Optica Applicata, Vol. XX, No. 3, 1990

Optical anisotropy in fibres with irregular cross-sections and having a skin-core structure

A. A. HAMZA, T. Z. N. SOKKAR, M. A. KABEEL

Department of Physics, Faculty of Science, Mansoura University, Mansoura, Egypt.

Mathematical expressions are derived for the case of two-beam interference fringes crossing fibres having an irregular cross-sectional core surrounded by an irregular skin. These expressions are used for the determination of refractive indices and birefringence of nylon 6 fibres. The Pluta polarizing interference microscope was used with white and monochromatic light of wavelength 546 nm. Microinterferograms are given for illustration.

1. Introduction

Most fibres are anisotropic materials as a result of fibre spinning and drawing. Two-beam and multiple-beam microinterferometry are used efficiently for studying optical anisotropy in polymer fibres. These techniques were used to determine the refractive indices and birefringence of fibres (see, for example, [1]–[7]). Interferometric studies on fibres throw light on the molecular orientation and density fluctuations in fibres. Homogeneous and heterogeneous fibres can be studied interferometrically [8].

Accurate results of refractive indices and birefringence measurements of fibres having irregular cross-sections are obtained when considering the area under the interference fringe shift representing the optical path difference integrated across the fibres [9], [4], [5]. The two-beam interferometric technique [10], [3] is a quick method for studying the optical properties of fibres.

The aim of this work is to describe an interferometric method to determine the refractive indices and birefringence of every layer of heterogeneous fibres of irregular cross-sections and having a skin-core structure using the Pluta microscope.

2. Theoretical considerations

Mathematical expressions of two-beam interference fringes crossing fibres with irregular transverse sections and having a skin-core structure are derived. The necessary formulae for calculating refractive indices and birefringence of the fibres core are given.

Figure 1 shows the cross-section of a fibre having an irregular cross-sectional core surrounded by an irregular skin. The refractive indices of the immersion liquid, the skin of the fibre and core are n_L, n_s and n_c, respectively. Using the Pluta

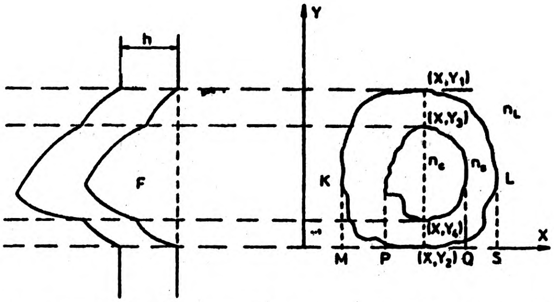

Fig. 1. Fibre with irregular cross-section having a skin-core structure and immersed in a liquid of refractive index n_L. A schematic representation of resulting fringes, using interference microscope, is shown

microscope with monochromatic light of wavelength λ, the interference fringes suffer shifts as they cross the fibre.

The cross-sectional area of the fibre in the X-Y plane is A and is given by

$$A = \int_{M}^{S} (Y_1 - Y_2) dx, \tag{1}$$

Y_1 and Y_2 are the intersection points of a line parallel to Y axis with the circumference of the fibre cross-section. This line lies in the region $S \geqslant X \geqslant M$. The cross-sectional area of the core of the fibre is B and is given by

$$B = \int_{P}^{Q} (Y_3 - Y_4) dx, \tag{2}$$

Y_3 and Y_4 are the intersection points of a line parallel to the Y axis, with the circumference of the core cross-section. This line lies in the $Q \geqslant X \geqslant P$. The shape of the two-beam interference fringes in the X-Z plane crossing the fibre is derived in the following.

If $n_a^{\parallel}$ and n_a are the mean refractive indices of the fibre for plane polarized light vibrating parallel and perpendicular to the fibre axis, respectively, then the optical path length differences, $\Delta\Gamma_{\parallel}$ and $\Delta\Gamma_{\perp}$ between the specimen and the immersion liquid are given by:

$$\Delta\Gamma_{\parallel} = (n_a^{\parallel} - n_L)t, \tag{3}$$

$$\Delta\Gamma_{\perp} = (n_a - n_L^{\perp})t \tag{4}$$

where $\Delta\Gamma_{\parallel}$ and $\Delta\Gamma_{\perp}$ are given in units of length (mm) and t, the mechanical thickness of the fibre, in the same unit. Referring to Fig. 1, the optical path length difference $\Delta\Gamma_{\parallel}$ can be expressed in the following equation:

$$\Delta\Gamma_{\parallel} = [(Y_1 - Y_2) - (Y_3 - Y_4)] n_s^{\parallel} + (Y_3 - Y_4) n_c^{\parallel} - n_L (Y_1 - Y_2), \tag{5}$$

$\Delta\Gamma_{\parallel}$ is also given by the formula

$$\Delta\Gamma_{\parallel} = \frac{d^{\parallel}}{h} \lambda \tag{6}$$

where $d^{\parallel}$ – interference fringe shift inside the fibre in units of length (mm), and h – interfringe spacing. Therefore

$$\frac{\Delta\Gamma_{\parallel}}{\lambda} = \frac{d^{\parallel}}{h} = \frac{1}{\lambda}\{[(Y_1-Y_2)-(Y_3-Y_4)]\; n_s^{\parallel}+(Y_3-Y_4)n_c^{\parallel}-n_L(Y_1-Y_2)\},$$

$$d^{\parallel} = \frac{h}{\lambda} = \{[(Y_1-Y_2)-(Y_3-Y_4)]n_s^{\parallel}+(Y_3-Y_4)n_c^{\parallel}-n_L(Y_1-Y_2)\}. \tag{7}$$

Integrating Equation (7) in the region $M \geqslant X \geqslant S$ gives the area under the fringe shift F, thus

$$\left.\begin{aligned} F^{\parallel} &= \frac{h}{\lambda}\,[(A-B)n_s^{\parallel}+Bn_c^{\parallel}-n_L A] \\ &= \frac{h}{\lambda}\,[(n_s^{\parallel}-n_L)A+(n_c^{\parallel}-n_s^{\parallel})B] \end{aligned}\right\}. \tag{8}$$

Therefore, the values of refractive indices of the fibre for plane polarized light vibrating parallelly and perpendicularly to the fibre axis are given, respectively, by

$$\left.\begin{aligned} \frac{\lambda}{h}F^{\parallel} &= (n_s^{\parallel}-n_L)A+(n_c^{\parallel}-n_s^{\parallel})B \\ \frac{\lambda}{h}F &= (n_s^{\perp}-n_L^{\perp})A+(n_c^{\perp}-n_s^{\perp})B \end{aligned}\right\}. \tag{9}$$

For homogeneous fibres with mean refractive index n_a, Equation (9) can be rewritten in the following form by putting $n_s = n_c = n_a$:

$$\frac{\lambda}{h}F = (n_a-n_L)A. \tag{10}$$

Equation (10) was derived earlier by Hamza [5].

When applying Equation (9) to fibres having a skin-core structure, the area under the fringe shift F, the cross-sectional area of the fibre A, the cross-sectional area of the core B and the interfringe spacing h are measured on the interferogram. The Becke line method [11] can be used to determine the skin refractive index n_s.

3. Experimental results and discussion

Figures 2**a** and **b** give microinterferograms for nonduplicated position of nylon 6 fibres using the Pluta microscope with white and monochromatic light of wavelength 546 nm, respectively. Figs. 3**a** and **b** are microinterferograms showing the totally duplicated images of a sample of nylon 6 fibre obtained using the Pluta microscope. White light and monochromatic light of wavelength $\lambda = 546$ nm were used, respectively.

The described method is used to determine the refractive indices and birefringence of both the skin and core of nylon 6 fibres. The results are given in the Table.

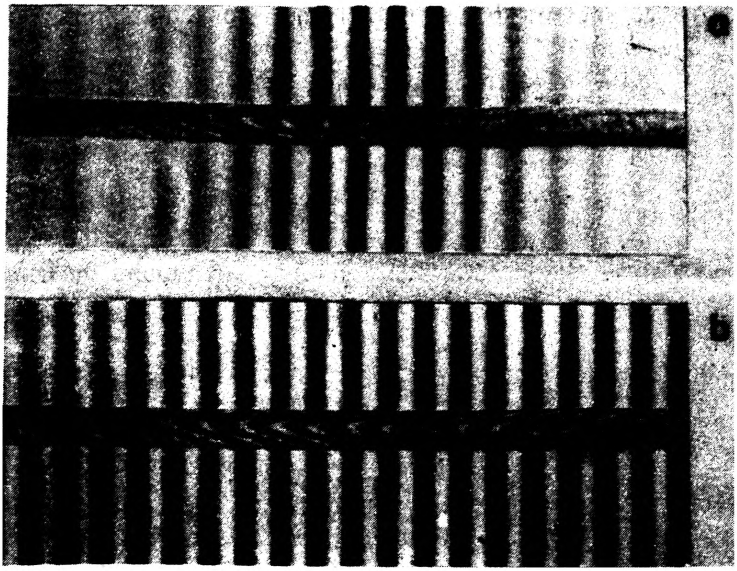

Fig. 2. Microinterferograms of non-duplicated images of nylon 6 fibres using the Pluta microscope with white (a) and monochromatic (b) light of wavelength $\lambda = 546$ nm, respectively

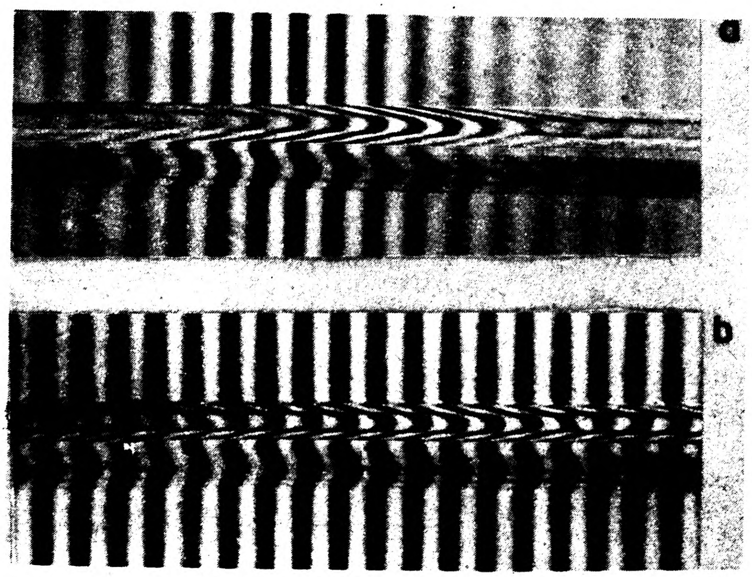

Fig. 3. Microinterferograms showing totally duplicated images for a sample of nylon 6 fibres using white (a) and monochromatic (b) light of wavelength $\lambda = 546$ nm, respectively

Refractive indices and birefringence of skin and core of nylon 6 fibres using white light* (n_L = 1.5810 at 22° C)

Area under the fringe shift $\times 10^{-4}$ [mm]^2		$h \times 10^{-4}$ [mm]	$A \times 10^{-4}$ [mm]^2	$B \times 10^{-4}$ [mm]^2
$F^{\parallel}$	$F^{\perp}$			
1.6	20	293	7.068	4.18

Refractive indices and birefringence of skin**			Refractive indices and birefringence of core		
$n_s^{\parallel}$	$n_s^{\perp}$	n_s	$n_c^{\parallel}$	$n_c^{\perp}$	n_c
1.5808	1.5303	0.0505	1.588	1.534	0.054

* The wavelength $\lambda = 550$ nm is taken as an average for white light.
** Measured by the Becke line method.

4. Conclusions

Two-beam interference microscopes with the derived formulae present suitable methods for studying optical anistropy in fibres of irregular cross-sections and having a skin-core structure. Refractive indices and birefringence of homogeneous and heterogeneous fibres having regular or irregular cross-sections can be determined by two-beam microinterferometry. The range and detail of the structural information provided by the interferometric methods are of great potential importance in quality control in the fibre and textile industries.

References

[1] FAUST R. C., Quart. J. Microsc. Sci. **97** (1956), 569.
[2] BARAKAT N., Textile Res. J. **41** (1971), 167.
[3] PLUTA M., J. Microsc. **96** (1972), 309.
[4] HAMZA A. A., Textile Res. J. **50** (1980), 731.
[5] HAMZA A. A., J. Microsc. **142** (1986), 35.
[6] HAMZA A. A., KABEEL M. A., J. Phys. D: Appl. Phys. **19** (1986), 1175.
[7] HAMZA A. A., EL-FARAHATY K. A., HELALY S. A., Opt. Appl. **18** (1988), 133.
[8] HAMZA A. A., SOKKAR T. Z. N., KABEEL M. A., J. Phys D: Appl. Phys. **18** (1985), 2321.
[9] SIMMENS S. C., Nature **181** (1958), 1260.
[10] PLUTA M. Opt. Acta **18** (1971), 661.
[11] HARTSHORNE N. H., STUART A., *Crystals and the Polarising Microscope*, Edward Arnold, London 1970, pp. 258–272.

Received April 12, 1990

Оптическая анизотропия в фибрах структуры сердечник-скин-слой с нерегулярным сечением

Введены математические выражения для случая полосок двухлучевой интерферометрии при пересеченных полосах нерегулярного сечения сердечника и нерегулярного скин-слоя. Эти выражения применяется для определения коэффициента преломления и двойного лучепреломления в фибрах нейлон 6. Применен поляризационно-интерференционный микроскоп Плюты с белым и монохроматическим светом длины волны 546 nm. Для иллюстрации даны микроинтерферограммы.