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Analytical relations for double-layer antireflection coatings on absorbing substrates at normal incidence

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Analytical relations for layer thicknesses of double-layer antireflection coatings on absorbing substrates at normal incidence are given. They are useful for antireflection coatings on transparent optical components in UV or IF high power laser and integrated optical devices.

1. Introduction

The transparent materials used as substrates in thin-layer coatings could become absorbing in UV or IF spectral ranges. Especially, for high power laser devices the substrate absorption should be taken into account in an antireflection coating design to avoid the optical component destructions.

In this note analytical relations for layer thicknesses of a double-layer antireflection coating on absorbing substrate at normal incidence are given. General and particular cases are considered. The relations deduced are more simple and general than those given explicitly in [1] only in particular cases. Similar relations for nonabsorbing substrates could be found in [2]–[4]. They are very useful in material studies for high power laser optics and integrated optical components.

2. General case

Let us denote the refractive index of ambient medium by n_0, and the complex refractive index of the substrate by $n_s - ik_s$. The refractive indices of thin dielectric layers are noted with n_1 for the outer layer and n_2 for the inner layer which is closest to the substrate. The phase thickness φ_i corresponding to the geometrical thickness d_i of the layer i-th is defined as

$$\varphi_i = 2\pi n_i d_i/\lambda. \tag{1}$$

We consider the normal-incidence reflection of monochromatic light of wavelength λ travelling in ambient by the system of two dielectric thin films on an absorbing substrate. From the antireflection condition one obtains

$$\begin{aligned}\tan^2\varphi_1 = {} & n_1^2[(n_s-n_0)(n_0n_s-n_2^2)+n_0k_s^2]/\\ & [(n_1^2-n_0n_s)(n_1^2n_s-n_0n_2^2)-k_s^2n_0n_1^2],\end{aligned} \tag{2}$$

$$\tan^2\varphi_2 = (A\tan^2\varphi_1 - B\tan\varphi_1 + C)/(D\tan^2\varphi_1 - B\tan\varphi_1 + E) \tag{3}$$

where:

$$A = n_2(n_0n_2+n_1^2)[(n_s-n_2)(n_0n_s-n_1^2)+n_0k_s^2],$$

$$B = 2n_1n_2^2k_s(n_1^2-n_0^2),$$

$$C = n_1^2 n_2(n_2+n_0)[(n_s-n_0)(n_s-n_2)+k_s^2], \tag{4}$$
$$D = (n_0 n_2+n_1^2)[(n_1^2 n_s-n_0 n_2^2)(n_s-n_2)+n_1^2 k_s^2],$$
$$E = n_1^2(n_2+n_0)[(n_0 n_s-n_2^2)(n_s-n_2)+n_0 k_s^2].$$

These relations for layer thicknesses of double-layer antireflection coatings on absorbing substrates were verified by a damped least square computer method. They could be expressed in terms of Fresnel coefficients $r_i = (n_i-n_{i-1})/(n_i+n_{i-1})$, $i = 1, 2$, and $r_s = r_s^r + i r_s^i$. One obtains:

$$\tan^2\varphi_1 = (a^2-c^2|r_s|^2)/(c^2|r_s|^2-b^2), \tag{5}$$

$$\tan^2\varphi_2 = [b(b-cr_s^r)\tan^2\varphi_1+2cr_s^i\tan\varphi_1+a(a+cr_s^r)]/ [b(b+cr_s^r)\tan^2\varphi_1-2cr_s^i\tan\varphi_1+a(a-cr_s^r)] \tag{6}$$

where:

$$a = 1+r_1 r_2, \quad b = 1-r_1 r_2, \quad c = (r_1+r_2)/|r_s|^2. \tag{7}$$

The following relations could be used to minimize the coating reflectance for a given φ_1 or φ_2:

$$\tan 2\varphi_1 = (1-r_2^2)(r_s^i\cos 2\varphi_2-r_s^r\sin 2\varphi_2)/[r_2(1+|r_s|^2) + (1+r_2^2)(r_s^r\cos 2\varphi_2+r_s^i\sin 2\varphi_2)], \tag{8}$$

$$\tan 2\varphi_2 = \{r_2 r_s^i(1+r_1^2)+r_1[r_s^i(1+r_2^2)\cos 2\varphi_1 - r_s^r(1-r_2^2)\sin 2\varphi_1]\}/\{r_2 r_s^r(1+r_1^2) + r_1[r_s^r(1+r_2^2)\cos 2\varphi_1+r_s^i(1-r_2^2)\sin 2\varphi_1]\}. \tag{9}$$

These relations are obtained from the conditions $\partial R/\partial(2\varphi_1) = 0$ and $\partial R/\partial(2\varphi_2) = 0$, where R is the coating reflectance.

Relations (2) and (3) (or (5) and (6)) are useful for antireflecting coatings on transparent optical components in UV or IF high power laser and integrated optical devices. They are valid only for optical constants resulting in positive values of their right hand sides. An illustration is given for antireflection double-layer coating on silicon solar cells. The relations are applied to the air–Si_3N_4–SiO_2–Si system at the (He-Ne laser) wavelength $\lambda = 632.8$ nm and at a shorter (He-Cd laser) UV wavelength $\lambda = 325$ nm [5]. The refractive indices of Si_3N_4, SiO_2 and Si are taken as $n_1 = 1.98$, $n_2 = 1.46$, $n_s = 3.85-i0.02$, respectively at $\lambda = 632.8$ nm, and $n_1 = 2.01$, $n_2 = 1.482$ and $n_s = 5.063-i3.218$, respectively at $\lambda = 325$ nm. At $\lambda = 632.8$ nm one obtains $\varphi_1 = 77.7122°$ ($d_1 = 68.99$ nm) and $\varphi_2 = 7.7503°$ ($d_2 = 9.33$ nm). When silicon absorption is neglected ($k_s = 0$) one obtains $\varphi_1 = 87.2789°$ ($d_1 = 77.48$ nm) and $\varphi_2 = 7.8891°$ ($d_2 = 9.50$ nm), resulting in a non-zero theoretical reflectance value of 1.5%. At $\lambda = 325$ nm the semiconductor silicon substrate behaves effectively as a metal and the right hand side of relation (2) becomes negative. The bilayer-substrate system becomes, in general, more highly reflecting [5].

3. Particular cases

Some particular cases of double-layer antireflection coatings on absorbing substrates can be distinguished with analogy to the nonabsorbing substrate cases.

i) Outside layer one-quarter wavelength thick, $\varphi_1 = (2m-1)\pi/2$, where m is an integer number, when:

$$n_0 n_2^2 = n_2^1 n_s - k_s^2 n_0 n_1/(n_1^2 - n_0 n_s), \tag{10}$$

and

$$\tan^2\varphi_2 = n_2^2(n_1^2 - n_0 n_s)^2/(n_1^4 k_s^2). \tag{11}$$

For nonabsorbing substrates ($k_s = 0$) it becomes the particular case of an antireflection coating with each layer one-quarter wavelength thick [2], [4], when $n_0 n^2 = n_1^2 n_s$ and $\varphi_1 = \varphi_2 = (2m-1)\pi/2$.

ii) Outside layer one-half wavelength thick, $\varphi_1 = m\pi$, when:

$$n_2^2 = n_0 n_s + n_0 k_s^2/(n_s - n_0), \tag{12}$$

and

$$\tan^2\varphi_2 = n_2^2(n_s - n_0)^2/(n_0^2 k_s^2). \tag{13}$$

For nonabsorbing substrates it becomes the particular case of a double-layer antireflection coating with outside layer one-half, and inside layer one-quarter wavelength thick, when $n_2^2 = n_0 n_s$, $\varphi_1 = m\pi$ and $\varphi_2 = (2m-1)\pi/2$.

iii) Outside layer one-quarter, and inside layer one-half-quarter wavelength thick, $\varphi_1 = 2\varphi_2 = (2m-1)\pi/2$, when:

$$n_2^2 = n_s^2 + k_s^2 \quad \text{and} \quad n_1^2 = n_0 n_2(n_2 \pm k_s)/n_s. \tag{14}$$

For nonabsorbing substrates, it becomes the particular case of one-quarter wavelength thick single-layer antireflection coating because $n_2 = n_s$, $n_1 = \sqrt{n_0 n_s}$ and $\varphi_1 = (2m-1)\pi/2$.

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Аналитические отношения для двухслойных противоореольных покрытий на абсорбирующем основании при вертикальном падении света

Даны аналитические зависимости для толщины слоев двухслойных покрытий на абсорбционных основаниях при вертикальном падении света. Они пригодны для избежения оптических элементов с противоореольными слоями, облучаемыми инфракрасным или ультрафиолетовым излучениями из лазеров большой мощности.

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