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Holographic method of determination of refractive index of solutions

An attempt has been made to determine the refractive index of solutions by holographic method. Equal inclination and equal thickness fringes have been formed holographically. An expression was derived to compare the refractive index of distilled water with that of a solution connecting the radii of curvature of circular fringes. Results obtained by this method favourably compare with those determined by classical methods. The special advantage of this method is its applicability to the dynamically varying situations which cannot be realised in the classical methods.

1. Introduction

In principle, fringes of equal inclination can be formed if two parallel wavefronts, one lagging behind the other, are made to interfere, whereas fringes of equal thickness can be obtained when the interfering wavefronts are inclined to one another. Holographic interferometric method has been employed here to form such fringes. In the case of formation of fringes of equal inclination, a rectangular cell with a liquid of different concentrations, was used in the path of the object beam to produce phase lag between the interfering parallel wavefronts. Employing this method an attempt has been made to compare refractive index of liquid at different concentrations.

By keeping two plane parallel plates, one fixed and the other movable to various inclinations in the path of the object wave, different sets of equal thickness fringes were formed. Alternatively these equal thickness fringes can also be produced by keeping a small wedge cell containing a liquid of different concentrations in the path of the object beam.

The speciality of holographic interferometry is that the superposing wavefronts need not exist simultaneously, as it is the basic requirement in classical interferometry. Sequential recording of wavefronts holographically generates interference fringes. This property of holographic interferometry made it possible the comparison of two wavefronts, one differing from the other.

2. Equal inclination fringes

The circular fringes can be analysed on the basis of double exposed hologram. Let E_o, be the complex amplitude of the object wave that passes through the cell containing distilled water, and E_R the reference beam, then the intensity I_1 of the first exposure is

$$I_1 = |E_R+E_o|^2 = E_R^2+E_o^2+E_R^*E_o+E_RE_o^*. \qquad (1)$$

If E_P is the complex amplitude of the retarded wave recorded in the second exposure, the intensity I_2 with the same reference wave is

$$I_2 = E_R^2+E_P^2+E_R^*E_P+E_RE_P^*. \qquad (2)$$

Thus the total intensity of the double exposed hologram is

$$I = I_1+I_2 = 2E_R^2+E_o^2+E_P^2+E_R(E_o^*+E_P^*)+ +E_R^*(E_o+E_P). \qquad (3)$$

In expression (3) the last term which denotes the resultant field at a given point is of interest. On reconstruction we have this term in the form

$$E_RE_R^*(E_o+E_P).$$

On assuming $E_RE_R^*$ to be unity, it is only the sum of the complex amplitudes which appears (E_o+E_P) if one sets

$$E_o = a\exp(i\Phi)$$

and

$$E_P = a\exp\big(i(\Phi+\sigma)\big)$$

where

$$\sigma = \left(\frac{2\pi}{\lambda}\right)d\cos\Theta.$$

Here, d is the path difference between the wavefronts, and Θ the angle the line of observation makes with the common axial line of the wavefronts. Then the intensity is given by

$$I = 2a^2(1+\cos\sigma) = 2a^2\left[1+\cos\left(\frac{2\pi d\cdot\cos\Theta}{\lambda}\right)\right]. \qquad (4)$$

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From (4) it can be seen that the intensity I is a periodic function, the argument of which being also a periodic function of the inclination angle Θ. Consequently, the fringes obtained are circular. Hence it may be interpreted that the circular fringes obtained are of a constant inclination type.

3. Fringes of equal thickness

In this case let Θ be the angle of tilt or inclination between the wavefronts, and let l be the distance of point of observation from the thin end of the wedge. Then the path difference at that point is $l\Theta$. If n is the fringe order, then we have a relation $l\Theta = n\lambda$ and the phase difference at that point is

$$\sigma = \left(\frac{2\pi}{\lambda}\right) l\Theta .$$

In this case the intensity distribution in the interference fringes can be also expressed as

$$I = 4a^2 \cos\left(\frac{\sigma}{2}\right), \tag{5}$$

where σ is a linear function of Θ. Here evidently the fringes formed should be straight, therefore, they can be considered as fringes of equal thickness.

4. Estimate of refractive index of solution at different concentrations

In the case of distilled water we consider the following equation [1]:

$$\mu t \cos\Theta_1 = n_1 \lambda, \tag{6}$$

where μ is the refractive index of distilled water, t is the length of the liquid path, n_1 is the number of the fringe, λ is the wavelength of light used.

In case of a solution of particular concentration the corresponding equation is

$$\mu' t \cos\Theta_1' = n_1 \lambda, \tag{7}$$

where μ' is the refractive index of the solution.

We may write

$$r_1 \cot\Theta_1 = r_1' \cot\Theta_1', \tag{8}$$

where r_1 and r_1' are the radii of n_1-th ring in each case, respectively.

In each case we choose another ring of order n_2 and the equations (6), (7) and (8) with subscript 2.

From equation (6) we have

$$\cos\Theta_1 = \frac{n_1\lambda}{\mu t},$$

$$\sin\Theta_1 = \frac{(\mu^2 t^2 - n_1^2\lambda^2)}{\sqrt{\mu t}},$$

$$\cot\Theta_1 = \frac{n_1\lambda}{\sqrt{\mu^2 t^2 - n_1^2\lambda^2}},$$

$$\cot\Theta_1' = \frac{n_1\lambda}{\sqrt{\mu'^2 t^2 - n_1^2\lambda^2}}.$$

Then equation (8) takes the form

$$\frac{r_1 n_1 \lambda}{\sqrt{\mu^2 t^2 - n_1^2\lambda^2}} = \frac{r_1' n_1 \lambda}{\sqrt{\mu'^2 t^2 - n_1^2\lambda^2}}.$$

Squaring on both sides we get

$$\therefore r_1^2(\mu'^2 t^2 - n_1^2\lambda^2) = r_1'^2(\mu^2 t^2 - n_1^2\lambda^2),$$

$$t^2(r_1^2\mu'^2 - r_1'^2\mu^2) = n_1^2\lambda^2(r_1^2 - r_1^{2\prime}),$$

$$\therefore t^2 = \frac{n_1^2\lambda^2(r_1^2 - r_1'^2)}{(r_1^2\mu'^2 - r_1'^2\mu^2)}. \tag{9}$$

Similarly, from the second set of readings

$$t^2 = \frac{n_2^2\lambda^2(r_2^2 - r_2'^2)}{(r_2^2\mu'^2 - r_2'^2\mu^2)} \tag{10}$$

(9) and (10) give t, whence μ and μ' are known.

$$n_1^2(r_1^2 - r_1'^2)(r_2^2\mu'^2 - r_2'^2\mu^2) = n_2^2(r_2^2 - r_2'^2)\times$$
$$\times(r_1^2\mu'^2 - r_1'^2\mu^2),$$

$$\therefore \mu'^2[n_1^2 r_2^2(r_1^2 - r_1'^2) - n_2^2 r_1^2(r_2^2 - r_2'^2)]$$
$$= \mu^2[n_1^2 r_2'^2(r_1^2 - r_1'^2) - n_2^2 r_1'^2(r_2^2 - r_2'^2)],$$

$$\therefore \frac{\mu'^2}{\mu^2} = \frac{n_1^2 r_2^2(r_1^2 - r_1'^2) - n_2^2 r_1^2(r_2^2 - r_2'^2)}{n_1^2 r_2'^2(r_1^2 - r_1'^2) - n_2^2 r_1'^2(r_2^2 - r_2'^2)}. \tag{11}$$

This expression allows to compare the refractive indices of solutions with different concentrations.

5. Experimental set-up

In the usual of-axis holographic arrangement a liquid cell filled with distilled water was placed in the path of the object beam as shown in fig. 1a. After giving the first exposure, distilled water in the cell was replaced by a salt solution of a known concentration. The wavefront that passes through the solution is naturally retarded in view of the increase in refractive index. In the second exposure the retarded wavefront was recorded, thus making a double exposed hologram. On reconstruction, circular fringes were observed. Further holograms were made similarly each time using solutions of different concentration. Variation in number of fringes with different concentrations are shown in fig. 2. The photographs of the circular fringes for two different concentrations can be seen in figs. 3 and 4.

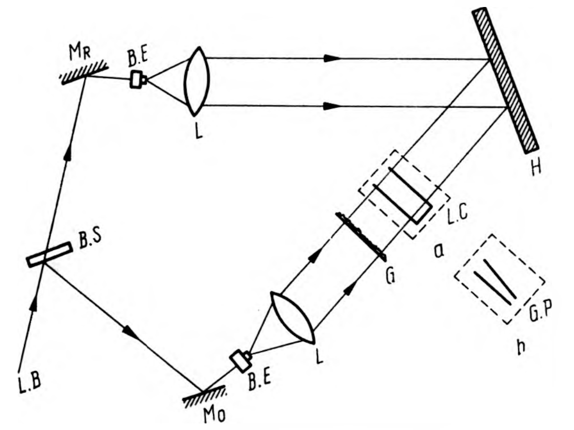

Fig. 1. Schematic diagram of the experimental set-up for making a) equal inclination fringes, and b) equal thickness fringes

L. B. — laser beam, *B. S.* — beam splitter, M_O, M_R — mirrors in object and reference beams, *B.E.* — beam expanders, *L* — collimating lenses, *G* — ground glass plate, *L.C.* — liquid cell, *G.P.* — plane glass plates, *H* — photographic plate

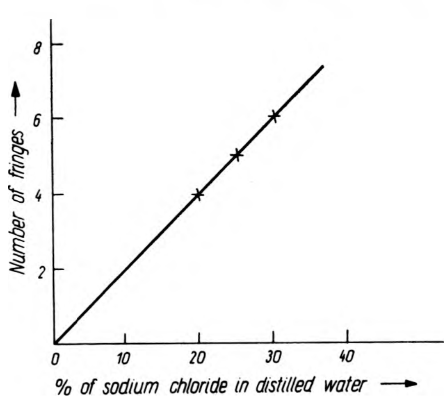

Fig. 2. Variation in number of fringes, with different concentrations of sodium chloride in distilled water

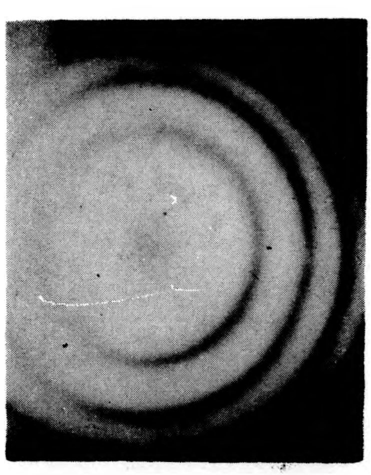

Fig. 3. Equal inclination fringes for 30% content of sodium chloride in distilled water

In the second set-up, as shown in fig. 1b, two plane parallel glass plates were kept in the object beam in such a way that the plates were perpendicular to the wavefront normal, and the first exposure was

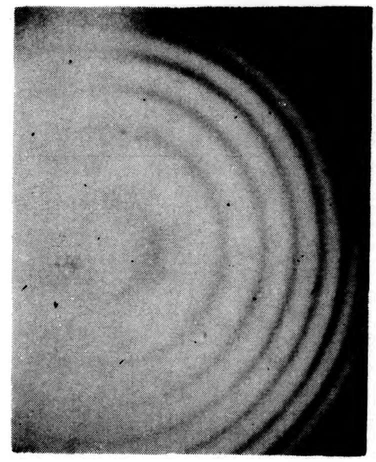

Fig. 4. Equal inclination fringes for 40% content of sodium chloride in destilled water

recorded. Then one glass plate was rotated through a measured angle, in order the wavefront passing through it be tilted with respect to the first recorded one. The recording of the tilted wavefront was also made on the same photographic plate. The reconstruction of the hologram revealed a set of straight fringes. Holograms with various tilts were recorded and on reconstruction it was observed that in each case the number of fringes was proportional to the angle of

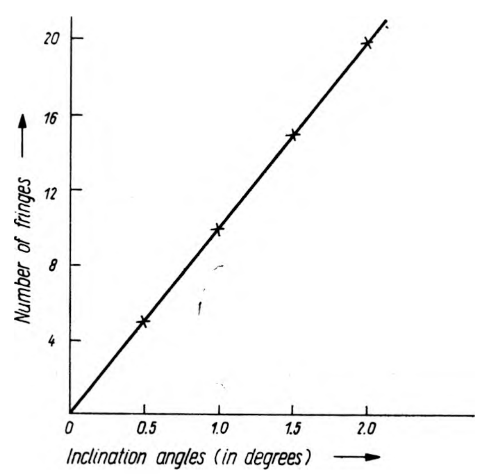

Fig. 5. Variation in number of fringes, with different angles between the glass plates

tilt. Variation in number of fringes with different angles are shown in fig. 5. The photographs of these fringes for two sets of tilts are displayed in figs. 6 i 7.

6. Results

The following are the values obtained for different variables of the equations (6), (7) and (8).

1. For 20 grams of sodium chloride in 100 c.c. of distilled water

$$n_1 = 2, \qquad n_2 = 5$$

$$r_1 = 1.3 \text{ cms}, \; r_2 = 3.0 \text{ cms}.$$

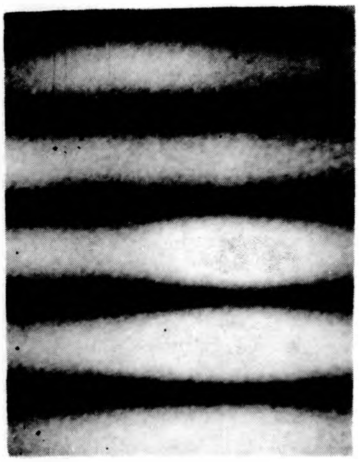

Fig. 6. Equal thickness fringes for 0.5° tilt between the glass plates

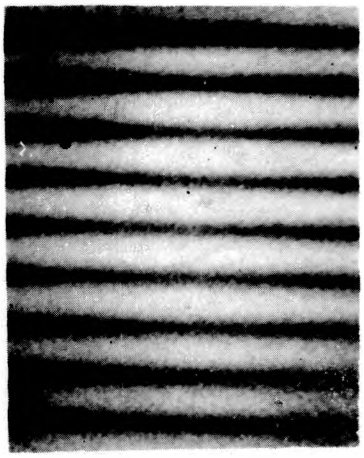

Fig. 7. Equal thickness fringes for 1.0° tilt between the glass plates

2. For 40 grams of sodium chloride in 100 c.c. of distilled water

$$n_1 = 2, \qquad n_2 = 5$$

$$r_1' = 1.2, \qquad r_2' = 2.8 \text{ cms}.$$

By inserting the above values to equation (11) we get

$$\frac{\mu'}{\mu} = 1.085.$$

Thus the result appears to be consistent within the order of a magnitude with that expected, as the value obtained for the same solutions with Jamin's interferometer is 1.078.

This method is applicable not only to the solutions of different concentrations but also to gases of different pressures and temperatures. The advantage of this method lies in its applicability to the dynamically varying situations using pulsed laser.

Голографические методы определения коэффициента преломления раствора

Попытались определить коэффициент преломления растворов голографическим методом. Полосы одинакого наклона и одинаковой толщины были получены голографически. Выведено выражение для сопоставления коэффициента преломления дистиллированной воды с коэффициентом преломления раствора, которое связывает соответствующие радиусы круговых полос. Результаты, полученные этим методом, считают положительными по сравнению с классическими методами. Особенная польза этого метода заключается в возможности его применения в ситуациях динамически переменных, которые не могут быть исследованы классическими методами.

References

[1] Wood R. W., *Physical Optics*, The Macmillan Company, New York 1959.

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