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Spatial filtering application to image comparison

K. Kołodziejczyk (member of BiOS), L. Wolf (member of SPIE)

Technical University of Łódź, Institute of Physics, ul. Wólczańska 219/223, 93–005 Łódź, Poland.

The Fourier-transforming properties of coherent imaging system and digital filtering system as well as their application in spatial filtering technique are well known and continuously developed. Most of these systems used in image comparison are only able to detect whether the images are identical or not. They do not provide information about the localization of differences. We present a new spatial filtering method able to detect and indicate the places where any differences occur in compared images. This method may be adopted both in coherent imaging optical systems and digital imaging systems. The sensitivity of the method is limited only by image resolution and its dynamics. Several simulation examples are provided to demonstrate the possibilities of this technique.

1. Background

The main idea of spatial filtering is to find a Fraunhofer diffraction pattern of predefined object. Assuming that the effect of Fraunhofer diffraction on the complex light amplitude is described by a Fourier transformation [1], we can calculate this by the following formula:

$$\mathcal{F}\{g(x,y)\} = \iint\limits_{-\infty}^{+\infty} g(x,y)\exp[-2\pi i(x\nu_x+y\nu_y)]\,dx\,dy = G(\nu_x,\nu_y) \tag{1}$$

where: $g(x,y)$ – light amplitude distribution at the two dimensional object,
x, y – spatial coordinates in the object plane.

The connection between the reduced coordinates (ν_x, ν_y) named spatial frequencies and the genuine coordinates (x_f, y_f) in the diffraction plane is:

$$\nu_x = x_f/\lambda f, \quad \nu_y = y_f/\lambda f,$$

where: λ – wavelength of the light,
f – focal length of the lens used to produce the Fourier hologram.

Next, we have to find the complex amplitude

$$G(\nu_x,\nu_y) = A(\nu_x,\nu_y)\exp[(i\varphi(\nu_x,\nu_y)], \tag{2}$$

and the way of its implementation.

In computer calculations $g(x,y)$ and $G(\nu_x,\nu_y)$ are spatially sampled. Generally, each sample of $G(\nu_x,\nu_y)$ has a complex value. Assume that we calculate $G(\nu_x,\nu_y)$ at $\nu_x = m\times\Delta\nu$, $\nu_y = n\times\Delta\nu$, $m,n = 0, 1, 2, \ldots$, $\Delta\nu = \text{const}$. The sampled hologram is then

$$\hat{G}(\nu_x, \nu_y) = \Sigma_m \Sigma_n G(m\Delta\nu, n\Delta\nu)\delta(\nu_x - m\Delta\nu, \nu_y - n\Delta\nu) \tag{3}$$

where $\delta(x, y)$ is the two-dimensional unit impulse function [2]. There are several methods of recording this Fourier transform hologram, to mention, as an example, the Lohmann or Lee method of computer generated holograms [3]. The amplitude factor $A(\nu_x, \nu_y)$ can be realized either as a dot size or the dot grey-tone level on photographic plate. The phase $\varphi(\nu_x, \nu_y)$ can be realized by translation of these dots in the plane of photographic plate. In the case of digital image processing, the amplitude factor and the phase are stored in computer memory.

2. Spatial filtering

The basic setup for optical realization of a Fourier transform is shown in Figure 1. A transparency with amplitude transmittance $p(x, y)$ is located in the front focal plane of the lens, where x, y are the spatial coordinates in that plane. When a plane wave a with unit magnitude impinges on the transparency from the left hand side the

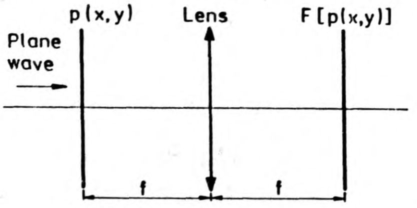

Fig. 1. Basic optical setup for Fourier transformation

Fourier transform of $p(x, y)$ is formed at the back focal plane of the lens. This Fourier transform may be used as a filter in spatial filtering [4]. To perform the Fourier transform in the computer, we use the sampled version of $p(x, y)$ and calculate its discrete Fourier transform, which gives a sampled version of $P(\nu_x, \nu_y)$.

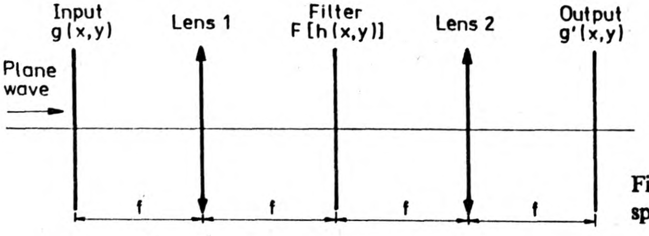

Fig. 2. Basic setup for coherent optical spatial filtering

Figure 2 shows the setup for coherent optical spatial filtering. The input image $g(x, y)$ is represented by the amplitude transmittance in the transparency at the front focal plane of the lens 1. The filter transparency has an amplitude transmittance $H(\nu_x, \nu_y)$. The light distribution at the back focal plane of lens 2 is specified by

$$g'(x, y) = g(x, y) \otimes h(x, y) \tag{4}$$

where $h(x, y)$ is the inverse Fourier transform $H(\nu_x, \nu_y)$. To perform the spatial filtering in the computer, we use the sampled version of $g(x,y)$, $h(x,y)$ and subsequently calculate three discrete Fourier transforms $G(\nu_x,\nu_y)$, $H(\nu_x,\nu_y)$ and $g'(x,y) = \mathscr{F}(G\times H)$.

Spatial filtering is applied in many areas, such as image enhancement, matched filtering and code translation [5], [6]. When the model object is represented by $h(x,y)$, the above described setup may be used to find identical elements of compared object and the model. But in many cases of image comparison, it is necessary to detect and show the places of difference occurrence.

3. Experimental method

The scheme measurement system is shown in Figure 3. The image is captured by a CCD camera, digitalized and stored in the computer memory. All digital image files have 128×128 pixels with 64 gray-scale levels. A fast Fourier transform (FFT) Cooley–Tukey algorithm is applied to the image file to obtain a Fourier transform also having 128×128 pixels, also stored in the computer.

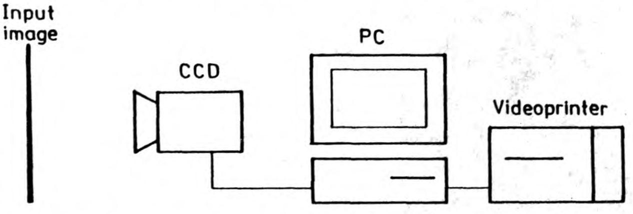

Fig. 3. Scheme system for image comparison

For the comparison of the two images, a total number of differences of spatial frequencies' amplitudes is computed. When this number is different from zero, the compared images are not the same. To produce the difference image (DI) with marked out places where differences occurr, we propose to compute a Fourier transform of spatial frequencies' difference spectrum. We define the amplitude of this difference spectrum as

$$A(\nu_x,\nu_y) = \begin{cases} 0 & \text{if} \quad A_1(\nu_x,\nu_y) \leqslant A_2(\nu_x,\nu_y), \\ A_1(\nu_x,\nu_y) - A_2(\nu_x,\nu_y) & \text{in other case.} \end{cases} \tag{5}$$

$A_1(\nu_x,\nu_y)$ and $A_2(\nu_x,\nu_y)$ are the amplitude factors of compared images of Fourier transforms, and $A_1(\nu_x,\nu_y)$ represent the amplitude of Fourier transform of the model image. The result of the comparison is shown on the PC screen and can be printed on the Mitsubishi CP-100B videoprinter.

4. Experimental results

The comparison of the three rectangles, one undefective and two with defects, is made to evaluate the effectiveness of this system. The undefective rectangle is treated as the model image and the rectangles are located in different places of image plane. Each rectangle is compared with all the others. The table shows the results of the

Table. Rectangles comparison result (+ indicates that images are identical, − indicates that they are different)

Rectangle	R1	R2	R3
R1	+	−	−
R2	−	+	−
R3	−	−	+

comparison. A plus sign indicates that compared images are identical, while a minus sign indicates that they are different. R1 represents an undefective rectangle, R2 and R3 represent defective ones. Figure 4 shows the compared rectangles, their Fourier transforms, and difference image before and after digital filtering (DI and DFDI, respectively). Places with maximum intensity of DI indicate the differences' localization. Other visible traces are caused by spatial frequencies corresponding to changed

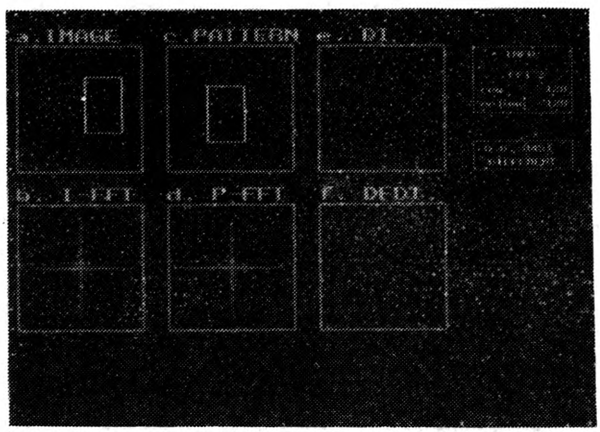

Fig. 4. Comparison of two different rectangles: a — rectangle R2 with defect in right bottom corner, b — FFT of R2, c — undefective rectangle R1, d — FFT of R1, e — difference image, f — difference image after digital filtering

dimensions of defective rectangle. These traces can be removed by application of digital filtering. Figure 4b shows the DI after cutting of background at the level of 25% maximum intensity. Rectangles R1 and R2 are different by one pixel in right bottom corner, and the presented method of comparison detects and localizes this difference. It shows that the method sensitivity is limited only by image resolution. In many applications, for example fingerprints comparison [7], such high sensitivity is undesirable. To decrease the sensitivity, a division factor is introduced, and all the differences of spatial frequencies' amplitudes are divided by this factor before summing up. It permits fixing a tolerance interval within which the images are recognized as identical.

For checking the effectiveness of the method in the case of irregular objects, two mosaics with three-pixels difference are compared. Figure 5 shows the results of

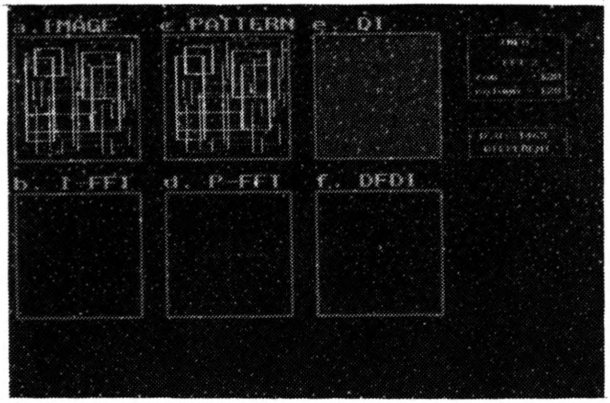

Fig. 5. Comparison of two different mosaics: **a** — mosaic M2 with three-pixels defect, **b** — FFT of M2, **c** — model mosaic M1, **d** — FFT of M1, **e** — difference image, **f** — difference image after digital filtering

this comparison. The place of difference occurrence is detected, located and clearly marked, even without digital filtering.

The method described may also be realized in coherent optical system. In this case a spatial filter must be performed as a negative binary Fourier hologram of image pattern.

5. Conclusions

We have demonstrated an experimental system which optically reads images, digitally stores them in a computer and compares them using modified spatial filtering and Fourier analysis method. The employment of Fourier transform to difference spectrum of spatial frequencies enables obtaining an image with places of differences clearly marked out. This difference image may be improved using digital filtering. The sensitivity of the demonstrated system is limited only by image resolution and makes it possible to detect the differences of even one-pixel.

References

[1] Nabil H. Farhat [Ed.], *Advances in Holography*, Marcel Dekker Inc., 1975, New York and Basel, Vol. 1.
[2] Wai Hon Lee, Appl. Opt. 9 (1970), 639.
[3] Huang T. S., Proc. IEEE 59 (1971), 1335.
[4] Brown B. R., Lohmann A. W., Appl. Opt. 5 (1966), 967.
[5] Man Mohan Sondhi, Proc. IEEE 60 (1972), 842.
[6] Javidi B., Ruiz C., Ruiz J., Proc. SPIE 1211 (1990), 313.
[7] Fielding K. H., Horner J. L., Kakekau C. K., Opt. Eng. 30 (1991), 1958.

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