Optica Applicata, Vol. XLIV, No. 3, 2014
DOI: 10.5277/oa140305

Optical solitons in birefringent fibers with parabolic law nonlinearity

QIN ZHOU[1, 2*], QIUPING ZHU[1], ANJAN BISWAS[3]

[1] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan, 430212, P.R. China

[2] School of Physics and Technology, Wuhan University, Wuhan, 430072, P.R. China

[3] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah-21589, Saudi Arabia

[*] Corresponding author: qinzhou@whu.edu.cn

This paper studies the propagation of optical solitons through birefringent fibers with parabolic law nonlinearity. The Hamiltonian perturbations that are inter-modal dispersion, self-steepening, third-order dispersion and nonlinear dispersions are taken into account. Both, Riccati equation expansion method and Jacobian elliptic equation expansion method are used. Finally, analytical solutions that are Jacobian elliptic periodic traveling wave solutions, periodic solutions, unbounded solutions, singular solutions, bright and dark soliton solutions are obtained under several constraint conditions.

Keywords: solitons, parabolic law nonlinearity, birefringent fibers, Jacobian elliptic equation, Riccati equation.

1. Introduction

Optical solitons, the most ideal carriers of information, have important application features in the optical communications and ultra-fast signal processing systems [1–5]. Most of the existing papers mainly focus on the optical solitons in the polarization preserving fibers, while there are very few papers that study the optical solitons in the birefringent fibers [6–14]. So the key idea of this paper is to seek exact soliton solutions to the birefringent fibers with Hamiltonian perturbations and parabolic law nonlinearity.

Birefringence is a natural phenomenon that occurs in optical fibers [6, 8]. The optical pulse will split into two orthogonally polarized pulses that have different propagation constants and group velocities, because it is very difficult to have delicate circularly symmetry for optical fibers [8].

In the presence of strong Hamiltonian type perturbations, the governing equation for the propagation of optical solitons through birefringent fibers with parabolic law nonlinearity is given by the following Hirota equations:

$$iq_t + a_1 q_{xx} + \left(b_1|q|^2 + c_1|r|^2\right)q + \left(d_1|q|^4 + e_1|q|^2|r|^2 + f_1|r|^4\right)q + \\ + i\lambda_1 q_x + is_1\left(|q|^2 q\right)_x + i\mu_1\left(|q|^2\right)_x q + i\theta_1|q|^2 q_x + i\gamma_1 q_{xxx} = 0 \quad (1)$$

$$ir_t + a_2 r_{xx} + \left(b_2|r|^2 + c_2|q|^2\right)r + \left(d_2|r|^4 + e_2|r|^2|q|^2 + f_2|q|^4\right)r + \\ + i\lambda_2 r_x + is_2\left(|r|^2 r\right)_x + i\mu_2\left(|r|^2\right)_x r + i\theta_2|r|^2 r_x + i\gamma_2 r_{xxx} = 0 \quad (2)$$

In Equations (1) and (2), the unknown functions $q(x, t)$ and $r(x, t)$ are the optical wave profiles for the two components in birefringent fibers; x and t represent the spatial and temporal variables, respectively.

For $l = 1, 2$, the constant parameters a_l, b_l, c_l, λ_l, s_l and γ_l are, respectively, the parameters of the group velocity dispersion (GVD), self-phase modulation (SPM), cross--phase modulation (XPM), inter-modal dispersion (IMD), self-steepening and third-order dispersion (TOD) for the two polarized pulses. The terms with d_l, e_l, and f_l are associated with the quintic terms of the parabolic (cubic-quintic) law nonlinearity [7, 8]. Finally, μ_l and θ_l are the nonlinear dispersions.

The aim of the present work is to construct the Jacobian elliptic periodic traveling wave solutions, periodic solutions, unbounded solutions, singular solutions, singular, bright and dark soliton solutions in the birefringent fibers with Hamiltonian perturbations and parabolic law nonlinearity. The strong Hamiltonian type perturbations that are IMD, self-steepening, TOD and nonlinear dispersions are taken into consideration. The integration methods are the Riccati equation expansion method and Jacobian elliptic equation expansion method. Several constraint conditions for analytical solutions to exist are displayed.

In order to obtain exact solutions to Eqs. (1) and (2), making the hypothesis in the form [6–9]:

$$q(x, t) = A_1 P_1\left[\eta(x, t)\right]\exp\left[i\phi_1(x, t)\right] \quad (3)$$

$$r(x, t) = A_2 P_2\left[\eta(x, t)\right]\exp\left[i\phi_2(x, t)\right] \quad (4)$$

where $\eta = B(x - vt)$ and $\phi_l = -\kappa_l x + \omega_l t + \theta_l$; $P_l(\eta)$ and $\phi_l(x, t)$ for $l = 1, 2$ are the amplitude and phase components of the two solitons, respectively; A_l, B and v represent the amplitude, width and velocity of the solitons. Additionally, κ_l are frequencies of the two solitons, ω_l are the wave numbers, while θ_l are the phase constants.

Substituting (3) and (4) into (1) and (2), and separating the real and imaginary parts, respectively, one obtains

$$-\left(\omega_l - \lambda_l \kappa_l + a_l \kappa_l^2 + \gamma_l \kappa_l^3\right) P_l + c_l A_{\bar{l}}^2 P_l P_{\bar{l}}^2 + d_l A_l^4 P_l^5 + e_l A_l^2 A_{\bar{l}}^2 P_l^3 P_{\bar{l}}^2 + \\ + f_l A_{\bar{l}}^4 P_l P_{\bar{l}}^4 + (b_l + s_l \kappa_l + \theta_l \kappa_l) A_l^2 P_l^3 + (a_l + 3\gamma_l \kappa_l) B^2 P_l'' = 0 \tag{5}$$

$$\left(\lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 - v\right) P_l' + (3s_l + 2\mu_l + \theta_l) A_l^2 P_l^2 P_l' + \gamma_l B^2 P_l''' = 0 \tag{6}$$

for $l = 1, 2$ and $\bar{l} = 3 - l$.

2. Riccati equation expansion method

Assume that $P_l(\eta)$ satisfies

$$P_l'(\eta) = a + b P_l^2(\eta) \tag{7}$$

where a and b are the nonzero real constants. Equation (7) is the famous Riccati equation [15–17], the solutions of which are listed in Table 1.

T a b l e 1. Solutions to the Riccati equation (7).

$ab > 0$	$P_l(\eta) = \dfrac{\sqrt{ab}}{b}\tan(\sqrt{ab}\,\eta)$
	$P_l(\eta) = -\dfrac{\sqrt{ab}}{b}\cot(\sqrt{ab}\,\eta)$
$ab < 0$	$P_l(\eta) = -\dfrac{\sqrt{-ab}}{b}\tanh(\sqrt{-ab}\,\eta)$
	$P_l(\eta) = -\dfrac{\sqrt{-ab}}{b}\coth(\sqrt{-ab}\,\eta)$

Substituting the assumption (7) into Eqs. (5) and (6) yields

$$-\left(\omega_l - \lambda_l \kappa_l + a_l \kappa_l^2 + \gamma_l \kappa_l^3\right) P_l + c_l A_{\bar{l}}^2 P_l P_{\bar{l}}^2 + d_l A_l^4 P_l^5 + e_l A_l^2 A_{\bar{l}}^2 P_l^3 P_{\bar{l}}^2 + \\ + f_l A_{\bar{l}}^4 P_l P_{\bar{l}}^4 + (b_l + s_l \kappa_l + \theta_l \kappa_l) A_l^2 P_l^3 + (a_l + 3\gamma_l \kappa_l) B^2 \left(2abP_l + 2b^2 P_l^3\right) = 0 \tag{8}$$

$$\left(\lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 - v\right)\left(a + bP_l^2\right) + (3s_l + 2\mu_l + \theta_l) A_l^2 P_l^2 \left(a + bP_l^2\right) + \\ + \gamma_l B^2 \left(2a^2 b + 8ab^2 P_l^2 + 6b^3 P_l^4\right) = 0 \tag{9}$$

Then using the homogeneous balance principle, from Eqs. (8) and (9), setting the coefficients of each power of $P_l(\eta)$ to zero gives:

$$\omega_l = \lambda_l \kappa_l - a_l \kappa_l^2 - \gamma_l \kappa_l^3 + 2ab(a_l + 3\gamma_l \kappa_l)B^2 \tag{10}$$

$$(b_l + s_l \kappa_l + \theta_l \kappa_l) A_l^2 + 2b^2(a_l + 3\gamma_l \kappa_l) B^2 + c_l A_{\bar{l}}^2 = 0 \tag{11}$$

$$d_l A_l^4 + e_l A_l^2 A_{\bar{l}}^2 + f_l A_{\bar{l}}^4 = 0 \tag{12}$$

$$v = \lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 + 2ab\gamma_l B^2 \tag{13}$$

$$v = \lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 + 8ab\gamma_l B^2 + \frac{a(3s_l + 2\mu_l + \theta_l)}{b} A_l^2 \tag{14}$$

$$(3s_l + 2\mu_l + \theta_l) A_l^2 + 6b^2 \gamma_l B^2 = 0 \tag{15}$$

It needs to be noted that upon equating the two values of the solitons velocities from (13) and (14) also yields the same relation as given by (15).

Equating the two values of the soliton velocity v, for $l = 1, 2$, from Eq. (13) gives the width of the soliton as

$$B = \left[\frac{(\lambda_l - \lambda_{\bar{l}}) - 2(a_l \kappa_l - a_{\bar{l}} \kappa_{\bar{l}}) - 3(\gamma_l \kappa_l^2 - \gamma_{\bar{l}} \kappa_{\bar{l}}^2)}{2ab(\gamma_l - \gamma_{\bar{l}})} \right]^{1/2} \tag{16}$$

which introduces the constraint condition

$$ab(\gamma_l - \gamma_{\bar{l}}) \left[(\lambda_l - \lambda_{\bar{l}}) - 2(a_l \kappa_l - a_{\bar{l}} \kappa_{\bar{l}}) - 3(\gamma_l \kappa_l^2 - \gamma_{\bar{l}} \kappa_{\bar{l}}^2) \right] > 0 \tag{17}$$

From Eq. (15), the amplitude of the solitons are given by

$$A_l = \left\{ -\frac{3b\gamma_l \left[(\lambda_l - \lambda_{\bar{l}}) - 2(a_l \kappa_l - a_{\bar{l}} \kappa_{\bar{l}}) - 3(\gamma_l \kappa_l^2 - \gamma_{\bar{l}} \kappa_{\bar{l}}^2) \right]}{a(3s_l + 2\mu_l + \theta_l)(\gamma_l - \gamma_{\bar{l}})} \right\}^{1/2} \tag{18}$$

with the constraint condition

$$\gamma_l (3s_l + 2\mu_l + \theta_l) < 0 \tag{19}$$

Additionally, Equations (11) and (12) pose other two constraint conditions that are given by

$$\frac{3\gamma_l(b_l + s_l\kappa_l + \theta_l\kappa_l)}{3s_l + 2\mu_l + \theta_l} + \frac{3c_l\gamma_{\bar{l}}}{3s_{\bar{l}} + 2\mu_{\bar{l}} + \theta_{\bar{l}}} = a_l + 3\gamma_l\kappa_l \tag{20}$$

$$\frac{d_l\gamma_l^2}{(3s_l + 2\mu_l + \theta_l)^2} + \frac{e_l\gamma_l\gamma_{\bar{l}}}{(3s_l + 2\mu_l + \theta_l)(3s_{\bar{l}} + 2\mu_{\bar{l}} + \theta_{\bar{l}})} + \frac{f_l\gamma_{\bar{l}}^2}{(3s_{\bar{l}} + 2\mu_{\bar{l}} + \theta_{\bar{l}})^2} = 0 \tag{21}$$

Hence, finally the singular solutions, dark and singular soliton solutions for the birefringent fibers with parabolic law nonlinearity are obtained, which are listed as follows.

Case 1 – when $ab > 0$, Eqs. (1) and (2) admit the singular periodic solutions that are given by

$$q(x, t) = \frac{\sqrt{ab}}{b}A_1\tan\left[\sqrt{ab}\,B(x - vt)\right]\exp\left[i(-\kappa_1 x + \omega_1 t + \theta_1)\right] \tag{22}$$

$$r(x, t) = \frac{\sqrt{ab}}{b}A_2\tan\left[\sqrt{ab}\,B(x - vt)\right]\exp\left[i(-\kappa_2 x + \omega_2 t + \theta_2)\right] \tag{23}$$

$$q(x, t) = -\frac{\sqrt{ab}}{b}A_1\cot\left[\sqrt{ab}\,B(x - vt)\right]\exp\left[i(-\kappa_1 x + \omega_1 t + \theta_1)\right] \tag{24}$$

$$r(x, t) = -\frac{\sqrt{ab}}{b}A_2\cot\left[\sqrt{ab}\,B(x - vt)\right]\exp\left[i(-\kappa_2 x + \omega_2 t + \theta_2)\right] \tag{25}$$

Case 2 – when $ab < 0$, Eqs. (1) and (2) admit the dark soliton solutions that are given by

$$q(x, t) = -\frac{\sqrt{-ab}}{b}A_1\tanh\left[\sqrt{-ab}\,B(x - vt)\right]\exp\left[i(-\kappa_1 x + \omega_1 t + \theta_1)\right] \tag{26}$$

$$r(x, t) = -\frac{\sqrt{-ab}}{b}A_2\tanh\left[\sqrt{-ab}\,B(x - vt)\right]\exp\left[i(-\kappa_2 x + \omega_2 t + \theta_2)\right] \tag{27}$$

and the singular soliton solutions that are given by

$$q(x, t) = -\frac{\sqrt{-ab}}{b}A_1\coth\left[\sqrt{-ab}\,B(x - vt)\right]\exp\left[i(-\kappa_1 x + \omega_1 t + \theta_1)\right] \tag{28}$$

$$r(x, t) = -\frac{\sqrt{-ab}}{b}A_2\coth\left[\sqrt{-ab}\,B(x - vt)\right]\exp\left[i(-\kappa_2 x + \omega_2 t + \theta_2)\right] \tag{29}$$

where the amplitude and width of the solitons are given by Eqs. (18) and (16) respectively, while the velocity of the solitons are given by Eq. (13) or (14) and finally the wave numbers are given by Eq. (10). The constraint conditions for analytical solutions to exist are given by Eqs. (17) and (19)–(21).

3. Jacobian elliptic equation expansion method

Assume that $P_l(\eta)$ satisfies

$$P_l'^2(\eta) = g_0 + g_2 P_l^2(\eta) + g_4 P_l^4(\eta) \tag{30}$$

where g_0, g_2 and g_4 are the nonzero real constants. Eq. (30) is Jacobian elliptic equation, the solutions of which are listed in [2, 18–20].

Substituting the assumption (30) into Eqs. (5) and (6) yields

$$-\left(\omega_l - \lambda_l \kappa_l + a_l \kappa_l^2 + \gamma_l \kappa_l^3\right) P_l + c_l A_{\bar{l}}^2 P_l P_{\bar{l}}^2 + d_l A_l^4 P_l^5 + e_l A_l^2 A_{\bar{l}}^2 P_l^3 P_{\bar{l}}^2 + \\ + f_l A_{\bar{l}}^4 P_l P_{\bar{l}}^4 + (b_l + s_l \kappa_l + \theta_l \kappa_l) A_l^2 P_l^3 + (a_l + 3\gamma_l \kappa_l) B^2 \left(g_2 P_l + 2 g_4 P_l^3\right) = 0 \tag{31}$$

$$\left(\lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 - v\right)^2 + 2\left(\lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 - v\right)(3s_l + 2\mu_l + \theta_l) A_l^2 P_l^2 + \\ + (3s_l + 2\mu_l + \theta_l)^2 A_l^4 P_l^4 = \gamma_l^2 B^4 \left(g_2 + 6 g_4 P_l^2\right)^2 \tag{32}$$

Then using the homogeneous balance principle, from Eqs. (31) and (32), setting the coefficients of each power of $P_l(\eta)$ to zero gives

$$\omega_l = g_2 (a_l + 3\gamma_l \kappa_l) B^2 + \lambda_l \kappa_l - a_l \kappa_l^2 - \gamma_l \kappa_l^3 \tag{33}$$

$$(b_l + s_l \kappa_l + \theta_l \kappa_l) A_l^2 + c_l A_{\bar{l}}^2 + 2 g_4 (a_l + 3\gamma_l \kappa_l) B^2 = 0 \tag{34}$$

$$d_l A_l^4 + e_l A_l^2 A_{\bar{l}}^2 + f_l A_{\bar{l}}^4 = 0 \tag{35}$$

$$v = \lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 + g_2 \gamma_l B^2 \tag{36}$$

$$v = \lambda_l - 2a_l \kappa_l - 3\gamma_l \kappa_l^2 - \frac{6 g_2 g_4 \gamma_l^2 B^4}{(3s_l + 2\mu_l + \theta_l) A_l^2} \tag{37}$$

$$(3s_l + 2\mu_l + \theta_l) A_l^2 = 6 g_4 \gamma_l B^2 \tag{38}$$

It needs to be noted that equating the two values of the solitons velocities from (36) and (37) also yields the same relation as given by (38).

T a b l e 2. Jacobian elliptic periodic traveling wave solutions to Eqs. (1) and (2).

g_0	g_2	g_4	$q(x,t)$	$r(x,t)$
λ^2E^2	$-\lambda^2(1+m^2)$	λ^2m^2/E^2	$EA_1sn\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2sn\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$\lambda^2E^2(1-m^2)$	$\lambda^2(2m^2-1)$	$-\lambda^2m^2/E^2$	$EA_1cn\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2cn\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$-\lambda^2E^2(1-m^2)$	$\lambda^2(2-m^2)$	$-\lambda^2/E^2$	$EA_1dn\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2dn\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$\lambda^2m^2E^2$	$-\lambda^2(1+m^2)$	λ^2/E^2	$EA_1ns\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2ns\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$-\lambda^2m^2E^2$	$\lambda^2(2m^2-1)$	$\lambda^2(1-m^2)/E^2$	$EA_1nc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2nc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$-\lambda^2E^2$	$\lambda^2(2-m^2)$	$-\lambda^2(1-m^2)/E^2$	$EA_1nd\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2nd\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$\lambda^2(2-m^2)$	$\lambda^2(1-m^2)/E^2$	$EA_1sc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2sc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$\lambda^2(2m^2-1)$	$-\lambda^2m^2(1-m^2)/E^2$	$EA_1sd\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2sd\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$\lambda^2E^2(1-m^2)$	$\lambda^2(2-m^2)$	λ^2/E^2	$EA_1cs\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2cs\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$-\lambda^2(1+m^2)$	λ^2m^2/E^2	$EA_1cd\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2cd\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$-\lambda^2m^2(1-m^2)E^2$	$\lambda^2(2m^2-1)$	λ^2/E^2	$EA_1ds\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2ds\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
$\lambda^2m^2E^2$	$-\lambda^2(1+m^2)$	λ^2/E^2	$EA_1dc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2dc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$

T a b l e 3. Trigonometric periodic solutions to Eqs. (1) and (2).

g_0	g_2	g_4	$q(x,t)$	$r(x,t)$
λ^2E^2	$-\lambda^2$	0	$EA_1\sin\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\sin\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$-\lambda^2$	0	$EA_1\cos\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\cos\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$

Table 4. Unbounded solutions to Eqs. (1) and (2).

g_0	g_2	g_4	$q(x,t)$	$r(x,t)$
$-\lambda^2E^2$	λ^2	0	$EA_1\cosh\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\cosh\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	λ^2	0	$EA_1\sinh\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\sinh\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$

Table 5. Singular periodic solutions to Eqs. (1) and (2).

g_0	g_2	g_4	$q(x,t)$	$r(x,t)$
0	$-\lambda^2$	λ^2/E^2	$EA_1\csc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\csc\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
0	$-\lambda^2$	λ^2/E^2	$EA_1\sec\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\sec\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$2\lambda^2$	λ^2/E^2	$EA_1\tan\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\tan\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$2\lambda^2$	λ^2/E^2	$EA_1\cot\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\cot\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$

Table 6. Singular, dark and bright soliton solutions to Eqs. (1) and (2).

g_0	g_2	g_4	$q(x,t)$	$r(x,t)$
λ^2E^2	$-2\lambda^2$	λ^2/E^2	$EA_1\coth\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\coth\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
0	λ^2	λ^2/E^2	$EA_1\operatorname{csch}\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\operatorname{csch}\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
λ^2E^2	$-2\lambda^2$	λ^2/E^2	$EA_1\tanh\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\tanh\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$
0	λ^2	$-\lambda^2/E^2$	$EA_1\operatorname{sech}\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_1x+\omega_1t+\theta_1)\right]$	$EA_2\operatorname{sech}\left[\lambda B(x-vt)\right]\exp\left[i(-\kappa_2x+\omega_2t+\theta_2)\right]$

Equating the two values of the soliton velocity v, for $l = 1, 2$, from Eq. (36) gives the width of the soliton as

$$B = \left[\frac{(\lambda_l - \lambda_{\bar{l}}) - 2(a_l\kappa_l - a_{\bar{l}}\kappa_{\bar{l}}) - 3(\gamma_l\kappa_l^2 - \gamma_{\bar{l}}\kappa_{\bar{l}}^2)}{g_2(\gamma_l - \gamma_{\bar{l}})}\right]^{1/2} \tag{39}$$

which poses the constraint condition

$$g_2(\gamma_l - \gamma_{\bar{l}})\left[(\lambda_l - \lambda_{\bar{l}}) - 2(a_l\kappa_l - a_{\bar{l}}\kappa_{\bar{l}}) - 3(\gamma_l\kappa_l^2 - \gamma_{\bar{l}}\kappa_{\bar{l}}^2)\right] > 0 \tag{40}$$

From Eq. (38), the amplitudes of the solitons are given by

$$A_l = \left\{\frac{6g_4\gamma_l\left[(\lambda_l - \lambda_{\bar{l}}) - 2(a_l\kappa_l - a_{\bar{l}}\kappa_{\bar{l}}) - 3(\gamma_l\kappa_l^2 - \gamma_{\bar{l}}\kappa_{\bar{l}}^2)\right]}{g_2(3s_l + 2\mu_l + \theta_l)(\gamma_l - \gamma_{\bar{l}})}\right\}^{1/2} \tag{41}$$

with the constraint condition

$$g_4\gamma_l(3s_l + 2\mu_l + \theta_l) > 0 \tag{42}$$

Additionally, Equations (34) and (35) pose other two constraint conditions that are given by

$$\frac{3\gamma_l(b_l + s_l\kappa_l + \theta_l\kappa_l)}{3s_l + 2\mu_l + \theta_l} + \frac{3c_l\gamma_{\bar{l}}}{3s_{\bar{l}} + 2\mu_{\bar{l}} + \theta_{\bar{l}}} + (a_l + 3\gamma_l\kappa_l) = 0 \tag{43}$$

$$\frac{d_l\gamma_l^2}{(3s_l + 2\mu_l + \theta_l)^2} + \frac{e_l\gamma_l\gamma_{\bar{l}}}{(3s_l + 2\mu_l + \theta_l)(3s_{\bar{l}} + 2\mu_{\bar{l}} + \theta_{\bar{l}})} + \frac{f_l\gamma_{\bar{l}}^2}{(3s_{\bar{l}} + 2\mu_{\bar{l}} + \theta_{\bar{l}})^2} = 0 \tag{44}$$

Hence, finally the explicit Jacobian elliptic periodic traveling wave solutions for the birefringent fibers with parabolic law nonlinearity are constructed (see Table 2). The amplitude and width of the solitons are given by Eqs. (41) and (39), respectively, while the velocity of the solitons are given by Eq. (36) or (37) and finally the wave numbers are given by Eq. (33). The constraint conditions for analytical solutions to exist are given by Eqs. (40) and (42)–(44).

It needs to be noted that when the modulus $m = 0$ and $m = 1$, the Jacobian elliptic periodic traveling wave solutions become trigonometric periodic solutions (see Table 3), unbounded solutions (see Table 4), singular solutions (see Table 5), singular, bright and dark soliton solutions (see Table 6).

4. Conclusion

The Hirota equation, describing the propagation of optical solitons through birefringent fibers with Hamiltonian perturbations and parabolic law nonlinearity, is studied analytically by employing the Riccati equation expansion method and Jacobian elliptic equation expansion method. We report the Jacobian elliptic periodic traveling wave solutions, periodic solutions, unbounded solutions, singular solutions, singular, bright and dark soliton solutions. We obtain the constraint conditions for these solutions to exist.

Acknowledgements – The work of the second author (Q.P. Zhu) was supported by the Scientific Research Fund of Hubei Provincial Education Department under Grant No. B2013193.

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Received June 10, 2014
in revised form July 17, 2014