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Contribution to the theory of the integrated optical loop resonator

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A relation between the output electric field amplitude and the input one is derived for the integrated optical loop resonator with the help of two coupled waves equations. Then the result relation is used to analyse some specific configurations of the loop resonator.

1. Introduction

The integrated optical loop resonator is formed by three strip planar waveguides lying close to one another (Fig. 1): two of them (I and II) being open and one in between (III) in a shape of a closed track.

The working principle of this device consists in the light coupling from the input waveguide I to the resonant loop II and then to the output waveguide III. The compatibility with optical fibers and simple construction enable this device to be employed in many different applications. For instance, it can work as a temperature, electric or magnetic field and acoustic wave sensors [1], [2].

In this paper, the dependence of the output electric field amplitude on the input amplitude is derived for the optical loop resonator. Up to the author's knowledge such characteristics for a similar kind of optical device have so far been presented in

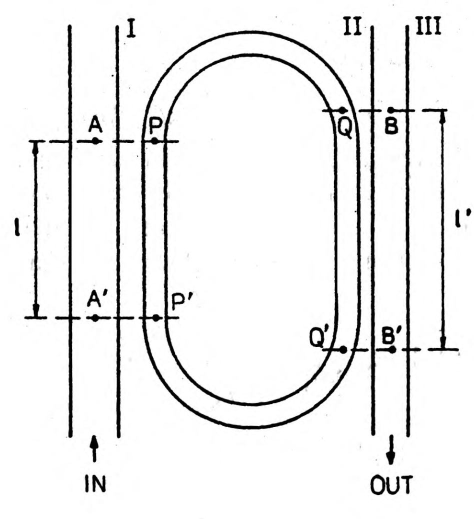

Fig. 1 Integrated optical loop resonator (top view): I is the input waveguide with complex propagation constant γ_1, II is the resonant loop waveguide with complex propagation constant γ_2, III is the output waveguide with complex propagation constant γ_3, l, l' are the interaction lengths

works [3], [4]. However, the present work seems to be more general because our resonator consists not only of the same waveguides. Additionally, the working principle of these two devices is not the same.

In order to obtain this relation, it is necessary to solve equations for two coupled waves (presented in Sect. 2), and then to apply the result to the analysis of the light propagation in the resonator configuration (presented in Sect. 3). The analysis of typical resonator structures is given in Sect. 4 and 5.

2. Two coupled waves equations

A comprehensive theory of coupled waves has been presented for instance in papers [5], [6] or [7]. Let us assume that all three waveguides are single mode, so coupling there is between two waves only. So, we can take into account two coupled waves equation [6]:

$$\frac{dE_1}{dz} = -\gamma_1 E_1 + c_{21} E_2, \tag{1a}$$

$$\frac{dE_2}{dz} = -\gamma_2 E_2 + c_{12} E_1 \tag{1b}$$

where: E_1, E_2 – complex wave amplitudes,
γ_1, γ_2 – complex propagation constants (small perturbations of γ_1 and γ_2 are neglected: we are within the framework of weak coupling approximation),
c_{12}, c_{21} – coupling functions (dependent on z),
z – direction of the wave propagation,

with initial conditions:

$$E_1(z=0) = E_{1\mathrm{w}},\quad E_2(z=0) = E_{2\mathrm{w}}. \tag{2}$$

The initial condition $E_2(z=0) = E_{2\mathrm{w}} \neq 0$ reflects the fact that the light in the loop waveguide (Fig. 1) does not vanish when it arrives at $z = 0$ after its round trip.

Additionally, let us assume that waveguides are parallel to each other along the interaction regions, so the values of functions c_{12} and c_{21} are constant and equal to $\bar{c}_{12}$, $\bar{c}_{21}$, respectively. Then, by imposing the conditions (2) on Eq. (1) we obtain the following solution of Eq. (1):

$$E_1 = \exp\left(-\frac{\gamma_1+\gamma_2}{2}z\right)\left[\frac{2\bar{c}_{21}E_{2\mathrm{w}}+(R-\gamma_1+\gamma_2)E_{1\mathrm{w}}}{2R}\exp\left(\frac{R}{2}z\right)\right.$$
$$\left.+\frac{-2\bar{c}_{21}E_{2\mathrm{w}}+(R+\gamma_1-\gamma_2)E_{1\mathrm{w}}}{2R}\exp\left(-\frac{R}{2}z\right)\right], \tag{3a}$$

$$E_2 = \exp\left(-\frac{\gamma_1+\gamma_2}{2}z\right)\left[\frac{2\bar{c}_{21}E_{2w}+(R+\gamma_2-\gamma_1)E_{1w}}{4\bar{c}_{21}R}(\gamma_1-\gamma_2+R)\exp\left(\frac{R}{2}z\right)\right.$$
$$\left.+\frac{-2\bar{c}_{21}E_{2w}+(R-\gamma_2+\gamma_1)E_{1w}}{4\bar{c}_{21}R}(\gamma_1-\gamma_2-R)\exp\left(-\frac{R}{2}z\right)\right] \tag{3b}$$

where $R=\sqrt{(\gamma_1-\gamma_2)^2+4\bar{c}_{21}\bar{c}_{12}}$.

Throughout this paper this result will be used to analyse the coupling between waveguides I and II and between waveguides II and III of the loop resonator (Fig. 1). In the first case the form (3) will be used. In the second one the change of propagation constants and coupling coefficients yields:

$$E'_1 = \exp\left(-\frac{\gamma_2+\gamma_3}{2}z\right)\left[\frac{2\bar{c}_{32}E'_{2w}+(R'-\gamma_2+\gamma_3)E'_{1w}}{2R'}\exp\left(\frac{R'}{2}z\right)\right.$$
$$\left.+\frac{-2\bar{c}_{32}E'_{2w}+(R'+\gamma_2-\gamma_3)E'_{1w}}{2R'}\exp\left(-\frac{R'}{2}z\right)\right], \tag{4a}$$

$$E'_2 = \exp\left(-\frac{\gamma_2+\gamma_3}{2}z\right)\left[\frac{2\bar{c}_{32}E'_{2w}+(R'-\gamma_2+\gamma_3)E'_{1w}}{4\bar{c}_{32}R'}(\gamma_2-\gamma_3+R')\exp\left(\frac{R'}{2}z\right)\right.$$
$$\left.+\frac{-2\bar{c}_{32}E'_{2w}+(R'+\gamma_2-\gamma_3)E'_{1w}}{4\bar{c}_{32}R'}(\gamma_2-\gamma_3-R')\exp\left(-\frac{R'}{2}z\right)\right], \tag{4b}$$

where: $R'=\sqrt{(\gamma_2-\gamma_3)^2+4\bar{c}_{23}\bar{c}_{32}}$,
$\bar{c}_{23}$, $\bar{c}_{32}$ – values of coupling coefficient for waveguides II and III.
These forms will be used when necessary.

3. Output/input characteristics

The solutions (3) and (4) of coupling Equations (1) enables us to analyse the performance of the integrated optical loop resonator and to calculate its output /input characteristics.

Let us consider a device as in Fig. 1. Its strip waveguides are characterized by their propagation constants γ_1, γ_2 and γ_3, respectively. A part of light propagating in the input waveguide I is coupled into the loop waveguide II along the interaction length l (from the point A' to A). During its round trip in the loop the light is again partially coupled into the waveguide III along the second interaction length l' (from B to B'). After many round trips in the loop a steady-state amplitude is attained in all three waveguides. High level of the output wave amplitude can be obtained in two following cases:

i) The efficiency of coupling is not 0% and the resonance exists in the loop:

$$\beta d = \pm 2n\pi \tag{5}$$

where d is the loop length, $\beta = \mathrm{Im}(\gamma)$, γ is the average value of propagation constant in the loop and n is natural.

ii) The efficiency of coupling from the waveguide I into II is not 0% and the efficiency of coupling from the waveguide II into III is 100% (no resonant condition is then necessary).

We will analyse the light propagation in the resonator under assumption that one of situations i) or ii) takes place.

The wave amplitudes will be denoted as follows:

$E_{\mathrm{IN}}(A')$ – the input complex wave amplitude (in the waveguide I at point A'),

E_{L} – the complex wave amplitude in the waveguide II,

$E_{\mathrm{OUT}}(B')$ – the output complex wave amplitude (in the waveguide III at point B').

Accordingly the notation $E_{\mathrm{L}}(P)_i$ is applied to the wave amplitude in the loop waveguide at point P during the i-th round trip of light, and $E_1\{E_{2\mathrm{w}} = x,\ E_{1\mathrm{w}} = y,\ z = w\}$ denotes the value of the function E_1 when $E_{2\mathrm{w}} = x$, $E_{1\mathrm{w}} = y$ and $z = w$.

In the beginning of our consideration the light is launched into the waveguide I. We take its amplitude at A' to be E_{IN}. Simultaneously there is no light in the loop waveguide II, so at P' (the beginning of the interaction region) $E_{2\mathrm{w}} = 0$. Then the wave is coupled into the loop and its amplitude at P is given by Eq. (3b) with $z = l$ (l is the length of an interaction region) and $E_{1\mathrm{w}} = E_{\mathrm{IN}}(A')$

$$E_{\mathrm{L}}(P)_1 = E_2\{E_{2\mathrm{w}} = 0,\ E_{1\mathrm{w}} = E_{1\mathrm{w}}(A'),\ z = l\}$$

$$= E_{\mathrm{IN}}(A')\exp\left(-\frac{\gamma_1+\gamma_2}{2}\,l\right)\frac{[(\gamma_1-\gamma_2)^2-R^2]}{4\bar{c}_{21}R}\left[\exp\left(-\frac{R}{2}\,l\right)-\exp\left(\frac{R}{2}\,l\right)\right]. \quad (6)$$

Continuing its first round trip the light propagates along the loop and its amplitude is reduced because of optical losses (the same occurs of course along interaction region) and attenuation on bends. Then it comes across the coupling path of waveguides II and III and is partially coupled into the output waveguide III, as given by Eq. (4a) with $E'_{2\mathrm{w}} = 0$, $E'_{1\mathrm{w}} = E_{\mathrm{L}}(Q)_1$ and $z = l'$ (l' is the length of the second interaction region). At the arrival back to P' its amplitude is given by

$$E_{\mathrm{L}}(P')_2 = s'\exp(-\gamma_2 w)\mathrm{E}'_1\{\mathrm{E}'_{2\mathrm{w}} = 0,\ E'_{1\mathrm{w}} = sE_{\mathrm{L}}(P)_1\exp(-\gamma_2 w'),\ \mathrm{z} = l'\}, \quad (7)$$

where: $w+w' = d-l-l'$, s, s' are coefficients responsible for the attenuation on the loop bends, and d is a loop length.

Noting that in the case of $E_{2\mathrm{w}} = 0$ the factor describing the wave amplitude at Q is common in all parts of Eq. (7), we can extract it and impose $E_{1\mathrm{w}} = 1$, so that the formula (7) can be reduced to the form

$$E_{\mathrm{L}}(P')_2 = sE_{\mathrm{L}}(P)_1\exp[-\gamma_2(d-l-l')]\mathrm{E}'_1\{\mathrm{E}'_{2\mathrm{w}} = 0,\ E'_{1\mathrm{w}} = 1,\ \mathrm{z} = l'\}. \quad (8)$$

During the second trip in the loop the situation is similar to that at the beginning: the wave amplitude is given by Eq. (3a) with $E_{1\mathrm{w}} = E_{\mathrm{IN}}(A')$ and $z = l$, but now there is light propagates in the loop, so $E_{2\mathrm{w}} = E_{\mathrm{L}}(P')_2$ and we have

$$E_L(P)_2 = E_2\{E_{2w} = E_L(P')_2,\ E_{1w} = E_{IN}(A'),\ z = l\}, \tag{9}$$

which, after simple conversions, can be rewritten as

$$E_L(P)_2 = E_L(P)_1 + BE_L(P)_1 \tag{10}$$

where

$$B = \frac{\exp\left(-\frac{\gamma_1+\gamma_2}{2}l\right)\exp[-\gamma_2(d-l-l')]}{2R} sE'_1\{E'_{2w} = 0,\ E'_{1w} = 1,\ z = l'\}$$
$$\times\left[(\gamma_1-\gamma_2+R)\exp\left(\frac{R}{2}l\right)-(\gamma_1-\gamma_2-R)\exp\left(-\frac{R}{2}l\right)\right]. \tag{11}$$

After many round trips in the loop the wave amplitude on its n–th arrival to P' is given as

$$E_L(P')_n = sE_L(P)_{n-1}\exp[-\gamma_2(d-l-l')]E'_1\{E'_{2w} = 0,\ E'_{1w} = 1,\ z = l'\} \tag{12}$$

The situation at P is as follows: the wave amplitude is given by Eq. (3a) with $E_{1w} = E_{IN}(A')$, $z = l$ and $E_{2w} = E_L(P')_n$, so we can write

$$E_L(P)_n = E_2\{E_{2w} = E_L(P')_n,\ E_{1w} = E_{IN}(A'),\ z = l\} = E_L(P)_1 + BE_L(P)_{n-1}$$
$$= E_L(P)_1 + BE_L(P)_1 + B^2E_L(P)_1 + \ldots + B^{n-1}E_L(P)_1. \tag{13}$$

Let us note that the constant B which is expressed by Eq. (11), has an evident physical meaning. From Eq. (10) we have

$$B = \frac{E_L(P)_2 - E_L(P)_1}{E_L(P)_1} = \frac{A\exp(i\varphi_1) + A\exp(-\alpha d + i\varphi_2) - A\exp(i\varphi_1)}{A\exp(i\varphi_1)} \tag{14}$$

where A is real, α is the optical loss coefficient in the loop, φ_1 is the phase of the wave coupled into the loop, and φ_2 is the phase of the wave after full lap. Assuming that there exists the resonance in the loop, $\exp(i\varphi_1)$ equals $\exp(i\varphi_2)$ and then from Eq. (14) we get

$$B = \exp(-\alpha d) < 1,\ \text{if}\ \alpha \neq 0. \tag{15}$$

Now, let us return to Eq. (13). Its right-hand side forms a geometrical series. Since $B < 1$ we can find a steady-state wave amplitude at P as a sum of infinite series

$$E_L(P) = E_L(P)_n = \lim_{n\to\infty}\sum_{k=1}^{n} E_L(P)_1B^{k-1} = \lim_{n\to\infty}\left[E_L(P)_1\frac{1-B^n}{1-B}\right] = \frac{E_L(P)_1}{1-B}. \tag{16}$$

Inserting B, given by Eq. (11), into Eq. (16) we obtain the following relation:

$$E_L(P) = \frac{N}{D}E_{IN}(A') \tag{17}$$

where:

$$N = [(\gamma_1-\gamma_2)^2-R^2]\left[\exp\left(-\frac{R}{2}l\right)-\exp\left(\frac{R}{2}l\right)\right]\exp\left(-\frac{\gamma_1+\gamma_2}{2}l\right),$$

$$D = 4\bar{c}_{21}R-2\bar{c}_{21}s\exp[-\gamma_2(d-l-l')]\exp\left(-\frac{\gamma_1+\gamma_2}{2}l\right)$$
$$\times\left[(\gamma_1-\gamma_2+R)\exp\left(\frac{R}{2}l\right)-(\gamma_1-\gamma_2-R)\exp\left(-\frac{R}{2}l\right)\right]$$
$$\times E'_1\{E'_{2w}=0,\ E'_{1w}=1,\ z=l'\}.$$

Now, using Eq. (4b) we can derive an output wave amplitude as a result of coupling from waveguide II into III

$$E_{\mathrm{OUT}}(B') = E'_2\{E'_{2w}=0,\ E'_{1w}=s'E_{\mathrm{L}}(P)\exp[-\gamma_2(w')],z=l'\}. \tag{18}$$

After some algebra we find the final form of the relation describes the wave amplitude propagates in the output waveguide

$$E_{\mathrm{OUT}}(z) = E_{\mathrm{IN}}(A')\frac{sN}{4\bar{c}_{32}DR'}\exp\left(-\frac{\gamma_2+\gamma_3}{2}l'\right)\exp[-\gamma_2(w')]$$
$$\times\left[(R'-\gamma_2+\gamma_3)(R'+\gamma_2-\gamma_3)\exp\left(\frac{R'}{2}l'\right)\right.$$
$$\left.+(R'+\gamma_2-\gamma_3)(-R'+\gamma_2-\gamma_3)\exp\left(-\frac{R'}{2}l'\right)\right]\exp(-\gamma_3 z). \tag{19}$$

4. Particular configurations

For symmetrical structure as in Fig. 2 the waveguides I and III are identical. Then we can write $\gamma_1=\gamma_3$, $l=l'$, $\bar{c}_{12}=\bar{c}_{23}$, $\bar{c}_{21}=\bar{c}_{32}$, in a consequence, Eq. (19) may be reduced to the form

$$E_{\mathrm{OUT}}(z) = E_{\mathrm{IN}}(A')\frac{N'}{D'}\exp(-\gamma_1 z) \tag{20}$$

where:

$$N' = s\exp\left(-\gamma_1 l-\gamma_2\frac{d}{2}\right)[(\gamma_1-\gamma_2)^2-R^2]^2\left[\exp\left(-\frac{R}{2}l\right)-\exp\left(\frac{R}{2}l\right)\right]^2,$$

$$D' = (4\bar{c}_{21}R)^2-4\bar{c}_{21}{}^2s^2(\gamma_1-\gamma_2+R)^2-[(\gamma_1-\gamma_2)^2-R^2]$$
$$\times[\exp(Rl)+\exp(-Rl)]+(\gamma_1-\gamma_2-R)^2\exp[-\gamma_1 l-\gamma_2(d-l)].$$

Another particular case is a symmetrical structure with a loop waveguide the same like the others (Fig. 3). Then $\gamma_1=\gamma_2=\gamma_3=\gamma$ and coupling functions are both

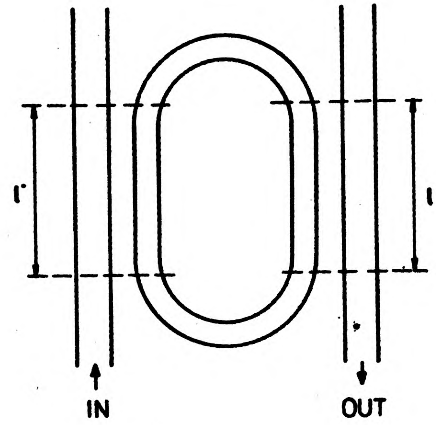

Fig. 2. Symmetrical integrated optical resonator

constant and equal to ic [6]. Thus Eq. (19) takes the following form:

$$E_{\mathrm{OUT}}(z) = \frac{s\sin^2(cl)\exp\left[-\gamma\left(l+\frac{d}{2}\right)\right]}{s^2\cos^2(cl)\exp(-\gamma d)-1}E_{\mathrm{IN}}(A')\exp(-\gamma z). \tag{21}$$

Now, let us consider the attenuations on the bends, formally represented in our equations by the parameter s. They can be neglected when [8]

$$r > \frac{24\pi^2|\xi|^3}{\lambda^2} \tag{22}$$

where: λ – wavelength in the vacum,
ξ – distance at which the wave amplitude in the neighbouring medium decreases $1/e$ times,
r – radius of the outside bend curvature.

For example: for $\lambda = 0.63$ μm and the refractive index of the single-mode waveguide $n_1 = 1.5$, the 1% attenuation along the distance of 1 mm takes place when $r = 1$ mm [8].

The radius r in such structures is usually of order of 0.5 mm or grater (for instance, see [9]). Assuming that bend losses may be neglected Eq. (21) can be written as

$$E_{\mathrm{OUT}}(z) = \frac{\sin^2(cl)\exp\left[-\gamma\left(l+\frac{d}{2}\right)\right]}{\cos^2(cl)\exp(-\gamma d)-1}E_{\mathrm{IN}}(A')\exp(-\gamma z). \tag{23}$$

5. Discussion of the dependence of output wave power on resonator's parameters

Let us examine the behaviour of the light propagates in the output waveguide in the dependence on the resonator's parameters c and l. c is strongly dependent on the separation distance w between two coupled strip waveguides [10]

$$c = C\exp(-w/\Omega) \tag{24}$$

where C, Ω are coefficients.

For convenience, we examine a power of output wave instead of the wave amplitude, which may have sometimes a complex value (see Eq. (19)). We take into account the simplest case, e.g., a relation (23), where we impose $z = 0$ (point B').

The Figure 3 illustrates the functional dependence of $P_{OUT} = |E_{OUT}(B')|^2$ on c with fixed value of d and l, for $\alpha = \mathrm{Re}(\gamma) = 0.1$, 0.01, and 0.001, respectively. Let us note periodical behaviour of this characteristic with period equal to π/l, and the maximum value $P_{max} = \exp[-2\alpha(l\text{-}d/2)]$, when $P_{IN}(A')$ is normalized to unity. We obtain an exact repeatibility of energy transfer with the increasing c. It means that the coupling efficiency has also exact repeatibility, which is surprising since we know that increasing c means decrising the distance between waveguides. The characteristic has a relatively wide plateaus about its minimum. It is related to lack of transfer light into the loop waveguide. The light travels only in the input waveguide.

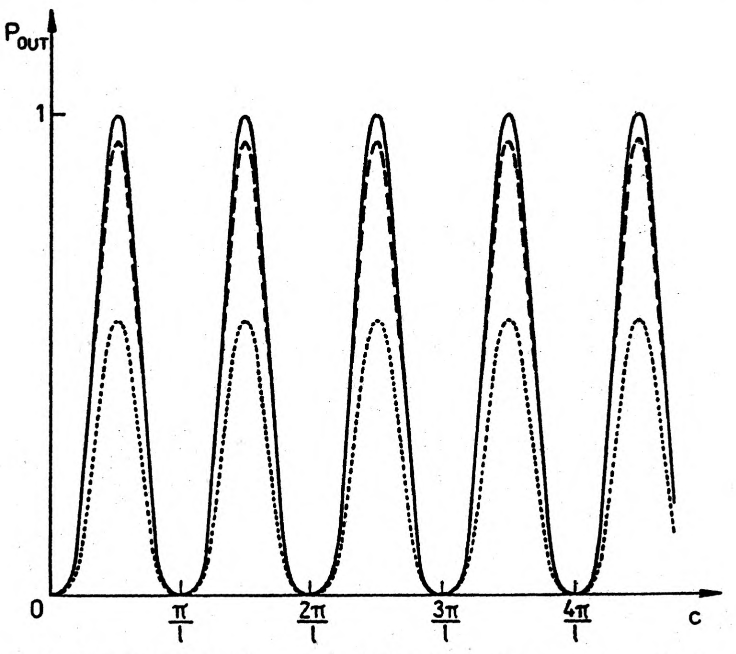

Fig. 3. Output power of the symmetrical loop resonator vs the coupling coefficient c (Eq. (23)). The dotted, dashed and solid lines correspond to the values of the loss coefficient $\alpha = 0.1$, 0.01, 0.001 cm^{-1}, respectively; $l = 0.3$ cm is an interaction length. The assumed value of the loop length d is 5 cm, $\beta = 150000\ \mathrm{cm}^{-1}$

The curves describing P'_{OUT} in terms of l (Fig. 4) are in general similar to those in terms of c, but these two characteristics (Fig. 3 and Fig. 4) are not the same. For c increasing, the curves in Fig. 4 have exactly the same shape of period. On the order hand, when l increases, the successive values of P'_{OUT}, related to the same points of period, decrease. It may be explained as follows. For each point of curve, there are two parts of light. One part makes its trip round the loop. When l increases, with step equal to period, the route of this part is always of the same length ($d =$ const), thus attenuation is the same, too. But the other part of the light propagates only along $l + d/2$. When l increases, and becomes the larger part of d (see Fig. 3), the route of this light is longer, hence its attenuation is larger and P'_{OUT} smaller.

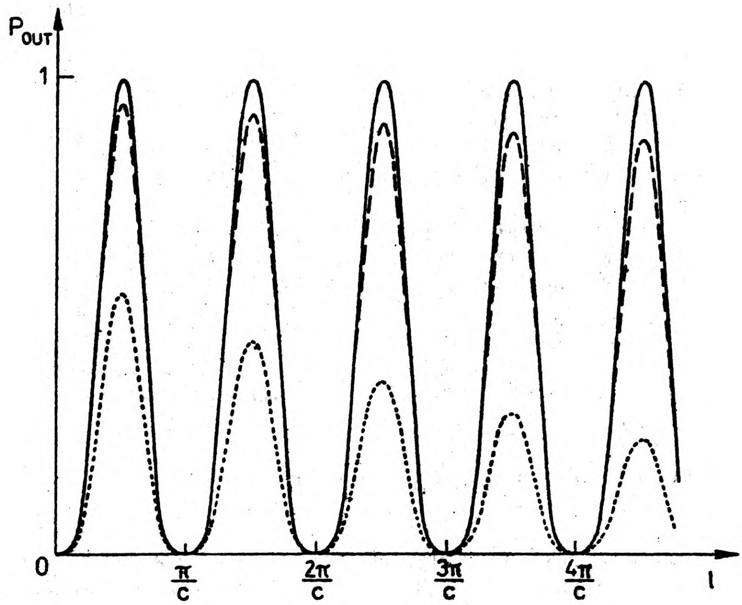

Fig. 4. Output power of the symmetrical loop resonator vs interaction length l (Eq. (23)). The dotted, dashed, and solid lines correspond to the values of the loss coefficient $\alpha = 0.1$, 0.01, 0.001 cm^{-1}, respectively; $c = 3$ cm^{-1} is the coupling coefficient. The assumed value of the loop length d is 5 cm, $\beta = 150000$ cm^{-1}

6. Summary

The expression of the output wave amplitude as a function of the input wave amplitude, propagations constants, coupling coefficients and length of interaction region, has been derived for an integrated optical coupler with waveguide loop resonator, assuming that the resonance condition in the loop is satisfied or that the efficiency of coupling from the waveguide II into III is 100%. It is shown that the output wave amplitude is proportional to the input one.

This relation has been derived with a help of coupled wave equations for two waves. Then the relation between input and output light power, and some resonator's parameters were discussed for two simple configurations of the optical loop resonator.

We hope that the results of this paper may be useful in designing such devices.

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Доклады к теории интегрального оптического резонатора с кольцевым волноводом

Для интегрального оптического резонатора, на основании уравнения связанных волн, определили зависимость между входной и выходной амплитудами света. Сделали анализ, пользуясь выведенной зависимостью для некоторых конфигураций резонаторов.