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Theoretical analysis of a hollow laser beam transmitting in an off-axis Cassegrain optical antenna system

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The optical model of a Cassegrain optical antenna with a confocal double-parabolic reflector structure has been designed, and the propagation characteristics of a hollow laser beam, which could avoid the loss of energy caused by the subreflector center reflection in the optical antenna, has been researched in this paper. By detailed analysis and numerical calculations of a receiving Cassegrain antenna with different deflection angles, the coupling efficiency curve and 3-D distributions of the receiving light intensity for different inclined angles have been obtained.

Keywords: Cassegrain optical antenna, off-axis hollow laser beam, receiving efficiency.

1. Introduction

With the rapid advancement of space laser communications technology and optical devices, optical communication has developed much faster in recent years [1]. Plenty of studies have been accomplished and much more manpower and material resources have been and will be invested in those researches [2]. Gaussian optical beams have also attracted much attention when designing the high power of SLED, and they have been widely used in optical communication systems [3, 4]. A double-parabolic reflector Cassegrain optical antenna structure, which possesses many advantages, including the bigger aperture, less aberration and more choices of different wavelengths, has been often employed in space optical communication systems [5]. However, the loss of energy caused by the subreflector center reflection in the transmitting Cassegrain optical antenna and the off-axis error of a receiving antenna have seriously hindered the transmission efficiency in the space optical communication system [6]. Therefore, a hollow beam could almost avoid the loss of energy caused by the subreflector center reflection, which can be seen from the calculation results.

In this work, the effect of the off-axis antenna structure on the hollow beam propagation efficiency has been investigated. The results will offer the fundamental research for the propagation efficiency enhancement of the free space optical communication.

2. The comparison between the transmitting Gaussian beam and hollow beam

The Cassegrain antenna is mainly made up of a primary reflector and a subreflector. We chose the aperture diameters of the primary reflector and the subreflector as $2a = 150$ mm and $2b = 30$ mm, respectively. The distance between the vertices of two reflectors is $d = 300$ mm. The transmitting antenna and the receiving antenna with a symmetrical structure are shown in Fig. 1.

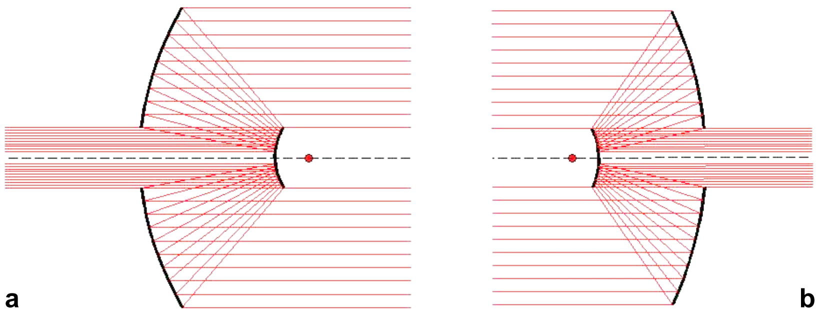

Fig. 1. The Cassegrain antenna system in space optical communication. The structure of a transmitting optical antenna (**a**). The structure of a receiving optical antenna (**b**).

Because those two antennas are symmetric, only the parameters of the transmitting antenna were considered in this paper. According to the properties of the double-parabolic reflection Cassegrain system, the geometric optical paths of the transmitting op-

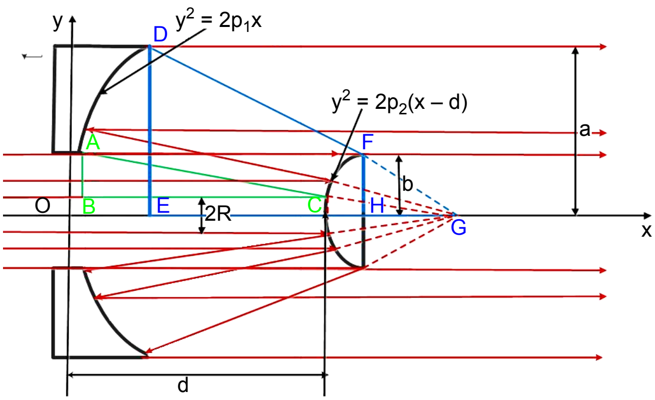

Fig. 2. The geometric optical path of transmitting Cassegrain antenna.

tical Cassegrain antenna are shown in Fig. 2. The primary mirror satisfies the equation $y^2 = 2p_1x$, and the equation of the subreflector is $y^2 = 2p_2(x - d)$

$$d = \frac{p_1}{2} - \frac{p_2}{2} \tag{1}$$

In the triangle ΔABC,

$$\overline{AC} = \left(\frac{b^2}{2p_1} + \frac{p_1}{2}\right) - \left(\frac{R^2}{2p_2} + \frac{p_2}{2}\right) = \frac{b^2}{2p_1} - \frac{R^2}{2p_2} + d \tag{2}$$

$$\overline{BC} = \frac{R^2}{2p_2} + d - \frac{b^2}{2p_1} \tag{3}$$

$$\overline{AB} = b - R \tag{4}$$

It can be deduced that

$$\left(\frac{b^2}{2p_1} - \frac{R^2}{2p_2} + d\right)^2 = \left(\frac{R^2}{2p_2} + d - \frac{b^2}{2p_1}\right)^2 + (b - R)^2 \tag{5}$$

By solving two simultaneous equations (Eqs. (1) and (5)), the radius of the light which was reflected backward at the center of the subreflector is

$$R = \frac{p_2}{p_1} b \tag{6}$$

In the triangles ΔDEG and ΔFHG

$$\frac{\overline{HG}}{\overline{EG}} = \frac{\overline{FH}}{\overline{DE}} \Rightarrow \frac{p_2/2 - b^2/2p_2}{p_1/2 - a^2/2p_1} = \frac{b}{a} \tag{7}$$

By solving Eqs. (1) and (7), the focal length of the primary reflector could be written as

$$\frac{p_1}{2} = \frac{a}{a - b} d \tag{8}$$

and the focal length of the subreflector is

$$\frac{p_2}{2} = \frac{b}{a - b} d \tag{9}$$

According to Eqs. (8) and (9), Eq. (6) can be simplified as

$$R = b^2/a \tag{10}$$

Supposing the incident light is a Gaussian beam with a wavelength of $\lambda = 830$ nm. From the theoretical analysis, the light intensity of the beam is [5]

$$I_1(r, z) = \frac{C^2}{\omega^2(z)} \exp\left[-\frac{2r^2}{\omega^2(z)}\right] \tag{11}$$

where

$$\omega(z) = \omega_0 \sqrt{1 + \left(\frac{\lambda z}{\pi \omega_0^2}\right)^2} \tag{12}$$

representing the spot radius of the Gaussian beam, z denotes the propagation distance, and ω_0 is the light waist of the Gaussian beam.

From Eq. (6), the power attenuation of the Gaussian beam can be written as

$$\frac{P_1}{P} = \frac{\int_0^{2\pi} \frac{C^2}{\omega^2(z)} \mathrm{d}\theta \int_0^{R} \exp\left[-\frac{2r^2}{\omega^2(z)}\right] r \mathrm{d}r}{\int_0^{2\pi} \frac{C^2}{\omega^2(z)} \mathrm{d}\theta \int_0^{\omega(z)} \exp\left[-\frac{2r^2}{\omega^2(z)}\right] r \mathrm{d}r} = \frac{\exp\left[-\frac{2r^2}{\omega^2(z)}\right]\Bigg|_0^R}{\exp\left[-\frac{2r^2}{\omega^2(z)}\right]\Bigg|_0^{\omega(z)}} \tag{13}$$

Because of the properties of the Gaussian beam, $\omega(z)$ denotes the radius of a spot reaching the subreflector. Supposing $\omega(z) = b$ and $R = b^2/a = 3$ mm, the incident light is expanded and aligned to match with the antenna. Therefore,

$$\frac{P_1}{P} = \frac{\exp\left[-\frac{2r^2}{b^2}\right]\Bigg|_0^3}{\exp\left[-\frac{2r^2}{b^2}\right]\Bigg|_0^{15}} = 0.0889 \tag{14}$$

which showed that the power attenuation was as much as 8.89%. If the incident beam is a dark hollow laser beam whose dark-part radius was R, by theoretical conclusion, the light intensity of the beam is [6]

$$I(r, z) = \frac{P_0}{\pi \omega^2(z)} \exp\left[-\frac{2(r - R_0)^2}{\omega^2(z)}\right] \tag{15}$$

where

$$\omega(z) = \omega_0 \sqrt{1 + \frac{\left[z + (r - R_0)^2 / \tan(\theta)\right]^2}{z_0^2}} \tag{16}$$

$$R_0 = L\sin(2\theta)\left[1 - \frac{\sin(\theta)}{\sqrt{n^2 - \cos^2(\theta)}}\right] \tag{17}$$

and L denotes the length of the double cone prism which is used to create a hollow beam, θ is the vertex angle of the double cone prism, z means the propagation distance, and n stands for the refractive index of the double cone prism.

The power attenuation can be written as

$$\frac{P_1'}{P} = \frac{\dfrac{P_0}{\pi\omega^2(z)}\pi R^2 e^{-2}}{\displaystyle\int_0^{2\pi} d\theta \int_R^b I(r, z) r dr} \tag{18}$$

Supposing, $\omega(z) = (b - R)/2 = 6$ mm, $R_0 = (b + R)/2 = 9$ mm. By calculation,

$$\frac{P_1'}{P} = \frac{\dfrac{P_0}{\pi 6^2}\pi R^2 e^{-2}}{\displaystyle\int_0^{2\pi} d\theta \int_9^{15} \frac{P_0}{\pi 6^2} \exp\left[-\frac{2(r-40)^2}{6^2}\right] r dr} = 0.0094 \tag{19}$$

which suggested that the power attenuation was only 0.94%. In comparison, it was much more efficient to transmit the hollow beam than the Gaussian beam.

3. Analysis of power and coupling efficiency of an antenna with an inclined optical axis

Assuming the transmitting Cassegrain antennas are precisely aligned and the inclined angle between the receiving Cassegrain antenna and the optical axis is γ, the schematic diagram of the off-axis antenna system is as shown in Fig. 3.

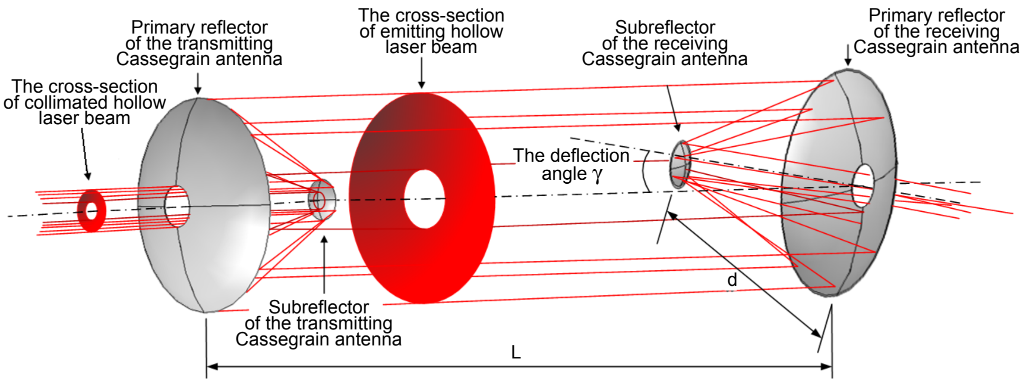

Fig. 3. The schematic diagram of an off-axis antenna system.

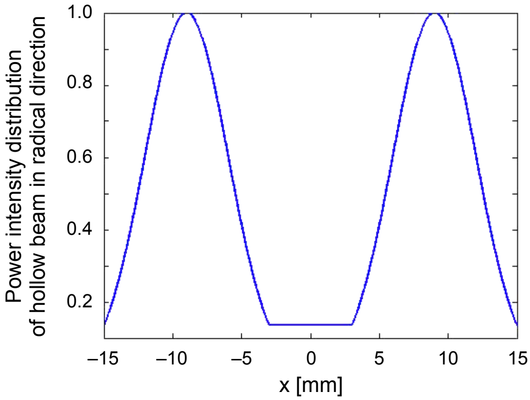

Fig. 4. The light intensity distribution of a hollow beam in the radical direction.

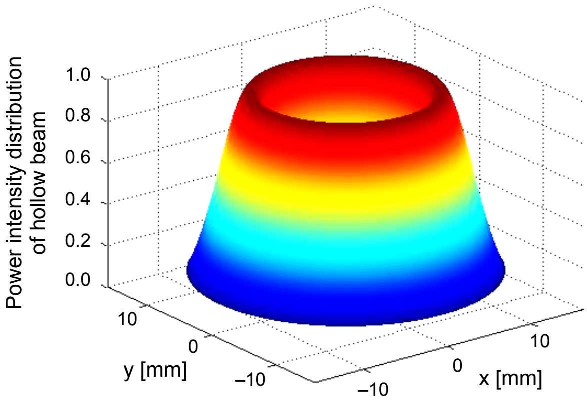

Fig. 5. The 3-D image light intensity distribution of a hollow beam.

Supposing the incident beam is a hollow laser beam that is parallel to the optical axis of the system, the light intensity distribution of the hollow beam in the radial direction can be obtained, as shown in Fig. 4. The 3-D image light intensity distribution of the hollow beam is shown in Fig. 5.

By the theoretical analysis, when transmitting a hollow beam, the receiving power has been obtained [7].

As shown in Fig. 6**a**, when $\gamma \geq \mathrm{atan}(2a/d)$, there is no light received, hence $P_r = 0$. As shown in Fig. 6**b**, when $\mathrm{atan}[(a+b)/d] \leq \gamma < \mathrm{atan}(2a/d)$, the receiving power is

$$P_r = \int_{-\sqrt{a^2-[d\tan(\gamma)/2]^2}}^{\sqrt{a^2-[d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)/2}^{\sqrt{a^2-x^2}} I(x,y)\,\mathrm{d}y\,\mathrm{d}x + \\ + \int_{-\sqrt{a^2-[d\tan(\gamma)/2]^2}}^{\sqrt{a^2-[d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)-\sqrt{a^2-x^2}}^{d\tan(\gamma)/2} I(x,y)\,\mathrm{d}y\,\mathrm{d}x \tag{20}$$

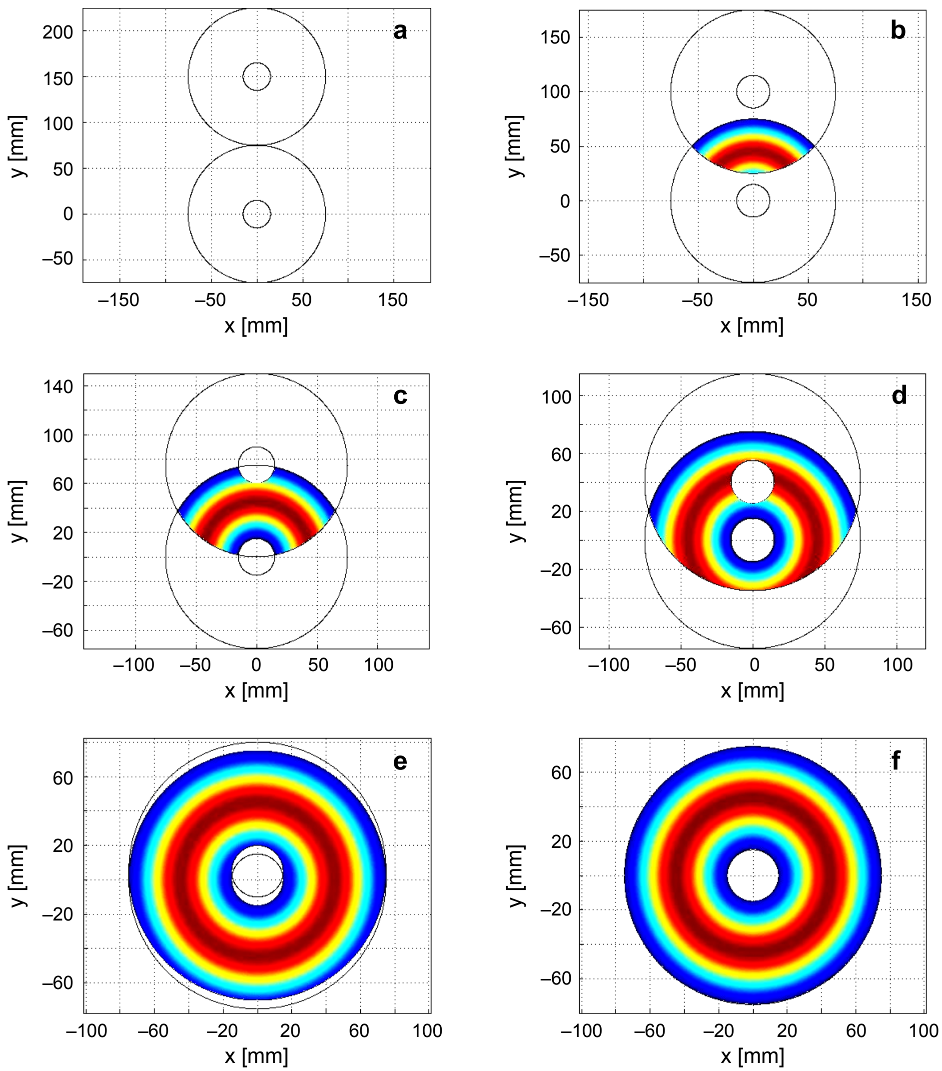

Fig. 6. The distribution of a hollow beam light spot on the receiving plane (see text for explanation).

As shown in Fig. 6**c**, when $\operatorname{atan}[(a-b)/d] \le \gamma < \operatorname{atan}[(a+b)/d]$, the receiving power is

$$P_r = \int_{-\sqrt{a^2-[d\tan(\gamma)/2]^2}}^{\sqrt{a^2-[d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)/2}^{\sqrt{a^2-x^2}} I(x,y)\mathrm{d}y\mathrm{d}x +$$

$$+ \int_{-\sqrt{a^2-[d\tan(\gamma)/2]^2}}^{\sqrt{a^2-[d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)-\sqrt{a^2-x^2}}^{d\tan(\gamma)/2} I(x,y)\mathrm{d}y\mathrm{d}x +$$

$$- \int_{-\sqrt{a^2 - y_1^2}}^{\sqrt{a^2 - y_1^2}} \int_{y_1}^{\sqrt{a^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x - \int_{-\sqrt{a^2 - y_1^2}}^{\sqrt{a^2 - y_1^2}} \int_{d\tan(\gamma) - \sqrt{b^2 - x^2}}^{y_1} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$- \int_{-\sqrt{a^2 - y_2^2}}^{\sqrt{a^2 - y_2^2}} \int_{y_2}^{d\tan(\gamma) - \sqrt{a^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x - \int_{-\sqrt{a^2 - y_2^2}}^{\sqrt{a^2 - y_2^2}} \int_{\sqrt{b^2 - x^2}}^{y_2} I(x, y) \mathrm{d}y \mathrm{d}x \tag{21}$$

where $y_1 = \dfrac{a^2 - b^2 + d^2\tan^2(\gamma)}{2d\tan(\gamma)}$ and $y_2 = \dfrac{b^2 - a^2 + d^2\tan^2(\gamma)}{2d\tan(\gamma)}$.

As shown in Fig. 6**d**, when $\mathrm{atan}(2b/d) \le \gamma < \mathrm{atan}[(a - b)/d]$, the receiving power is

$$P_r = \int_{-\sqrt{a^2 - [d\tan(\gamma)/2]^2}}^{\sqrt{a^2 - [d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)/2}^{\sqrt{a^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$+ \int_{-\sqrt{a^2 - [d\tan(\gamma)/2]^2}}^{\sqrt{a^2 - [d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma) - \sqrt{a^2 - x^2}}^{d\tan(\gamma)/2} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$- \int_{-b}^{b} \int_{-\sqrt{b^2 - x^2}}^{\sqrt{b^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x - \int_{-b}^{b} \int_{d\tan(\gamma) - \sqrt{b^2 - x^2}}^{d\tan(\gamma) + \sqrt{b^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x \tag{22}$$

As shown in Fig. 6**e**, when $0 \le \gamma < \mathrm{atan}(2b/d)$, the receiving power is

$$P_r = \int_{-\sqrt{a^2 - [d\tan(\gamma)/2]^2}}^{\sqrt{a^2 - [d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)/2}^{\sqrt{a^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$+ \int_{-\sqrt{a^2 - [d\tan(\gamma)/2]^2}}^{\sqrt{a^2 - [d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma) - \sqrt{a^2 - x^2}}^{d\tan(\gamma)/2} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$- \int_{-b}^{b} \int_{-\sqrt{b^2 - x^2}}^{\sqrt{b^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x - \int_{-b}^{b} \int_{d\tan(\gamma) - \sqrt{b^2 - x^2}}^{d\tan(\gamma) + \sqrt{b^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$+ \int_{-\sqrt{b^2 - [d\tan(\gamma)/2]^2}}^{\sqrt{b^2 - [d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma)/2}^{\sqrt{b^2 - x^2}} I(x, y) \mathrm{d}y \mathrm{d}x +$$

$$+ \int_{-\sqrt{b^2 - [d\tan(\gamma)/2]^2}}^{\sqrt{b^2 - [d\tan(\gamma)/2]^2}} \int_{d\tan(\gamma) - \sqrt{b^2 - x^2}}^{d\tan(\gamma)/2} I(x, y) \mathrm{d}y \mathrm{d}x \tag{23}$$

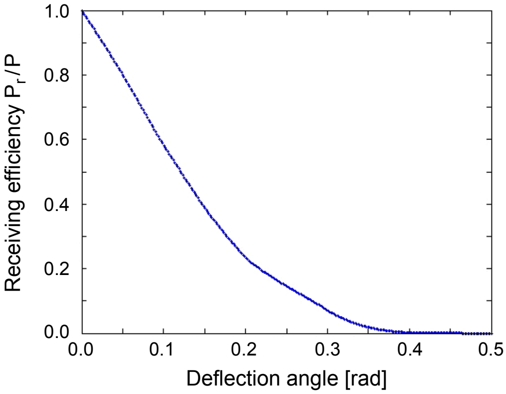

Fig. 7. The receiving efficiency curve.

Because the integration $\int e^{(r-R)^2} r \mathrm{d}r$ is non-integrable, the receiving efficiency curve was shown in Fig. 7 by numerical calculation and simulation. In this paper, we chose the normalized amplitude of $I(x, y)$ and $\omega(z) = (a + b)/2$, which kept the intensity distribution of the receiving spot on the receiving plane the same with the ideal hollow beam's.

It is clear that the receiving power will be zero, if the deflection angle is more than 0.38 rad. The receiving power is less than 80% of the total power, when the deflection

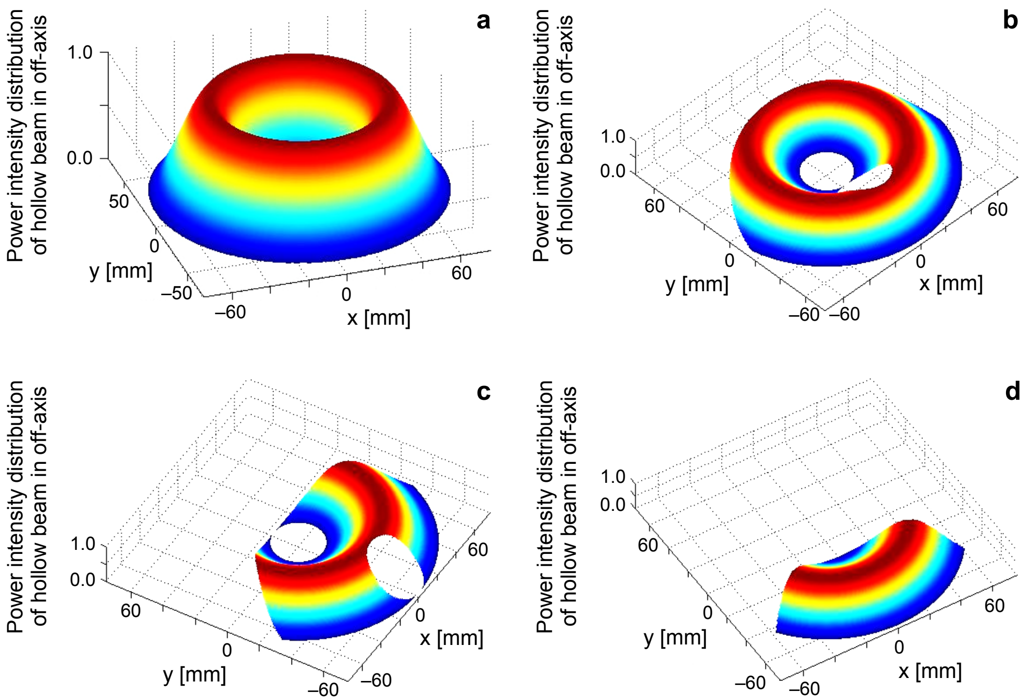

Fig. 8. The 3-D images of light intensity distribution with inclined angles $\gamma = 0$ rad (**a**), $\gamma = 0.0997$ rad (**b**), $\gamma = 0.1974$ rad (**c**) and $\gamma = 0.2915$ rad (**d**).

angle is more than 0.05 rad. And it can be seen that the ratio of power attenuation when the inclined angle is $\gamma \in [0, 0.2]$ fiercer than the case when the inclined angle $\gamma \in [0.2, 0.5]$. Meanwhile, the 3-D images of light intensity distribution with several different inclined angles are shown in Fig. 8.

By the theoretical analysis of an off-axis Cassegrain optical system, we found that the distance of the centers of the small circle and ellipse satisfied the equation $\overline{OO'} = d\tan(\gamma)$. Therefore, we changed the distance $\overline{OO'}$ to get the images of Fig. 8, and the inclined angles were calculated according to the equation $\gamma = \mathrm{atan}(\overline{OO'}/d)$.

4. Conclusion

The characteristics of a double-parabolic reflector Cassegrain optical antenna has been analyzed, and the relationships among the focal length of a primary reflector and sub-reflector ($0.5p_1$, $0.5p_2$), the aperture of two reflectors (a, b), and the distance of two reflectors d have been obtained. This work will provide some theoretical references for those manufacturing Cassegrain optical antennas. When the inclined angle is less than 0.05 rad, the receiving power is beyond 80%, which is satisfied with the Strehl standard. To keep the receiving efficiency higher than 80%, the inclined angle must be kept less than 0.05 rad. Therefore, the technology of aligning the optical system needs to be improved much more. From the receiving power curve, we can adjust the antennas to make the off-axis Cassegrain optical system aligned more precisely according to the receiving power. The investigation results will offer the fundamental research for the propagation efficiency enhancement of the free space optical communication.

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References

[1] Han Sun, Huajun Yang, Yi Peng, *3-D simulation research for off-axis Cassegrain optical antenna and coupling systems*, Optoelectronics and Advanced Materials – Rapid Communications **6**(1–2), 2012, pp. 284–287.

[2] Huilin Jiang, Guojun Liu, Fuchang Yin, Zhi Liu, *Laser communications technology with airborne platform*, Proceedings of SPIE **6031**, 2005, article 603102.

[3] Zhigang Zang, Minato T., Navaretti P., Hinokuma Y., Duelk M., Velez C., Hamamoto K., *High power (>110 mW) superluminescent diodes by using active multimode interferometer*, IEEE Photonics Technology Letters **22**(10), 2010, pp. 721–723.

[4] Zhigang Zang, Mukai K., Navaretti P., Duelk M., Velez C., Hamamoto K., *Thermal resistance reduction in high power superluminescent diodes by using active multi-mode interferometer*, Applied Physics Letters **100**(3), 2012, article 031108.

[5] Posyniak M., Stacewicz T., Miernecki M., Jagodnicka A.K., Malinowski S.P., *Multiwavelength micropulse lidar for atmospheric aerosol investigation*, Optica Applicata **40**(3), 2010, pp. 623–632.
[6] Kai Huang, Huajun Yang, Tuohui Li, Chenghong Li, Quan Xu, Kang Xie, *Analysis of space off-axis and performance of Cassegrain optical antenna system*, Optik – International Journal for Light and Electron Optics **121**(18), 2010, pp. 1688–1692.
[7] Zihao Chen, Huajun Yang, Xinyang Wang, Jing Wang, Xiaoping Huang, *Theoretical analysis and test for off-axis Cassegrain optical antenna*, Optik – International Journal for Light and Electron Optics **123**(3), 2012, pp. 268–271.

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