Optica Applicata, Vol. XLI, No. 1, 2011

Theoretical spectroscopic scan of the sensitivity of asymmetric slab waveguide sensors

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An extensive theoretical analysis is carried out to investigate the variation of the sensitivity of optical slab waveguide sensors with the wavelength of the guided wave. We consider a three-layer waveguide as an optical sensor. The sensitivity for both polarizations of light: *s*-polarized light (TE) and *p*-polarized light (TM), is derived using the characteristic equation of the structure. The dispersion of the materials is taken into account to study the sensitivity spectroscopic scan over the near IR-range from 1.2–2 μm. It is found that an optimum wavelength exists for each guiding layer thickness and this optimum value increases linearly with the thickness of the guiding layer.

Keywords: slab waveguides, optical sensors, sensitivity.

1. Introduction

Optical waveguide sensing is a rapidly growing area of research due to the potential applications in chemistry, biochemistry, and biology. Some of the main activities are to quantify protein adsorption, affinity-based recognition and attachment of bacteria or living cells. The optical waveguide sensors make use of guided modes in planar waveguides for sensing applications. In particular, the fundamental modes TE_0 and TM_0 in very thin slab waveguides of high refractive index are used in this noncommunication application of waveguides. The principle of operation of optical waveguide sensors depends on the evanescent field extended into the cladding and the substrate. The evanescent field of the guided mode interacts with the sample to be detected (analyte) and it senses changes in the refractive index of the cladding. Thus changes in the effective refractive index N of the guided mode are induced. A variety of optical sensors based upon evanescent wave sensing techniques have been proposed such as surface plasmon resonance sensors [1], integrated optical waveguide sensors [2], resonant mirrors [3], and differential interferometry sensors [4].

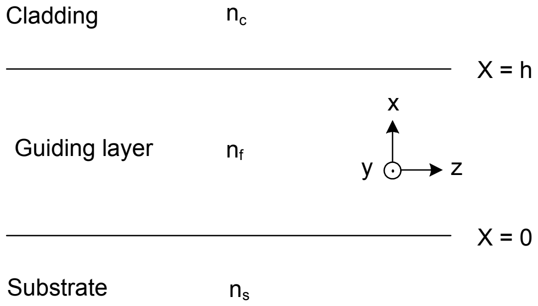

Fig. 1. A schematic diagram of the waveguide structure under consideration.

The conventional optical waveguide sensor consists of a three-layer structure; a thin film as a guiding layer surrounded by a gaseous or a liquid cladding and a substrate. A normalized analysis for this structure was carried out and the condition for the maximum achievable sensitivity was also derived for both TE and TM polarizations [5]. TAYA *et al.* [6–9] presented an extensive theoretical analysis for such a structure when one or both of the surrounding media has an intensity dependent refractive index. In this article, we investigate the behavior of the sensitivity of asymmetrical slab waveguide sensors with the wavelength of the guided wave for the two types of light polarizations. We also study the dependence of the sensitivity on the thickness of the guiding layer.

2. Theory

A schematic structure of the waveguide under consideration is illustrated in Fig. 1. It consists of a thin optically linear dielectric guiding layer of thickness h and refractive index n_f sandwiched between a linear cladding and a linear substrate of refractive indices n_c and n_s, respectively. The waves in the guiding layer are assumed to travel in the z-direction.

Helmholtz equation for TE and TM modes is given by

$$\frac{\partial^2 A_y(x)}{\partial x^2} + \left(k_0^2 n_i^2 - \beta^2\right) A_y(x) = 0 \tag{1}$$

where $A_y(x)$ stands for $E_y(x)$-field in TE modes and for $H_y(x)$-field in TM modes while k_0 is the free space wave number. The refractive index n_i is n_c, n_f, or n_s depending on which region we are defining the field in. The propagation constant β can be written as $\beta = k_0 N$, where N is the modal effective index of the waveguide.

The solution of Eq. (1) in each layer can be written as

$$A_y(x) = a \exp\left[-\gamma_c (x - h)\right], \qquad x > h \tag{2}$$

$$A_y(x) = b_1 \sin(\gamma_f x) + b_2 \cos(\gamma_f x), \qquad 0 < x < h \tag{3}$$

$$A_y(x) = c\exp(\gamma_s x), \qquad 0 < x \tag{4}$$

where $\gamma_c = \sqrt{\beta^2 - k_0^2 n_c^2}$, $\gamma_f = \sqrt{k_0^2 n_f^2 - \beta^2}$, and $\gamma_s = \sqrt{\beta^2 - k_0^2 n_s^2}$.

Using Equations (2)–(4) all the nonvanishing field components can be calculated for the two modes by applying Maxwell's equations. The continuity of the tangential components of the fields gives rise to the characteristic equation which can be written as [5]

$$\gamma_f h = \operatorname{atan}\left(\frac{\gamma_s}{\gamma_f [a_s]^{\rho}}\right) + \operatorname{atan}\left(\frac{\gamma_c}{\gamma_f [a_c]^{\rho}}\right) + m\pi \tag{5}$$

where $\rho = 1$ for TM and $\rho = 0$ for TE modes, m is the mode order, and a_s and a_c are two asymmetry parameters given by $a_s = n_s^2/n_f^2$, $a_c = n_c^2/n_f^2$.

When the analyte is homogeneously distributed in the cladding, the sensitivity of the optical waveguide sensor S is defined as the rate of change of the effective refractive index under an index change of the cover [5, 6]. Differentiating Eq. (5) with respect to N and calculating S as $(\partial n_c/\partial N)^{-1}$ we obtain,

$$S = \frac{\sqrt{a_c}}{X_c\sqrt{1 + X_c^2}\,\sqrt{a_c + X_c^2}\left(\gamma_f h + \dfrac{1}{X_s} + \dfrac{1}{X_c}\right)}, \qquad \text{for TE modes} \tag{6}$$

$$S = \frac{\dfrac{2}{\sqrt{a_c q}} - \sqrt{a_c q}}{\left(1 + X_c^2\right) + rF}, \qquad \text{for TM modes} \tag{7}$$

where $X_s = \dfrac{\gamma_s}{\gamma_f}$, $X_c = \dfrac{\gamma_c}{\gamma_f}$, $q = \dfrac{1 + X_s^2}{a_s + X_s^2}$, $r = \dfrac{X_c}{a_c}\left(a_c^2 + X_c^2\right)$, $F = \gamma_f h + \dfrac{a_s(1 + X_s^2)}{X_s(a_s^2 + X_s^2)}$.

3. Results and discussion

In our calculations we consider the clad to be air, the guiding layer to be crystalline silicon (c-Si), and the substrate to be SiO_2. The refractive indices of the guiding layer and the substrate are taken from the handbook of optical constants of solids [10] in the spectrum range 1.2–2 μm. A computer program was written to solve the characteristic equation, given by Eq. (5), for N and the sensitivities were calculated using Eqs. (6) and (7). Only the fundamental mode ($m = 0$) will be considered since it corresponds to the highest sensitivity [5]. The resulting sensitivity curves as functions of the wavelength are shown in Fig. 2 for TE_0 modes and in Fig. 3 for TM_0 modes. Many interesting features can be seen in the two figures. For each guiding layer

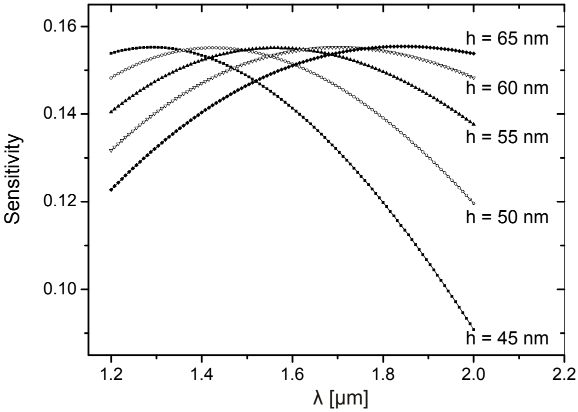

Fig. 2. The sensitivity as a function of the wavelength for different guiding layer thicknesses for TE_0 mode.

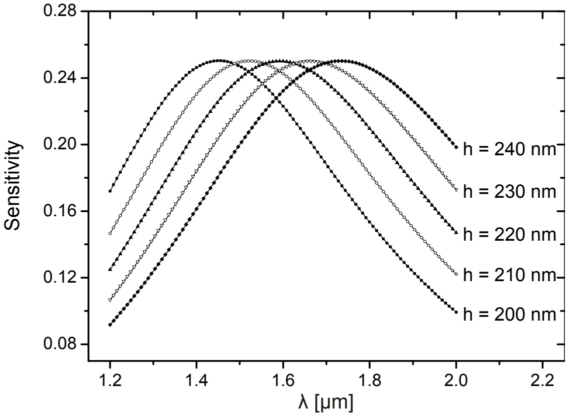

Fig. 3. The sensitivity as a function of the wavelength for different guiding layer thicknesses for TM_0 mode.

thickness, there is an optimum wavelength at which the optical waveguide sensor exhibits its maximum sensing sensitivity. The optimum value of λ is shifted toward higher values as the thickness of the guiding layer increases.

The dependence of the optical sensitivity of the waveguide sensor on light polarization is shown in Fig. 4 for TM_0 and TE_0 modes. The figure reveals that the sensitivity in TM_0 has a higher peak that appears at a lower value of λ. Therefore, TM mode is recommended.

The variation of the sensitivity as a function of the guiding layer thickness for different wavelengths is shown in Fig. 5 for TE_0 and TM_0 modes. It can be seen that the sensitivity approaches zero at cut-off thickness. In this limit, all the power of the mode propagates in the substrate due to the infinite penetration depth. Consequently, the sensor probes the substrate side only. For the sensitivity to have a nonzero

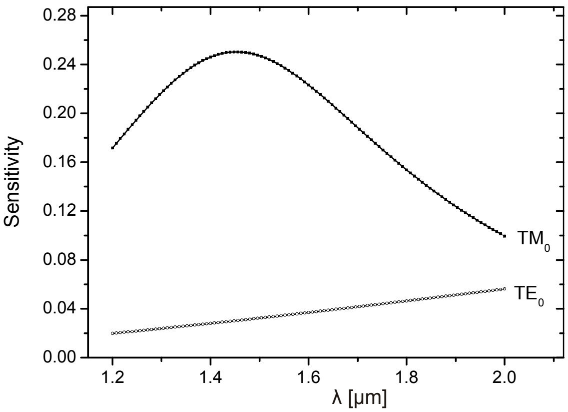

Fig. 4. The sensitivity as a function of the wavelength for different light polarizations for *h* = 200 nm.

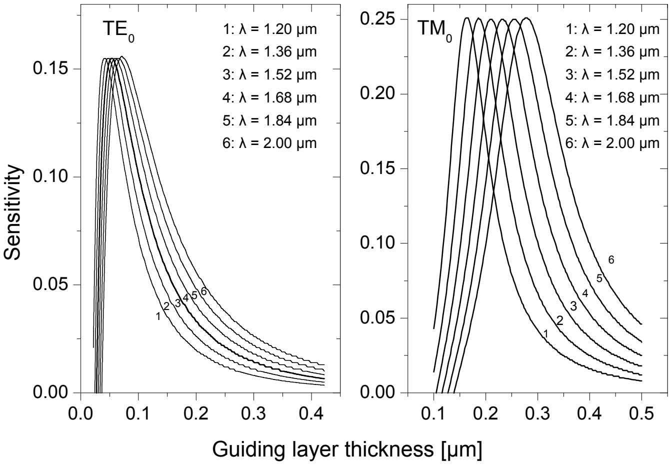

Fig. 5. The sensitivity as a function of the guiding layer thickness for different wavelengths for TE_0 and TM_0 modes.

value, the thickness of the guiding layer has to be greater than the cut-off thickness. In the other limit, far beyond the cut-off thicknesses, the effective waveguide thickness approaches the film thickness which means that all the power propagates in the film. In this case, the sensitivities approach zero again. Between these two limits, there is a maximum in the sensitivity curves, just above the cut-off thickness, representing an optimum where a relatively large part of the total mode power propagates in the covering medium [2, 6–9]. As the wavelength of the guided wave increases, the optimum value of the guiding layer thickness is shifted toward higher values.

The optimum wavelengths $\lambda_{\mathrm{optimum}}$ extracted from Figs. 2 and 3 are plotted in Fig. 6 with the thickness of the guiding layer for TE_0 and TM_0 modes. It is obvious that the optimum wavelength is linearly dependent upon the guiding layer thickness. The optimum thicknesses, extracted from Fig. 5, are illustrated with the wavelength

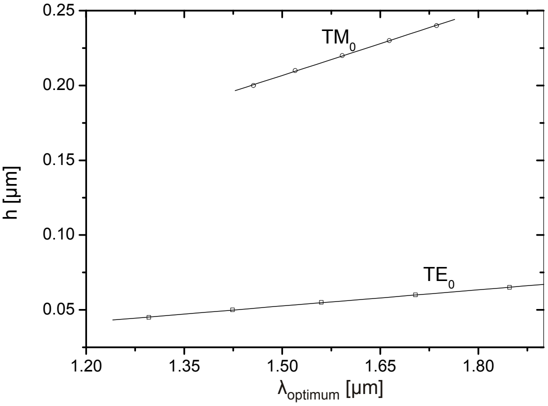

Fig. 6. Optimum wavelengths with the thickness of the guiding layer for TE$_0$ and TM$_0$ modes.

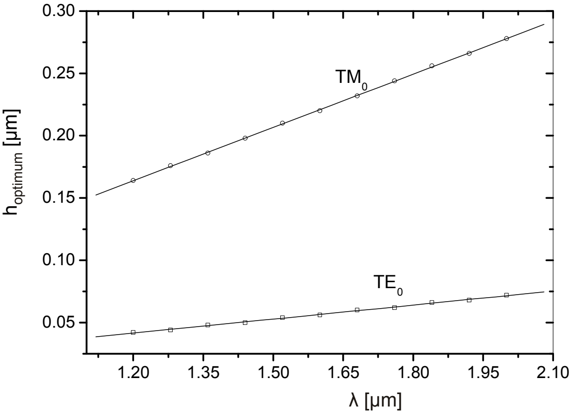

Fig. 7. Optimum thickness of the guiding layer versus the corresponding wavelength of the guided wave for TE$_0$ and TM$_0$ modes.

of the guided mode in Fig. 7. The optimum thickness also varies linearly with the wavelength of the guided mode. The special feature in Figs. 6 and 7 is that the slope calculated for TM$_0$ lines in the two figures is found to be the same (≈ 0.142) and so is the slope for TE$_0$ lines in the two figures (≈ 0.0365).

4. Conclusions

In this work we study the variation of the sensitivity of the three-layer asymmetrical slab waveguide sensor with the wavelength of the guided wave in the spectrum range 1.2−2 μm. We have found that there is an optimum wavelength at which the sensitivity is maximum. This optimum wavelength is determined by the guiding layer thickness and it is linearly dependent on the optical thickness of the guiding layer.

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Received June 29, 2010