

Effect of dispersion order on the spectral degree of polarization of stochastic electromagnetic pulsed beams

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The analytical expression for the cross-spectral density function of stochastic electromagnetic Bessel–Gauss pulsed beams through a dispersive aperture lens is derived and used to study the changes in the spectral degree of polarization in an optical focus system. The changes in the spectral degree of polarization at the focal plane and on the z -axis are performed in the case of dispersion-free, dispersion of the first, second, and higher orders, respectively. It is shown that the dispersion affects the peak value of the spectral degree of polarization, and the higher order dispersion leads to a more obvious effect on the peak value of the spectral degree of polarization at the focal plane. On the z -axis, the spectral degree of polarization in the dispersion-free case is different form that in the dispersion case, however, the dispersion of different orders almost has the same effect on the spectral degree of polarization. The results obtained in this paper may be crucial for high precision laser detection.

Keywords: stochastic electromagnetic Bessel–Gauss pulsed beams, a dispersive aperture lens, pulse duration, temporal coherence length.

1. Introduction

In 2003 WOLF proposed the unified theory of coherence and polarization of stochastic stationary electromagnetic beams [1–3]. Since then, lots of work has been made on the propagation of stochastic stationary electromagnetic beams [4–6]. Recently, a scalar model of spectrally partially coherent pulses, in which the correlation between different frequency components was taken into consideration, was introduced by PÄÄKKÖNEN *et al.* [7]. LAJUNEN *et al.* obtained the coherent-mode representation for spatially and spectrally partially coherent scalar pulses [8, 9]. Several methods for generating such partially coherent pulsed field have also been proposed [10, 11]. CHAOLIANG DING *et al.* extended the spatially and spectrally partially coherent pulses from the scalar model to the vectorial case, namely, stochastic electromagnetic pulsed beams [12]. And the changes in the spectral degree of polarization of stochastic elec-

tromagnetic pulsed beams propagating in dispersive medium and reflecting gratings have been investigated [13, 14]. Recently, stochastic electromagnetic pulsed beams propagating in the waveguide have attracted much attention for designing and fabricating high power superluminescent light emitting diodes (SLEDs), and the output power and bandwidth of SLEDs can be modulated by the parameters of stochastic electromagnetic pulsed beams [15–17]. Then, VOIPIO *et al.* introduced the partial polarization theory of stochastic electromagnetic pulsed beams [18].

In this paper we extend our former research about the spectral degree of polarization of stochastic electromagnetic pulsed beams to the optical focus system, and the influence of dispersion order on the spectral degree of polarization is emphasized. In Section 2, the analytical expression for the cross-spectral density matrix of stochastic electromagnetic Bessel–Gauss pulsed beams through a dispersive aperture lens is derived, and used to formulate the spectral degree of polarization of the pulsed beams. Numerical calculations are given in Section 3 to illustrate how the dispersion of different orders affects the spectral degree of polarization of stochastic electromagnetic pulsed beams for different values of pulse duration and temporal coherence length. Finally, the main results obtained in this paper are summarized in Section 4.

2. Theoretical formulation

Consider the optical system shown in Fig. 1, suppose that a stochastic electromagnetic Bessel–Gauss pulsed beam is incident upon an aperture lens with full width $2a$ and focal length f . In the space-time domain, the electric mutual coherence matrix of stochastic electromagnetic Bessel–Gauss pulsed beams at the source plane $z = 0$ is given by [1]

$$\overset{\leftrightarrow}{\Gamma}^0(\mathbf{r}'_1, \mathbf{r}'_2, t_1, t_2) = \left[\Gamma_{ij}^0(\mathbf{r}'_1, \mathbf{r}'_2, t_1, t_2) \right] = \left[\langle E_i^*(\mathbf{r}'_1, t_1) E_j(\mathbf{r}'_2, t_2) \rangle \right] \quad (1)$$

and $i = x, y; j = x, y$ unless otherwise stated.

To simplify the analysis, it is assumed that the electric vector components in the x and y directions are uncorrelated at the plane $z = 0$ [3, 19], *i.e.*,

$$\begin{aligned} \Gamma_{ii}^0(\mathbf{r}'_1, \mathbf{r}'_2, t_1, t_2) &= A_i J_0(\alpha_i \mathbf{r}'_1) J_0(\alpha_i \mathbf{r}'_2) \exp \left[-\frac{(\mathbf{r}'_1)^2 + (\mathbf{r}'_2)^2}{4w_0^2} \right] \times \\ &\times \exp \left[-\frac{t_1^2 + t_2^2}{2T_0^2} - \frac{(t_1 - t_2)^2}{2T_{ci}^2} + i\omega_0(t_1 - t_2) \right] \end{aligned} \quad (2)$$

$$\Gamma_{xy}^0(\mathbf{r}'_1, \mathbf{r}'_2, t_1, t_2) = \Gamma_{yx}^0(\mathbf{r}'_1, \mathbf{r}'_2, t_1, t_2) = 0 \quad (3)$$

where J_0 is the Bessel function of the first kind and order zero, α_i is the radial spatial frequency of the i component of the electric vector [20], \mathbf{r}'_i denotes the polar radius at

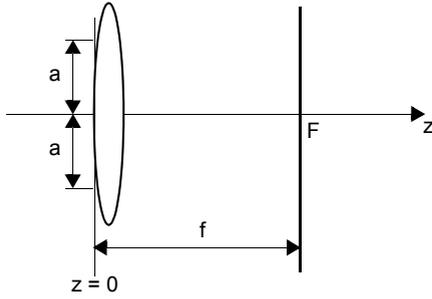


Fig. 1. Schematic illustration of stochastic electromagnetic Bessel-Gauss pulsed beams focused by an aperture lens.

the source plane, T_0 is the pulse duration and T_{ci} describes the temporal coherence length of the i component of the electric vector, w_0 denotes the waist of Gaussian beams, ω_0 is the carrier frequency.

By using the Fourier-transform

$$W_{ij}^0(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Gamma_{ij}^0(\mathbf{r}'_1, \mathbf{r}'_2, t_1, t_2) \exp[-i(\omega_1 t_1 - \omega_2 t_2)] dt_1 dt_2 \quad (4)$$

the cross-spectral density matrix at the plane $z = 0$ can be derived and given by

$$\overset{\leftrightarrow}{\mathbf{W}}(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) = \left[\langle W_{ij}^0(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) \rangle \right] \quad (5)$$

where

$$W_{ii}^0(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) = \frac{T_0 A_i}{2\pi \Omega_{0i}} J_0(\alpha_i \mathbf{r}'_1) J_0(\alpha_i \mathbf{r}'_2) \exp\left[-\frac{(\mathbf{r}'_1)^2 + (\mathbf{r}'_2)^2}{4w_0^2}\right] \times \\ \times \exp\left[-\frac{(\omega_1 - \omega_0)^2 + (\omega_2 - \omega_0)^2}{2\Omega_{0i}^2}\right] \exp\left[-\frac{(\omega_1 - \omega_2)^2}{2\Omega_{ci}^2}\right] \quad (6)$$

$$W_{xy}^0(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) = W_{yx}^0(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) = 0 \quad (7)$$

$$\Omega_{0i} = \sqrt{\frac{1}{T_0^2} + \frac{2}{T_{ci}^2}} \quad (8)$$

$$\Omega_{ci} = \frac{T_{ci}}{T_0} \Omega_{0i} \quad (9)$$

while Ω_{0i} and Ω_{ci} – spectral width and spectral coherence width of the i component of the electric vector, respectively.

According to the Collins formula [21], at the z plane, the cross-spectral density matrix of stochastic electromagnetic Bessel–Gauss pulsed beams focused by a dispersive aperture lens which is located at the source plane $z = 0$ is expressed as

$$\begin{aligned} \overleftrightarrow{\mathbf{W}}(\mathbf{r}_1, \mathbf{r}_2, z, \omega_1, \omega_2) &= \frac{\omega_1 \omega_2}{(2\pi cz)^2} \exp\left[\frac{i(\omega_2 - \omega_1)z}{c}\right] \int_0^a \int_0^{2\pi} \overleftrightarrow{\mathbf{W}}(\mathbf{r}'_1, \mathbf{r}'_2, \omega_1, \omega_2) \times \\ &\exp\left\{\frac{i\omega_2}{2cz} \left[\left(1 - \frac{z}{f}\right)(r'_2)^2 - 2r'_2 r_2 \cos(\varphi_2 - \varphi'_2) + r_2^2\right] + \right. \\ &\left. - \frac{i\omega_1}{2cz} \left[\left(1 - \frac{z}{f}\right)(r'_1)^2 - 2r'_1 r_1 \cos(\varphi_1 - \varphi'_1) + r_1^2\right]\right\} r'_1 r'_2 dr'_1 dr'_2 d\varphi'_1 d\varphi'_2 \end{aligned} \quad (10)$$

where r_i and φ denote the radial and azimuthal coordinates at the z plane, respectively, φ' is the azimuthal coordinate at the $z = 0$ plane, a is the radius of the aperture, f denotes the focal length of the lens, which is frequency-dependent and can be expanded about the central frequency ω_0 into the series

$$f(\omega) = \sum_{m=0}^{\infty} C_m (\omega - \omega_0)^2 = f_0 F(\omega) \quad (11)$$

and

$$F(\omega) = 1 + \xi_1 \left(\frac{\omega - \omega_0}{\omega_0}\right) + \xi_2 \left(\frac{\omega - \omega_0}{\omega_0}\right)^2 + \dots \quad (12)$$

$$\xi_1 = \frac{\omega_0}{f_0} \left. \frac{df(\omega)}{d\omega} \right|_0 = -\frac{\omega_0}{n(\omega) - 1} \left. \frac{dn(\omega)}{d\omega} \right|_0 \quad (13)$$

$$\begin{aligned} \xi_2 &= \frac{\omega_0^2}{2f_0} \left. \frac{d^2 f(\omega)}{d\omega^2} \right|_0 = \\ &= -\frac{\omega_0^2}{2[n(\omega) - 1]} \left. \frac{d^2 n(\omega)}{d\omega^2} \right|_0 + \frac{\omega_0^2}{[n(\omega) - 1]^2} \left(\left. \frac{d^2 n(\omega)}{d\omega} \right|_0 \right)^2 \end{aligned} \quad (14)$$

$$\frac{1}{f(\omega)} = [n(\omega) - 1] \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (15)$$

where f_0 is the focal length of the lens at the carrier frequency ω_0 , C_m is the coefficient of the series, ξ_1 and ξ_2 are material dispersive parameters of the lens, $n(\omega)$ is the re-

fraction index of the lens, R_1 and R_2 are curvature radii of the front and back surfaces of the lens. In the derivation of Eqs. (13) and (14), Eq. (15) has been used.

The hard-edged function

$$H(r') = \begin{cases} 1 & |r'| \leq a \\ 0 & |r'| > a \end{cases} \quad (16)$$

can be expressed as a finite sum of the complex Gaussian functions [22]

$$H(r') = \sum_{m=1}^M A_m \exp\left(-\frac{B_m r'^2}{a^2}\right) \quad (17)$$

where the coefficients A_m and B_m denote the Gaussian coefficients, which are given in [22].

On substituting from Eqs. (2) and (17) into Eq. (10), the elements of the cross-spectral density matrix of stochastic electromagnetic Bessel–Gauss pulsed beams at the plane $z > 0$ are given by

$$\begin{aligned} W_{ii}(\mathbf{p}_1, \mathbf{p}_2, z, \omega_1, \omega_2) &= \frac{T_0 A_i \delta^4}{2\pi \Omega_{0i}} \frac{\omega_1}{\omega_0} \frac{\omega_2}{\omega_0} \frac{z_0^2}{z^2} \exp\left[\frac{i(\omega_2 - \omega_1)z}{c}\right] \times \\ &\times \exp\left[\frac{iz_0}{z} \left(\frac{\omega_2}{\omega_0} \rho_2^2 - \frac{\omega_1}{\omega_0} \rho_1^2\right)\right] \exp\left[-\frac{(\omega_1 - \omega_0)^2 + (\omega_2 - \omega_0)^2}{2\Omega_{0i}^2}\right] \times \\ &\times \exp\left[-\frac{(\omega_1 - \omega_2)^2}{2\Omega_{ci}^2}\right] \sum_{m=1}^{10} \sum_{n=1}^{10} \frac{A_m A_n^*}{\beta_1 \beta_2} \times \\ &\times \exp\left\{-\frac{(\alpha_i \delta w_0)^2 + [2\rho_1 \delta(\omega_1/\omega_0)(z_0/z)]^2}{4\beta_1}\right\} I_0\left(\frac{\rho_1 \delta^2 \alpha_i w_0}{\beta_1} \frac{\omega_1}{\omega_0} \frac{z_0}{z}\right) \times \\ &\times \exp\left\{-\frac{(\alpha_i \delta w_0)^2 + [2\rho_2 \delta(\omega_2/\omega_0)(z_0/z)]^2}{4\beta_2}\right\} I_0\left(\frac{\rho_2 \delta^2 \alpha_i w_0}{\beta_2} \frac{\omega_2}{\omega_0} \frac{z_0}{z}\right) \end{aligned} \quad (18)$$

where:

$$W_{xy}(\mathbf{p}_1, \mathbf{p}_2, z, \omega_1, \omega_2) = W_{yx}(\mathbf{p}_1, \mathbf{p}_2, z, \omega_1, \omega_2) = 0 \quad (19)$$

$$\beta_1 = B_m + \delta^2 \left[1 + \frac{iz_0\omega_1}{\omega_0} \left(\frac{1}{z} - \frac{1}{f(\omega)} \right) \right] \quad (20)$$

$$\beta_2 = B_n^* + \delta^2 \left[1 - \frac{iz_0\omega_2}{\omega_0} \left(\frac{1}{z} - \frac{1}{f(\omega)} \right) \right] \quad (21)$$

$$z_0 = \frac{\pi w_0^2}{\lambda_0} \quad (22)$$

$$\delta = \frac{a}{w_0} \quad (23)$$

$$\rho_i = \frac{r_i}{w_0} \quad (24)$$

while z_0 denotes Rayleigh length, δ – truncation parameter, ρ_i – relative transversal coordinate, λ_0 – the central wavelength at the carrier frequency ω_0 .

Thus, the spectral degree of polarization of stochastic electromagnetic Bessel–Gauss pulsed beams at the plane $z > 0$ can be derived by [2]

$$P(\boldsymbol{\rho}, z, \omega) = \sqrt{1 - \frac{4\text{Det}[\overset{\leftrightarrow}{\mathbf{W}}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega)]}{\left\{ \text{Tr}[\overset{\leftrightarrow}{\mathbf{W}}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega)] \right\}^2}} = \left| \frac{W_{xx}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega) - W_{yy}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega)}{W_{xx}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega) + W_{yy}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega)} \right| \quad (25)$$

where

$$W_{ii}(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega, \omega) = \frac{T_0 A_i \delta^4}{2\pi \Omega_{0i}} \frac{\omega^2}{\omega_0^2} \frac{z_0^2}{z^2} \exp \left[-\frac{(\omega - \omega_0)^2}{\Omega_{0i}^2} \right] \times \\ \times \sum_{m=1}^{10} \sum_{n=1}^{10} \frac{A_m A_n^*}{\beta_1 \beta_2} I_0 \left(\frac{\rho \delta^2 \alpha_i w_0}{\beta_1} \frac{\omega}{\omega_0} \frac{z_0}{z} \right) I_0 \left(\frac{\rho \delta^2 \alpha_i w_0}{\beta_2} \frac{\omega}{\omega_0} \frac{z_0}{z} \right) \times \\ \times \exp \left\{ -\frac{1}{4} \left[(\alpha_i \delta w_0)^2 + \left(2\rho \delta \frac{\omega}{\omega_0} \frac{z_0}{z} \right)^2 \right] \left[\frac{1}{\beta_1} + \frac{1}{\beta_2} \right] \right\} \quad (26)$$

Equation (26) is the main analytical result obtained in this paper, which with Eq. (25) describe the changes in the spectral degree of polarization of stochastic electromagnetic Bessel–Gauss pulsed beams focused by a dispersive aperture lens from the $z = 0$ plane to the z -plane.

3. Illustrative examples

Numerical calculation results illustrate the changes in the spectral degree of polarization of stochastic electromagnetic Bessel–Gauss pulsed beams focused by a dispersive aperture lens. And, the influence of dispersion order on the spectral degree of polarization is emphasized. Assume that the electromagnetic pulsed beams are focused by a silica lens, whose refractive index $n(\lambda)$ is described by the Sellmeier relation [23]

$$n^2(\lambda) = 1 + \sum_{j=1}^3 \frac{B_j}{1 - \lambda_j^2/\lambda^2} \quad (27)$$

where $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.8974794$, $\lambda_1 = 0.0684043 \mu\text{m}$, $\lambda_2 = 0.1162414 \mu\text{m}$, $\lambda_3 = 9.896161 \mu\text{m}$, and $\lambda = 2\pi c/\omega$ is the wavelength in vacuum. The spectral degree of polarization $P(\mathbf{\rho}, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams *versus* the relative transversal coordinate ρ at the geometrical focal plane is plotted in Fig. 2, where the dispersion-free, dispersion of the first, second, and higher orders are considered, respectively. The calculation parameters are $\alpha_x = 2 \text{ mm}^{-1}$, $\alpha_y = 2\alpha_x$, $w_0 = 1 \text{ mm}$, $A_y/A_x = 1/2$, $T_0 = 2 \text{ fs}$, $T_{cx} = 5 \text{ fs}$, $T_{cy} = 2T_{cx}$, $\omega_0 = 3.04 \text{ rad}\cdot\text{fs}^{-1}$, $\omega/\omega_0 = 0.5$, $f_0 = 600 \text{ mm}$, $\delta = 1.0$. It is seen from Fig. 2 that the dispersion affects the peak value of $P(\mathbf{\rho}, z, \omega)$. As compared with the dispersion-free case, the dispersion results in a decrease in the peak value of $P(\mathbf{\rho}, z, \omega)$. Furthermore, the higher order dispersion leads to a further decrease in the peak value of $P(\mathbf{\rho}, z, \omega)$.

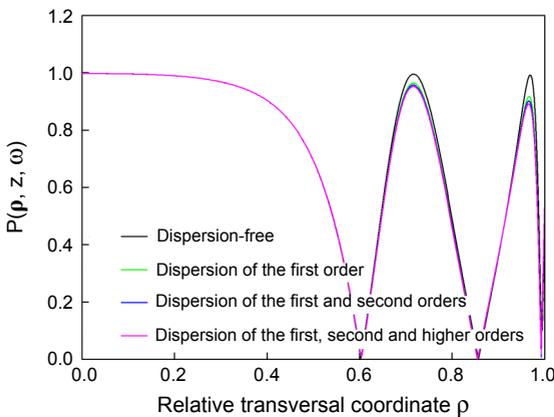


Fig. 2. The spectral degree of polarization $P(\mathbf{\rho}, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams *versus* the relative transversal coordinate ρ at the geometrical focal plane. The calculation parameters are $T_0 = 2 \text{ fs}$, $T_{cx} = 5 \text{ fs}$.

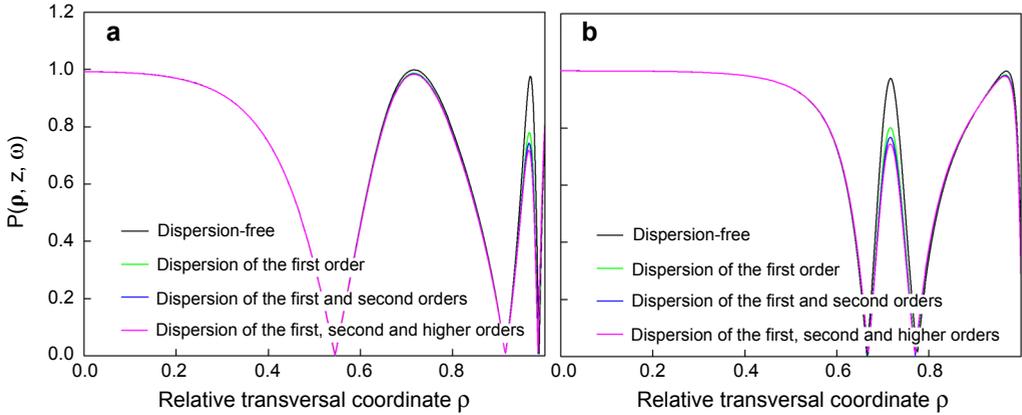


Fig. 3. The spectral degree of polarization $P(\boldsymbol{\rho}, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams versus the relative transversal coordinate ρ for different values of pulse duration: $T_0 = 1.5$ fs (a) and $T_0 = 2.5$ fs (b).

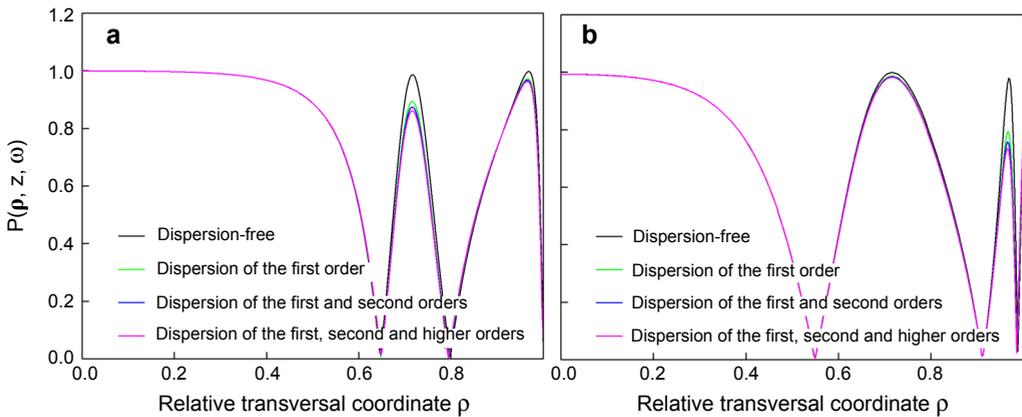


Fig. 4. The spectral degree of polarization $P(\boldsymbol{\rho}, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams versus the relative transversal coordinate ρ for different values of temporal coherence length: $T_{cx} = 3$ fs (a) and $T_{cx} = 9$ fs (b).

The spectral degree of polarization $P(\boldsymbol{\rho}, z, \omega)$ for different values of pulse duration $T_0 = 1.5$ fs and $T_0 = 2.5$ fs are shown in Figs. 3a and 3b, respectively (the other calculation parameters are the same as those in Fig. 2). Figure 3 shows that the dispersion has an opposite effect on the two peaks of $P(\boldsymbol{\rho}, z, \omega)$. From a comparison of Figs. 3 and 2, it can be seen that as the pulse duration T_0 increases, the influence of dispersion on the first and second peak of $P(\boldsymbol{\rho}, z, \omega)$ becomes greater and smaller, respectively.

The spectral degree of polarization $P(\boldsymbol{\rho}, z, \omega)$ for different values of temporal coherence length $T_{cx} = 3$ fs and $T_{cx} = 9$ fs are shown in Figs. 4a and 4b, respectively (the other calculation parameters are the same as those in Fig. 2). As can be seen in Figs. 4 and 2, for larger value of temporal coherence length T_{cx} the dispersion has great-

er influence on the second peak of $P(\boldsymbol{\rho}, z, \omega)$. However, for smaller value of temporal coherence length T_{cx} the dispersion has greater influence on the first peak of $P(\boldsymbol{\rho}, z, \omega)$.

The on-axis spectral degree of polarization $P(0, z, \omega)$ as a function of relative propagation distance z/f in the case of dispersion-free, dispersion of the first, second, and higher orders is shown in Fig. 5 ($\omega/\omega_0 = 1.5$, the other calculation parameters are the same as those in Fig. 2). It can be seen that the on-axis spectral degree of polarization $P(0, z, \omega)$ in the dispersion-free case is different form that in the dispersion case, however the dispersion of different orders almost has the same effect on the on-axis spectral degree of polarization. Therefore, the dispersion of the second, and higher orders plays a relatively minor role in the on-axis spectral degree of polarization $P(0, z, \omega)$.

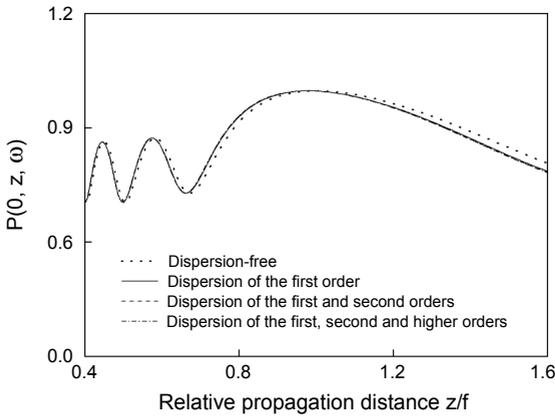


Fig. 5. The on-axis spectral degree of polarization $P(0, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams versus the relative propagation distance z/f .

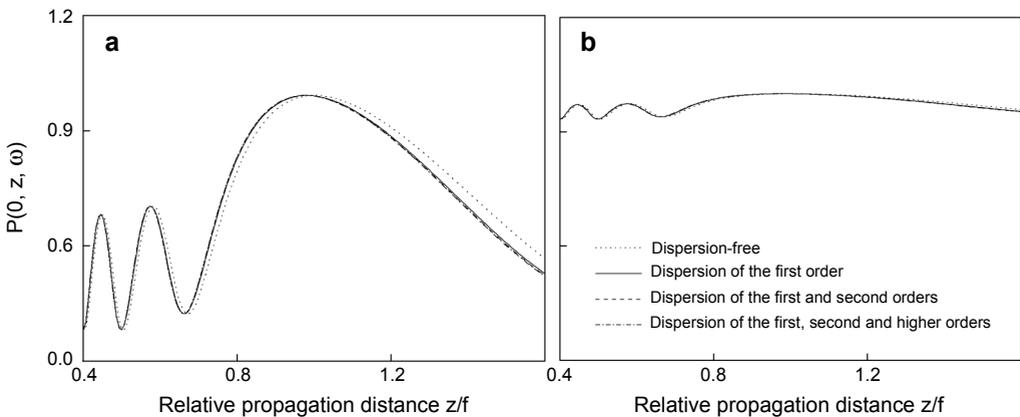


Fig. 6. The on-axis spectral degree of polarization $P(0, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams versus the relative propagation distance z/f for different values of pulse duration: $T_0 = 1.5$ fs (a) and $T_0 = 2.5$ fs (b).

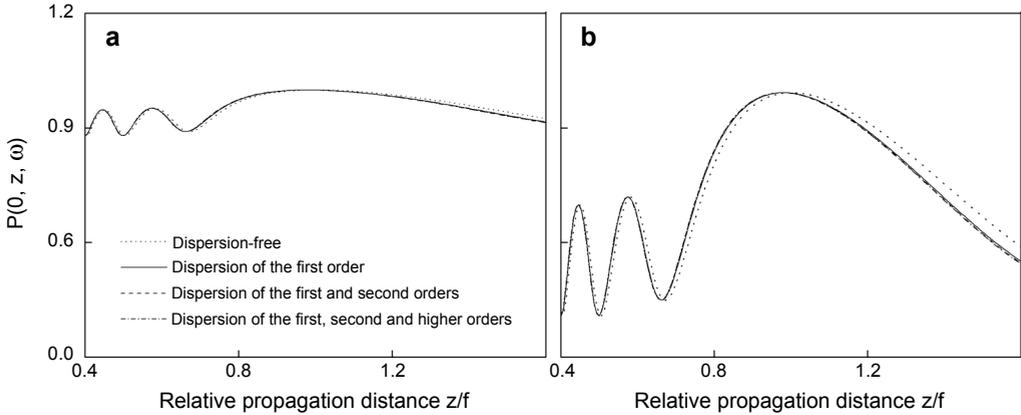


Fig. 7. The on-axis spectral degree of polarization $P(0, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams *versus* the relative propagation distance z/f for different values of temporal coherence length: $T_{cx} = 3$ fs (a) and $T_{cx} = 9$ fs (b).

Figure 6 gives the on-axis spectral degree of polarization $P(0, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams as a function of relative propagation distance z/f for different values of pulse duration T_0 ($\omega/\omega_0 = 1.5$, the other calculation parameters are the same as those in Fig. 3). As can be seen from Figs. 6 and 5, with a decrease in pulse duration T_0 , the effect of dispersion on the on-axis spectral degree of polarization $P(0, z, \omega)$ becomes more noticeable. In addition, the on-axis spectral degree of polarization $P(0, z, \omega)$ changes slightly with the relative propagation distance z/f as the pulse duration T_0 increases.

Figure 7 gives the on-axis spectral degree of polarization $P(0, z, \omega)$ of electromagnetic Bessel–Gauss pulsed beams *versus* the relative propagation distance z/f for different values of temporal coherence length T_{cx} . It is shown in Figs. 7 and 5 that the effect of dispersion on the on-axis spectral degree of polarization $P(0, z, \omega)$ becomes more noticeable with increasing temporal coherence length T_{cx} . And the on-axis spectral degree of polarization $P(0, z, \omega)$ changes slightly with the relative propagation distance z/f as the temporal coherence length T_{cx} decreases.

4. Conclusion

In this paper, we derive a closed-form analytical expression of the spectral degree of polarization $P(\mathbf{\rho}, z, \omega)$ of spatially and spectrally partially coherent electromagnetic Bessel–Gauss pulsed beams through a dispersive aperture lens. Numerical calculation results show the influence of dispersion on the spectral degree of polarization. As compared with the dispersion-free case, the dispersion results in a decrease in the peak value of $P(\mathbf{\rho}, z, \omega)$ and the higher order dispersion leads to further decrease in the peak value of $P(\mathbf{\rho}, z, \omega)$ at the focal plane. The dispersion has a greater influence on the first peak of $P(\mathbf{\rho}, z, \omega)$ for larger value of pulse duration T_0 or smaller value of

temporal coherence length T_{cx} . The dispersion has a greater influence on the second peak of $P(\mathbf{p}, z, \omega)$ for smaller value of pulse duration T_0 or larger value of temporal coherence length T_{cx} . The on-axis spectral degree of polarization $P(0, z, \omega)$ in the dispersion-free case is different form that in the dispersion case. With a decrease in pulse duration T_0 or an increase in temporal coherence length T_{cx} , the effect of dispersion on the on-axis spectral degree of polarization $P(0, z, \omega)$ becomes more noticeable. The on-axis spectral degree of polarization $P(0, z, \omega)$ changes slightly on propagation with increasing pulse duration T_0 or decreasing temporal coherence length T_{cx} . The results obtained in this paper may be crucial for high precision laser detection. And they can offer some theoretical guidance to the measurement of group delay dispersion of high numerical aperture objective lenses using two-photon excited fluorescence [24].

Acknowledgements – This work is supported by the National Natural Science Foundation of China under Grant Nos. 61275150, 61078077 and 61108090, the Education Department of Henan Province Project 13A140797, the Program for Science and Technology Innovation Talents in Universities of Henan Province (13HASTIT048) and the Program for Innovative Research Team (in Science and Technology) in University of Henna Province (Grant No. 13IRTSTHN020).

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*Received July 25, 2013
in revised form December 25, 2013*