Optica Applicata, Vol. XX, No. 1, 1990

Correction procedures of the immersion mismatching in interferometric determination of refractive index profile. Part IV. Correction of the interference orders for the transversal shearing case*

W. Kowalik, B. Dubik

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

In the method presented the orders of interference are subject to direct correction procedure which renders possibility of immediate usage of the algorithm employed for the case of perfect immersion-to-coat matching. The errors of the correction method have been analysed. The method of mismatching measurement for the refractive indices of the immersion liquid and the coat, respectively, has been proposed and accuracy of this measurement determined.

1. Introduction

This work is the fourth paper of the cycle [1]–[3] devoted to the problem of the accuracy with which the refractive index distribution in both the preforms and light waveguide may be determined in the case when the refractive indices in the immersion liquid and the coat of the measured object (preform, light waveguide) are mutually mismatched.

In the mathematical models used for wavefront reconstructions and calculations of refractive index distribution in the core of the measured object, it is usually assumed that: the wavefront is a continuous function, the object examined and the distribution of the refractive index inside the object are of cylindric symmetry and the refractive indices of both immersion liquid and the coat are usually equal to each other [4]. The deviations from those assumptions are the source of errors (sometimes very high).

In the present paper, the case will be considered when the refractive indices of the immersion liquid and the coat differ from each other.

* This work has been supported by CPBP 01.06.

2. Transversal shearing interferogram ($n_p \neq n_i$). Correction of the interference orders

The plane wavefront passing through the examined object perpendicularly to its symmetry axis suffers from the deformation within the regions of the core, the coat and the immersion liquid in which the object is suspended. The generated wavefronts

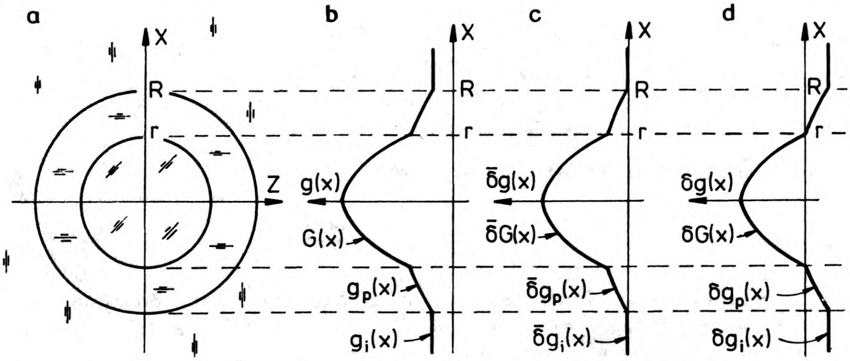

Fig. 1. Wavefronts after having passed through the object under test ($\delta n_i \neq 0$): **a** – object, **b** – wavefront, **c** – wavefront referred to the immersion, **d** – wavefront referred to the clad

(Fig. 1) referred to either the wavefront of the coat (Fig. 1**d**) or the immersion wavefront (Fig. 1**c**) gave the respective relative wavefronts:

– *in the core region* ($0 \leqslant |x| \leqslant r$)

$$\delta G(x) = \delta g(x) + \delta G_k(x), \qquad \bar{\delta} G(x) = \bar{\delta} g(x) + \bar{\delta} G_k(x); \tag{1}$$

– *in the coat region* ($r \leqslant |x| \leqslant R$)

$$\delta g_p(x) = 2\delta n_i[\sqrt{R^2-x^2} - \sqrt{R^2-r^2}], \qquad \bar{\delta} g_p(x) = 2\bar{\delta} n_i \sqrt{R^2-x^2}; \tag{2}$$

– *in the immersion liquid region* ($|x| \geqslant R$)

$$\delta g_i(x) = -2\delta n_i \sqrt{R^2-r^2}, \qquad \bar{\delta} g_i(x) = 0; \tag{3}$$

where $\delta G_k(x)$ and $\bar{\delta} G_k(x)$ are correcting wavefronts:

$$\delta G_k(x) = 2\delta n_i[\sqrt{R^2-x^2} - \sqrt{R^2-r^2}], \qquad \bar{\delta} G_k(x) = 2\bar{\delta} n_i \sqrt{R^2-x^2}. \tag{4}$$

The difference in the refractive indices of the coat n_p and the immersion liquid n_i has been denoted by δn_i

$$\delta n_i = n_p - n_i. \tag{5}$$

$\delta g(x)$ denotes the wavefront (referred to the coat) emerging from the core of the object in the case when $n_i = n_p$

$$\delta g(x) = 2\int_{0}^{z_r} \delta n(x)dz \tag{6}$$

where $\delta n(x)$ is the refractive index distribution in the object core (referred to the coat)

$$\delta n(x) = n(x) - n_p, \tag{6a}$$

and

$$z_r = \sqrt{r^2 - x^2}. \tag{6b}$$

All the above wavefronts (in the distinguished three regions), after having passed through the transverse shearing element, interfere with the same respective wavefronts shifted transversally by the value s with respect to the first ones [5]. Differences in optical paths between these wavefronts are recorded on the interferogram in form of interference fringes. The subject to the further analysis are the interference fringes produced by the wavefronts emerging from the region of the object core because of the information about the wavefront $\delta g(x)$ which is recorded there and necessary for calculation of the refractive index distribution $\delta n(x)$ in the object core.

In the region $-r+s \leqslant x \leqslant r$ (Fig. 2) either the wavefronts $\delta G_1(x)$ and $\delta G_2(x)$ (described in the coordinate system, as seen in Fig. 1**d**) or the wavefronts $\bar{\delta}G_1(x)$ and $\bar{\delta}G_2(x)$ (described in the coordinate system, as seen in Fig. 1**c**) interfere mutually. The optical path differences between those wavefronts amount, respectively, to:

$$\delta Z = \delta G(x-s) - \delta G(x), \quad \bar{\delta}Z = \bar{\delta}G(x-s) - \bar{\delta}G(x). \tag{7}$$

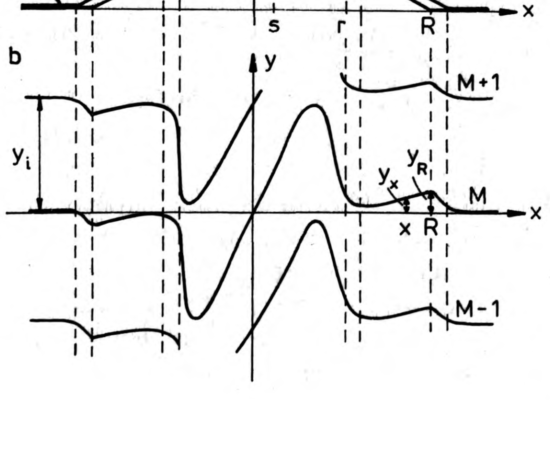

Fig. 2. Interfering wavefronts (**a**) and interferogram (**b**)

Taking account of the formula (1) and the relation betwen the optical path difference and the order of interference, it may be easily shown that the latter are described by the following expressions:

$$\delta M(x) = \delta m(x) + \delta M_k(x), \quad \bar{\delta} M(x) = \bar{\delta} m(x) + \bar{\delta} M_k(x) \tag{8}$$

where $\delta m(x)$ and $\bar{\delta} m(x)$ are the interference orders of the fringes recorded on the interferogram in the case when $n_p = n_i$:

$$\delta m(x) = (1/\lambda)(\delta g(x-s) - \delta g(x)), \quad \bar{\delta} m(x) = (1/\lambda)(\bar{\delta} g(x-s) - \bar{\delta} g(x)), \tag{9}$$

while $\delta M_k(x)$ is a correcting factor for the interference order of the fringes $\delta M(x)$ and $\bar{\delta} M(x)$ on the interferogram in the case when $n_p \neq n_i$. This correcting factor is identical in both the coordinate systems (Figs. 1**c**, **d**)

$$\delta M_k(x) = (2/\lambda)\delta n_i[\sqrt{R^2-(x-s)^2} - \sqrt{R^2-x^2}]. \tag{10}$$

Since the correcting factor $\delta M_k(x)$ for an interference order is proportional to both δn_i and R, therefore its course in Fig. 3 is given in the normed form $\delta M_k(x)/(\delta n_i R)$ for

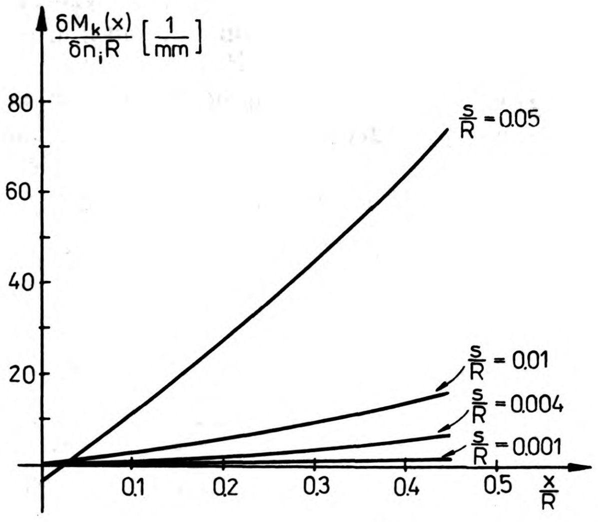

Fig. 3. Run of the function correcting the interference order $\delta M_k(x)$

different values of the parameter s/R (s – transversal shift of the wavefronts, R – radius core).

In order to find the correcting factor for the interference orders (10), it is necessary to know the value of mismatching δn_i of the refractive indices. The latter is usually found on the base of an additional measurement, the corresponding difference operator being denoted by $\bar{\delta}$ to distinguish it from the basic measurement. The additional measurement may be made in the same measuring system which was used for the basic one. In particular, δn_i may be determined from the same interferogram which was exploited in order to calculate $\delta n(x)$. The methods of the measurement of δn_i as well as the measuring setups are quite arbitrary. However, the most convenient way is to use the same setup for measuring δn_i and $\delta n(x)$.

3. Calculation of the difference δn_i between the refractive index of the immersion liquid and that of the phase object core, based on the additional interference measurement

Below, we will present the method of the measurement of δn_i performed in the setup used for the measurement of refractive index distribution in the object core $\delta n(x)$, i.e., in the transversal shearing interferometer [6]. In this measurement, the immersion liquid has been accepted as a reference level (and not the measured object coat as it was the case for basic measurement). In order to calculate δn_i, the information recorded on the transversal shearing interferometer has been exploited in the phase object core region. The optical path difference between the interfering wavefronts in this region $(r+\bar{s} \leqslant x \leqslant R)$ amounts to

$$\bar{\delta} z_p(x) = \bar{\delta} g_{p1}(x) - \bar{\delta} g_{p2}(x), \tag{11}$$

analogically as in Fig. 2a. By taking account of (11), the definition of the transversal shearing, as well as the relation between the optical path difference and the interference order, the following expression has been obtained:

$$\delta n_i = \frac{\lambda}{2} \frac{\bar{\delta} M_p(x)}{[\sqrt{R^2-(x-\bar{s})^2} - \sqrt{R^2-x^2}]}. \tag{12}$$

The changes of the interference order $\bar{\delta} M_p(x)$ have been measured by estimating the deviation of the fringes from rectilinearity

$$\bar{\delta} M_p(x) = y_x/y_i \tag{13}$$

where y_i is the interfringe spacing in the region of the immersion liquid, and y_x is the deviation of the fringe from the rectilinearity in the region of the object coat (Fig. 2a). In order the relative error of δn_i be minimal, the measurement is performed at the point where the deformation y_x of the fringe in the region of the object coat takes the maximum values, i.e., for $x = R$ (Fig. 2):

$$\delta n_i = \frac{\lambda}{2} \frac{\bar{\delta} M_p(R)}{\sqrt{\bar{s}(2R-\bar{s}}}, \quad \bar{\delta} M_p(R) = y_R/y_i. \tag{14}$$

4. Accuracy of the δn_i measurement

Since the accuracy of the measurement of the mismatching of the refractive indices of the immersion liquid and the measured object core δn_i influences the correction accuracy for both the wavefront $\delta g(x)$ [1], and the orders of interference $\delta m(x)$ it is important to analyse the factors affecting this accuracy. The absolute and relative errors of δn_i are defined as follows:

$$\Delta \delta n_i = \frac{\lambda}{2} \frac{\Delta \bar{\delta} M_p(R)}{\sqrt{\bar{s}(2R-\bar{s})}} + \delta n_i \frac{\bar{s} \Delta R + (R-\bar{s}) \Delta \bar{s}}{\bar{s}(2R-\bar{s})}, \tag{15}$$

$$\frac{\Delta\delta n_i}{\delta n_i} = \frac{\lambda}{2}\frac{\Delta\bar{\bar{\delta}} M_p(R)}{\sqrt{\bar{s}(2R-\bar{s})}}\frac{1}{\delta n_i} + \frac{\bar{s}\Delta R+(R-\bar{s})\Delta\bar{s}}{\bar{s}(2R-\bar{s})} \tag{16}$$

where $\Delta\bar{\bar{\delta}} M_p(R)$, ΔR, Δs are the respective accuracies of the measurements of the fringe interference order at the point $x = R$, radius of the coat R, and the transversal shift s of the interfering wavefronts. The purpose of the accuracy analysis carried out

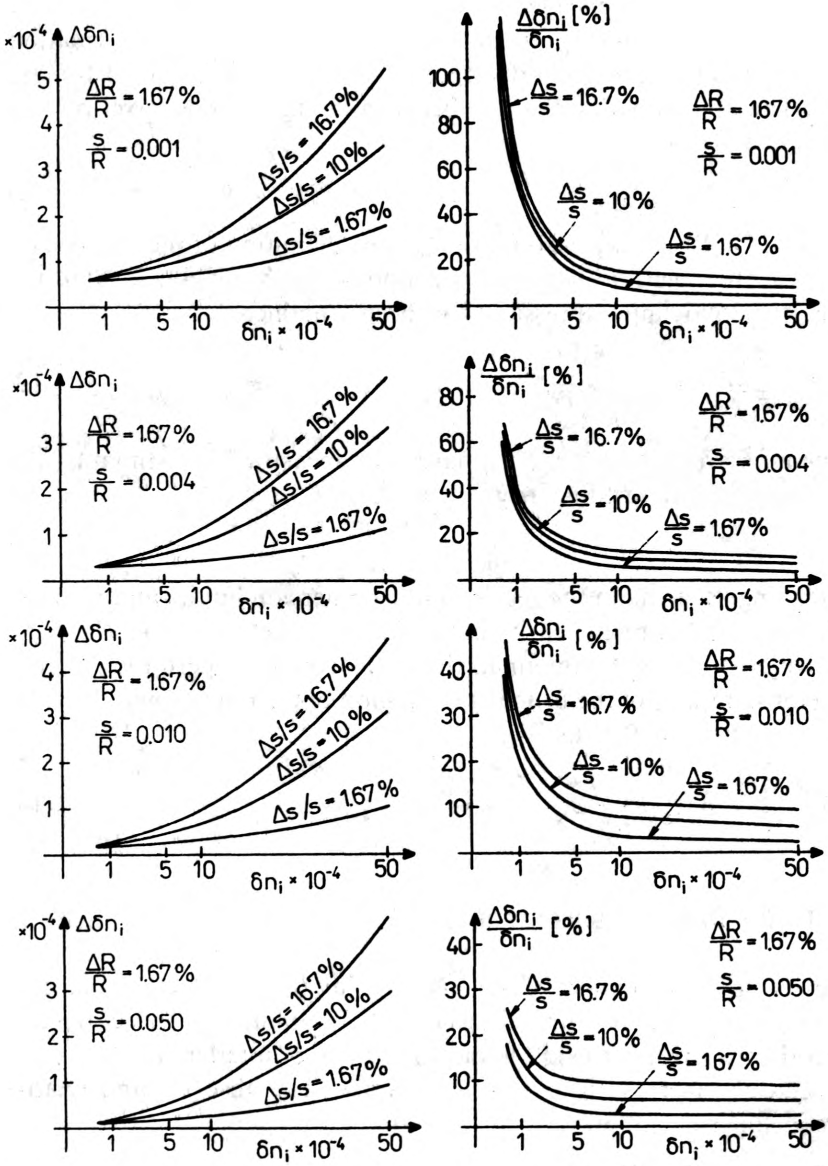

Fig. 4. Absolute and relative measurements errors for the refractive index difference δn_i for an established accuracy of the coat diameter measurement $\Delta R/R = 1.67\%$. Case of the preform $R = 6 \times 10^{-3}$ m

below, is to determine such measured parameters for which the measurement error is still within the admissible limits. The absolute errors increase nonlinearly, while the relative errors $\Delta\delta n_i/\delta n_i$ diminish nonlinearly with the increase of the measured mismatching δn_i. For the established errors of the object geometry $\Delta R/R$, the influence of the transversal shift s on the above errors is slight while there exists a distinct influence of the relative error $\Delta s/s$ (Fig. 4). The greatest influence on the

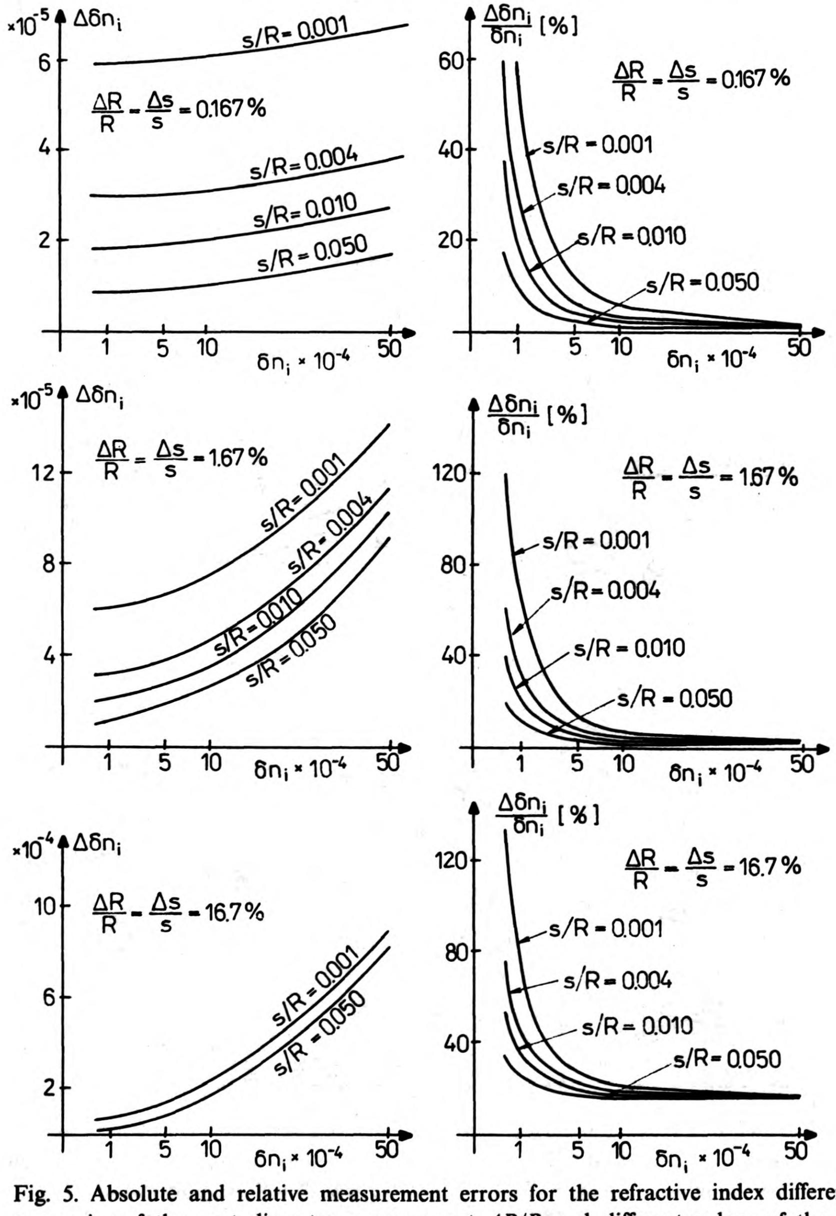

Fig. 5. Absolute and relative measurement errors for the refractive index difference δn_i for different accuracies of the coat diameter measurement $\Delta R/R$ and different values of the transversal shearing parameter $\Delta s/s$. Case of the preform $R = 6 \times 10^{-3}$ m

measurement errors is due to the measurement errors of the examined geometry $\Delta R/R$ (it has been assumed that $\Delta R/R = \Delta s/s$). The higher is the value $\Delta R/R$ the higher nonlinearity of the dependence of the error $\Delta \delta n_i$ on δn_i, being the greater the higher the value R (Figs. 5–7). From the given relations, it is possible to determine the conditions which must be fulfilled to attain the required accuracies. In the relations presented in Figs. 4–7, the errors introduced by the application of the zero-order approximation were not taken into account.

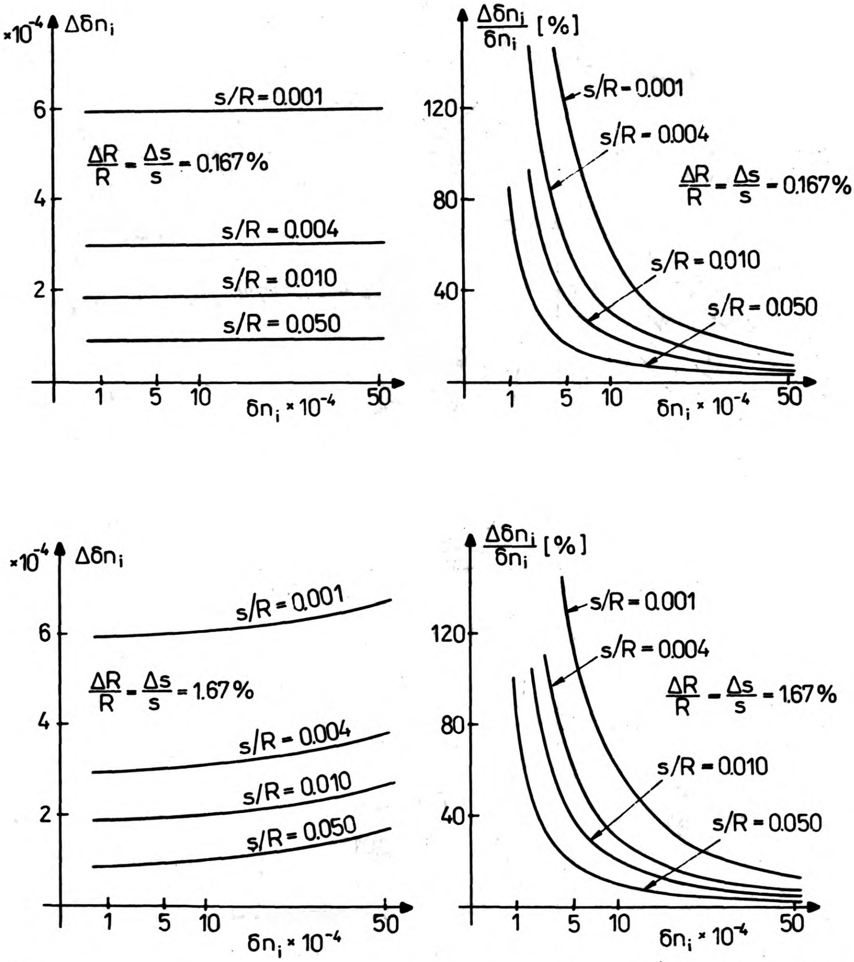

Fig. 6. Absolute and relative measurement errors of the refractive index difference δn_i for different accuracies of both the coat diameter measurement $\Delta R/R$ and the transversal shearing parameter $\Delta s/s$. Case of the thick-core waveguide $R = 6 \times 10^{-4}$ m.

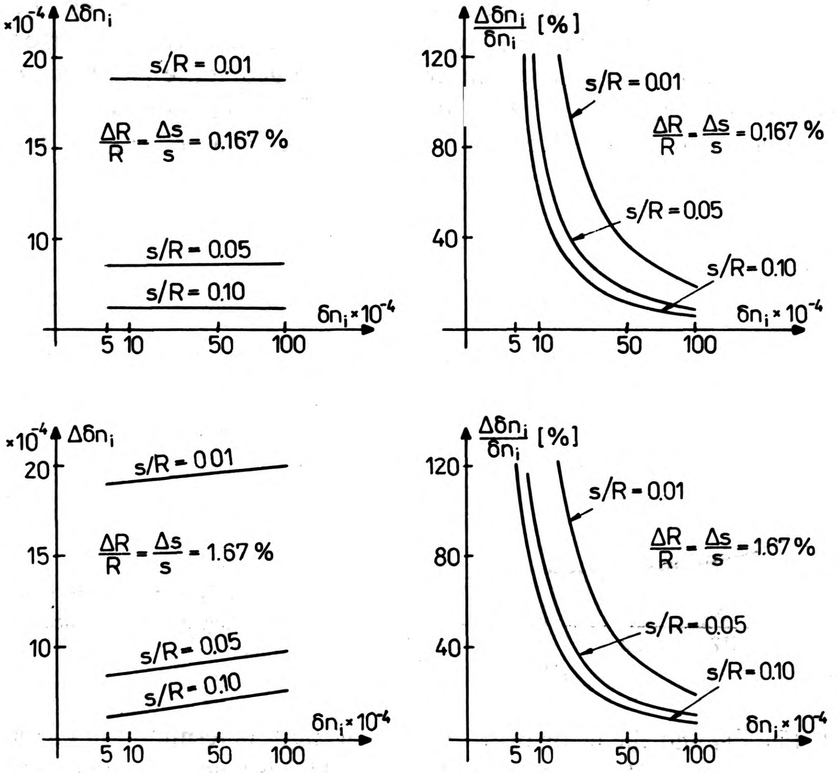

Fig. 7. Absolute and relative measurement errors of the refractive index difference δn_i for different accuracies of both the coat diameter measurement $\Delta R/R$ and the transversal shearing parameter $\Delta s/s$. Case of the light waveguide $R = 6\times 10^{-5}$ m

5. Correction accuracy for the interference orders

The relative error of the correcting factor $\delta M_k(x)$ is defined as follows:

$$\frac{\Delta\delta M_k(x)}{\delta M_k(x)} = \frac{\Delta\delta n_i}{\delta n_i} + \frac{\Delta\lambda}{\lambda} + A'\Delta R + B'\Delta x + C'\Delta s \tag{17}$$

where:

$$A' = \left|\frac{R}{\sqrt{R^2-(x-s)^2}-\sqrt{R^2-x^2}}\left[\frac{1}{\sqrt{R^2-(x-s)^2}}-\frac{1}{\sqrt{R^2-x^2}}\right]\right|,$$

$$B' = \left|\frac{1}{\sqrt{R^2-(x-s)^2}-\sqrt{R^2-x^2}}\left[\frac{x-s}{\sqrt{R^2-(x-s)^2}}-\frac{x}{\sqrt{R^2-x^2}}\right]\right|,$$

$$C' = \left|\frac{1}{\sqrt{R^2-(x-s)^2}-\sqrt{R^2-x^2}}\left[\frac{x-s}{\sqrt{R^2-(x-s)^2}}\right]\right|. \tag{18}$$

Its value has been determined for the class of the object characterized by the parameter s/R (Fig. 8). As it may be seen from (17), the relative error $\Delta\delta M_k(x)/\delta M_k(x)$ depends on the measurement errors of the geometric parameters, i.e., those of the coat radius ΔR, of the core radius Δr and of the position Δx, as well as on the mismatching of the refractive index of the object coat to that of the

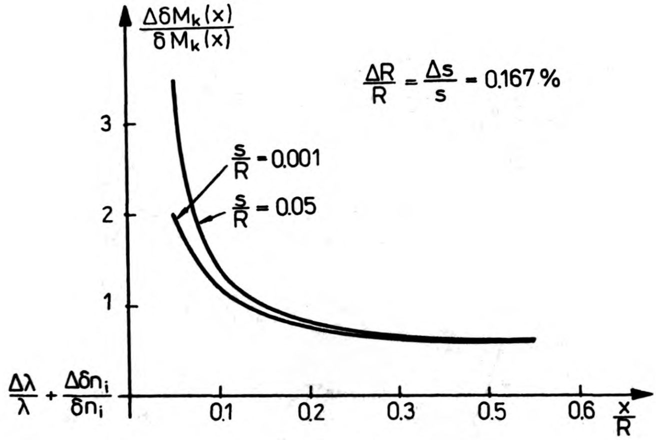

Fig. 8. Relative error of the function correcting the interference order $\delta M_k(x)$

immersion liquid $\Delta\delta n_i/\delta n_i$ and the error of $\Delta\lambda/\lambda$ of wavelength determination of the light used to the measurements. In most cases, the value $\Delta\lambda/\lambda$ is negligibly small and the value $\Delta\delta n_i/\delta n_i$ is estimated in the way shown in the previous section. The value of the error $\Delta\delta M_k(x)/\delta M_k(x)$ depends linearly on the accuracy with which the coat diameter and the transversal shift are measured, i.e., if $\Delta R/\mathrm{R} = \Delta s/s$ increases by an order of magnitude the error $\Delta\delta M_k(x)/\delta M_k(x)$ increases also by the same order of magnitude. As it may be seen from Fig. 8, the relative error of the correcting factor for the interference order $\delta M_k(x)$ decreases with the increase of x. Such dependence has been obtained under assumption that $x = (0.5)\Delta R$.

6. Summarizing remarks

The measurement of the refractive index profile carried out by using the methods of shearing interferometry is possible for both the cases of matching and mismatching the refractive index of the immersion liquid to that of the examined object (preform, waveguide) coat. The second case was worth considering since the perfect matching is difficult to achieve or employs some tedious procedures. The measurement method proposed in this work is simple, being based on correction of the interference orders measured on the interferogram. The presented analysis of the measurement accuracies enables us to select the measurement conditions which should be fulfilled in order to achieve the required accuracies.

The proposed method of calculation of the difference of the refractive indices of the immersion liquid and that of the coat of the examined object may be exploited also for the purposes other than those discussed in this work. The accuracy of the refractive index profile may be increased by applying more than accurate methods of wavefront calculation, for instance, those given in [1].

References

[1] Kowalik W., Dubik B., Opt. Appl. **20** (1990), 3.
[2] Kowalik W., Dubik B., Opt. Appl. **20** (1990), 17.
[3] Kowalik W., Dubik B., Opt. Appl. **20** (1990), 33.
[4] Kokubun Y., Iga K., Trans. IECE Jpn. **E 60** (1977), 702.
[5] Kowalik, W., Dubik B., Report no. SPR–209/87, Institute of Physics, Technical University of Wrocław, Wrocław 1987 (in Polish), unpublished.
[6] Kowalik W., Michalski K., SPIE **670** (1986), 82.

Received March 20, 1989
in revised form November 6, 1989

Процедуры, корректирующие несогласование иммерсии при интерферометрическом определении профиля коэффициента преломления. IV. Корректировка порядка интерференции для случая интерференции поперечный shearing

В представленном методе корректировка проводится непосредственно на порядках интерференции, что дает возможность выполнения расчетов по алгоритму, применяемому для примера согласования иммерсии оболочки исследуемого объекта. Проведен анализ погрешностей метода корректировки. Дан метод измерения величины несогласования коэффициентов преломления иммерсионной жидкости и оболочки исследуемого объекта, а также определена степень точности этого измерения.