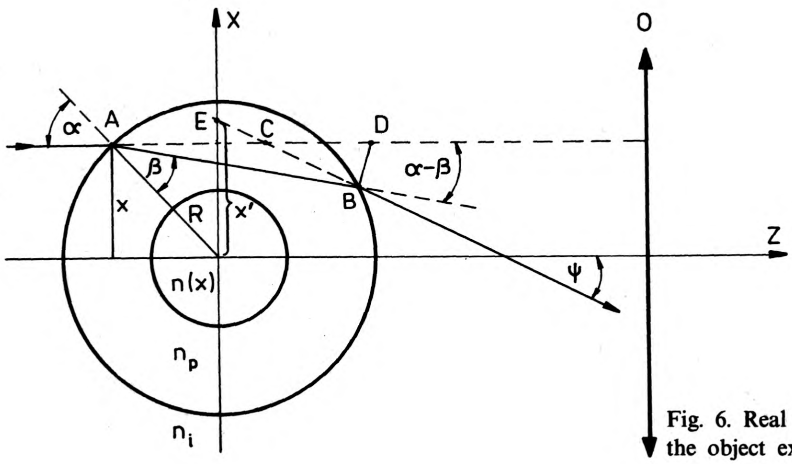

Fig. 6. Real run of the ray through the object examined

The calculated correcting wavefront δG_{kd} takes account of the refraction at the border of the media immersion-coat the influence of the core heterogeneity being neglected. This is admissible in practice, without making any serious errors effecting the correcting factor. The objective O does not image the plane X but the plane shifted slightly towards the objective. This results from the fact that the sharp adjustment is applied either to the center or to the border of the core [11]. The light ray entering the object coat at the height x parallely to the axis Z is incident on the immersion-coat border under the angle α to be then refracted under the angle β. At the exit surface from the coat to the immersion the ray is subject to another refraction, which results in deflection from the primary direction by an angle $\psi = 2(\alpha-\beta)$. In the interferometric measurements the examined object is imaged by the objective O (Fig. 6). The optical path difference between the wavefront deformed by δn_i and the plane reference wave may be calculated as the optical path difference between the ray emerging from the coat under the angle ψ to the axis Z and that ray from the reference beam which runs along the Z axis [11]

$$\delta G_{kd}(x) = ABn_p - ADn_i. \tag{14}$$

Both the ray considered enter the measurement system at the height x while the ray deviated due to the double refraction at the border of the media emerges apparently from the point C (Fig. 5). By assuming that an ideal imaging objective is used which introduces no additional optical path difference for two interfering rays and taking account of the relations following from both the refraction law and the following geometrical relations:

$$AC = r\cos\beta,$$

$$AB = 2r\cos\beta,$$

$$AC = CB,$$

$$\sin\alpha = x/R, \qquad \cos\beta = \sqrt{1-\left(\frac{n_i x}{n_p R}\right)^2},$$

$$\cos(\alpha-\beta) = r/(\cos\alpha+\sin\alpha\tan\beta), \tag{15}$$

and finally assuming that the point D lies in such a place that $CD = CB$ the relation $AD = 2AC$ has been obtained. After taking account of the above relations and (13), we get

$$\delta G_{kd}(x') = 2Rn_p\sqrt{1-(vx/R)^2} - \frac{v}{\sqrt{1-(x/R)^2}+\frac{v(x/R)^2}{\sqrt{1-(vx/R)^2}}} \tag{16}$$

where

$$v = 1-\delta n_i/n_p.$$

The objective imaging the preform or light waveguide "sees" the point C on the interferogram at the position E of apparent coordinate

$$x' = x/\cos\psi \tag{17}$$

where

$$\cos\psi = 2[v(x/r)^2+\sqrt{1-(vx/R)^2}\sqrt{1-(x/R)^2}]^2-1. \tag{18}$$

When knowing the value of the correcting order of interference at the point x' the value of $\delta M_{kd}(x)$ may be calculated by using method of either interpolation or approximation.

7. Determination accuracy of the wavefront correction factor (for real run of the ray)

The dependence (16) describing the error is both complex and inconvenient to direct analysis and therefore computer simulation has been applied. The following observations have been made:

1. The relative error of the correcting factor depends on neither the core diameter R, nor the difference of the refractive indices of the core and immersion δn_i, and practically is independent of the ratio of the radii of the core and coat r/R of the examined object.

2. Under the assumptions that the relative measurement errors for the coat and core are equal to each other ($\Delta r/r = \Delta R/R$), the error of measurement of coordinate within the core amounts to $\Delta x = \Delta r$ and that the error of the measurement of the

refractive index in the coat is $\Delta n_p = 1\times 10^{-4}$ the relative error may be expressed as follows:

$$\frac{\Delta(\delta G_{kd})}{\delta G_{kd}}100\% = a_0+a_1(x/R)+a_2(x/R)^2+\frac{\Delta(\delta n_i)}{\delta n_i}100\% \tag{19}$$

where:

$x/R \leqslant 0.5$,

$a_0 = 92.46(\Delta x/r)+185.69(\Delta x/r)^2-1107.13(\Delta x/r)^3$,

$a_1 = 0.003-20.07(\Delta x/r)+517.49(\Delta x/r)^2-4567.45(\Delta x/r)^3$,

$a_2 = 0.0003+165.1(\Delta x/r)-1065.7(\Delta x/r)^2-8693.9(\Delta x/r)^3$

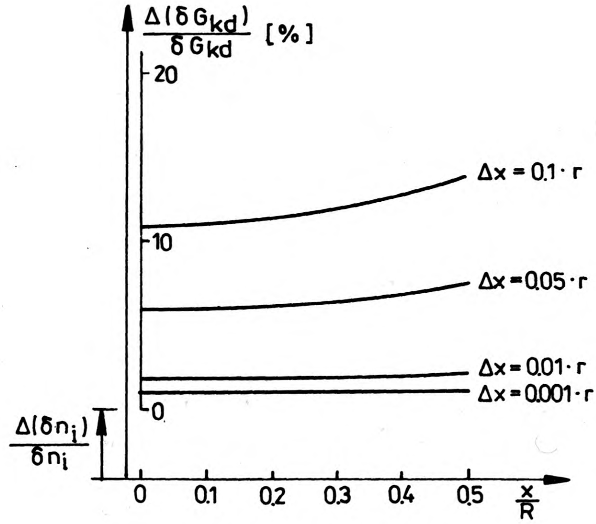

Fig. 7. Relative error of the correcting wavefront for real run of the ray through the examined object

3. The relative error depends on two factors: difference of the respective refractive indices of the coat and immersion and on the values of x/R, $\Delta x/r$. The higher are the values x/R, $\Delta x/r$ the more nonlinear the dependence (19), Fig. 7.

8. Summarizing remarks

In the paper the method of calculation of the wavefront generated by the examined preform or light waveguide has been presented in the case of refractive index mismatching of immersion and coat of the preform. This wavefront is a basis for refractive index distribution in the object examined. The proposed method is based on the due correction of the wavefront obtained in the case of perfect matching of the immersion to the object core. This method is simple and effective for calculation of the refractive index distribution in the object core for all the types of interference involved. The measurement conditions which may be defined from the accuracy carried out analysis should be fulfilled in order to reconstruct the wavefront with the given accuracy as well as the range of applicability of the zero order approximation.

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Процедуры, корректирующие несогласование иммерсии при интерферометрическом определении профиля коэффициента преломления. I. Корректировка волнового фронта

Представленный метод корректировки волнового фронта является универсальным для всех типов интерференции (напр. с плоской волной отнесения, радиальный и поперечный shearing). Ее положительная черта заключается в возможности применения в расчетах алгоритма, используемого в случае согласования иммерсии.