Optica Applicata, Vol. XLI, No. 4, 2011

A new spherical aberration coefficient C_4 for the Gaussian laser beam

ABDELHALIM BENCHEIKH*, MOUHAMED BOUAFIA, KOUIDER FERRIA

Applied Optics Laboratory, Institute of Optics and Precision Mechanics,
Ferhat Abbas University, Sétif, Algeria

*Corresponding author: haben76@hotmail.com

Laser beam quality is related to the aberration effect. Quartic phase aberration, more commonly known as spherical aberration, can result from aberrated optical components such as beam expanding telescopes, focusing or collimating lenses, or other conventional optical elements. In general, any kind of quartic aberration will lead to increased far field beam spread, degraded laser beam focusability and increased values of the beam quality. Currently, a well established quality parameter for laser beams is the M^2 factor which is proportional to the coefficient of quartic phase aberration denoted C_4. In many recent papers, authors used C_4 given in geometrical optics approach to evaluate the laser beam quality M^2 which belongs to the Gaussian beam optics and the two disciplines are not to be confused. In this paper, we present a new mathematical set for the spherical aberration coefficient C_4, especially for Gaussian beams in the context of Gaussian beam optics. A numerical analysis of a set of lenses is done to show the importance of the new C_4.

Keywords: laser beam quality factor, Gaussian beam, spherical aberration.

1. Introduction

The propagation and focusing of a Gaussian beam have been an important subject because of the widespread use of laser beams in the field of laser communication, laser interconnect and laser material processing, *etc*. [1–13]. Much of the work on propagation and focusing of Gaussian beams has been done for lenses of no-aberration type. In recent years, some papers dealing with focusing laser beams by lenses with spherical aberration SA were presented. It was shown that the beam quality of the laser beams may become bad, and the quality factor M^2 may become bigger and bigger, as shown by SIEGMAN [4]. The laser beam quality factor is proportional to the spherical aberration coefficient called C_4, which means that M^2 increases according to the C_4.

This paper presents a new mathematical set for the spherical aberration coefficient C_4 of Gaussian beams. The main idea comes from the estimation of the laser beam quality factor M^2 given by SIEGMAN [4] and others. We show that this coefficient concerns only the case of geometrical optics [4–8].

2. Development

The coefficient expressing spherical aberration of a thin lens is given by [4, 14]:

$$C_4 = \frac{1}{4}h^4K^3\left[\frac{n+2}{n(n-1)^2}X^2 - \frac{4(n+1)}{n(n-1)}XY + \frac{n^2}{(n-1)^2} + \frac{3n+2}{n}Y^2\right] \quad (1)$$

where:

$$X = \frac{c_1 + c_2}{c_1 - c_2}$$

$$Y = \frac{u_2' + u_1}{u_2' - u_1}$$

X stands for the Kodington factor, Y is the normalisation factor, c_1 and c_2 represent the curvature rays of the lens, h is the height of the lens, N is the refractive index of the lens and u_1, u_2' are the paraxial angles.

In the case of geometrical optics the object and the image distances are related by the conjugation law [14]:

$$\frac{1}{S_2} + \frac{1}{S_1} = \frac{1}{f} \quad (2)$$

The image distance can then be deduced as follows:

$$S_2 = \frac{S_1 f}{S_1 - f} \quad (3)$$

For a Gaussian beam, SELF [15] has derived an analogous formula by assuming that the waist of the input beam represents the object and the waist image of the output beam represents the image; the formula is expressed in terms of the Rayleigh range of the input beam by the relation:

$$\frac{1}{S_2} + \frac{1}{S_1 + \frac{Z_R^2}{S_1 - f}} = \frac{1}{f} \quad (4)$$

By the same way the waist image is expressed by:

$$S_2 = \frac{fs^2 - f^2s + fZ_R^2}{(s-f)^2 + Z_R^2} \quad (5)$$

where: $Z_R = \pi w_0^2/\lambda$ is the Rayleigh range, w_0 is the beam waist of the Gaussian beam and λ is the wavelength, f is the focal length of the lens and S_1, S_2 are the object and the image distances, respectively.

If one compares the two formulas (3) and (5) relating the object and the image, it can be found that the image distances for geometrical and Gaussian beam optics are not at the same position, and this makes a difference in the optical path between the two cases. This implies that the path difference between marginal and paraxial rays for geometrical optics is different for the case of Gaussian beam optics. Consequently, the two spherical aberration coefficients are also different.

2.1. The spherical aberration coefficient C_4

Let us examine the following relations. We have

$$Y = \frac{u_2' + u_1}{u_2' - u_1} \tag{6}$$

with $u_2' = h/S_2$ and $u_1 = h/S_1$.

This last relation gives rise to:

$$Y = \frac{S_1 + S_2}{S_1 - S_2} \tag{7}$$

If we replace the values of S_1 and S_2 given by relations (3) and (5) in relation (1), we will find new relations for the factors Y:

– For geometrical optics

$$Y = \frac{S_1}{S_1 - 2f} \tag{8}$$

– For Gaussian beam optics

$$Y = \frac{S_1^3 - fS_1^2 + S_1 Z_R^2 + fZ_R^2}{S_1^3 + fS_1^2 + (2f^2 + Z_R^2)S_1 + fZ_R^2} \tag{9}$$

If we compare both relations of Y, we notice that they are very different. Knowing that C_4 is a function of Y, this implies that the C_4 of geometrical optics should be different from the C_4 of Gaussian beam optics.

Substituting (7) and (8) into (1), we get the C_4 relations for geometrical and Gaussian beam optics:

– In the case of geometrical optics we obtain

$$C_4 = \frac{1}{4}h^4K^3\left[\frac{n+2}{n(n-1)^2}X^2 + \frac{4(n+1)}{n(n-1)}\,\frac{S_1}{S_1-2f}X + \right.$$
$$\left. + \frac{n^2}{(n-1)^2} + \frac{3n+2}{n}\left(\frac{S_1}{S_1-2f}\right)^2\right] \tag{10}$$

– In the case of Gaussian beam optics we get:

$$C_4 = \frac{1}{4}h^4K^3\left[\frac{n+2}{n(n-1)^2}X^2 + \frac{4(n+1)}{n(n-1)}\,\frac{S_1^3 - fS_1^2 + S_1Z_R^2 + fZ_R^2}{S_1^3 + fS_1^2 + (2f^2 + Z_R^2)S_1 + fZ_R^2}X + \frac{n^2}{(n-1)^2} + \frac{3n+2}{n}\left(\frac{S_1^3 - fS_1^2 + S_1Z_R^2 + fZ_R^2}{S_1^3 + fS_1^2 + (2f^2 + Z_R^2)S_1 + fZ_R^2}\right)^2\right] \quad (11)$$

where h is the height of the lens, as shown in Fig. 1.

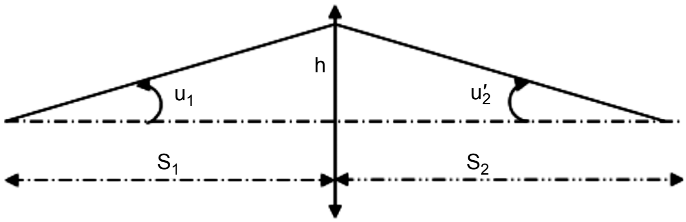

Fig. 1. Representation of the object and image plane relations.

We notice that this C_4 depends on the focal distance f, S_1 and Z_R, which characterizes the Gaussian laser beam.

2.2. Numerical results

To prove our approach experimentally, we simulate our proposal in the case of four different lenses (see the Table).

For the case of $X = 0$ (biconvex lenses), we have:

$$C_4 = \frac{1}{4}h^4K^3\left[\frac{n^2}{(n-1)^2} + \frac{3n+2}{n}Y^2\right] = C_0\left(8.54 + 4.31\,Y^2\right) \quad (12)$$

T a b l e. Lens characteristics.

No.	X	f [mm]	Refractive index n	Diameter D [mm]	Thickness [mm]
1	0	100	1.52	40	6.05
2	1	148	1.52	29	4.2
3	0	200	1.52	40	3.05
4	1	376	1.52	69	5.61

To simplify the calculus, we omit the constant C_0, and we just use the ratio:

$$C_4' = \frac{C_4}{C_0} = 8.54 + 4.31\,Y^2 \tag{13}$$

For the case of $X = 1$ (plano-convex lenses), we obtain:

$$C_4 = \frac{1}{4}h^4K^3\left[\frac{n+2}{n(n-1)} + \frac{n^2}{(n-1)^2} + \frac{4(n+1)}{n(n-1)}Y + \frac{3n+2}{n}Y^2\right] =$$
$$= C_1\left(17.1 + 12.75\,Y + 4.31\,Y^2\right) \tag{14}$$

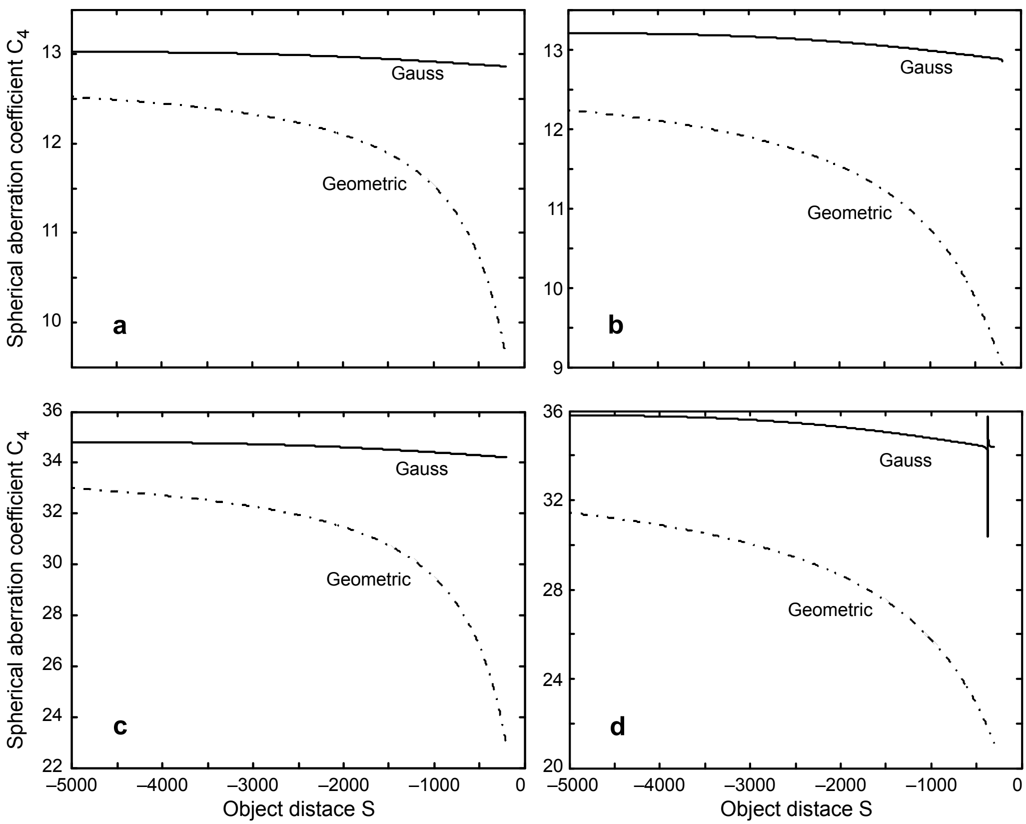

Fig. 2. Evolution of C_4 for the case of geometrical optics and Gaussian beam optics, for a biconvex lens with: $X = 0$, $f = 100$ mm, $w_0 = 1$ mm (**a**), $X = 0$, $f = 200$ mm, $w_0 = 1$ mm (**b**), $X = 1$, $f = 148$ mm, $w_0 = 1$ mm (**c**), and $X = 1$, $f = 376$ mm, $w_0 = 1$ mm (**d**).

The same as above, we use the ratio:

$$C_4' = \frac{C_4}{C_1} = 17.1 + 12.75Y + 4.31Y^2 \tag{15}$$

where in both cases C_0 and C_1 are constants.

Evolution of the C_4 according to the object distance is presented in Figures 2**a** through 2**d**, for four different lenses for the case of geometrical optics and Gaussian beam optics.

3. Discussion

If we look at the curves obtained and presented in Fig. 2 we first notice that the C_4 is very different between the Gaussian beam optics and geometrical optics for all the lenses, which is due to the difference that exists between the physics of light in the case of Gaussian beam optics and geometrical optics.

The principal difference is that the spherical aberration is based on the optical path difference between marginal and paraxial rays. This difference in path is not the same for Gaussian beam optics and geometrical optics.

Also, we note that in the paraxial region the coefficient of spherical aberration for the case of Gaussian beam optics is bigger than that of the geometrical optics, due to the focal depth introduced by the focusing lens. This depth of focus is a longitudinal spherical aberration, which is more important in Gaussian beams.

And the main result that we want to show is to prove that C_4 of Gaussian beam optics is different from ones of geometrical optics.

4. Conclusions

Starting from the conjugation law of geometrical and Gaussian beam optics, we showed numerically that the spherical aberration coefficient C_4 is different for the two cases. This is more observable on the plotting curves on which we can see the evolution as a function of the optical path. It is then possible to evaluate the spherical aberration coefficient for a Gaussian beam according to the object position for any optical system.

References

[1] WEBER H., *Some historical and technical aspects of beam quality*, Optical and Quantum Electronics **24**(9), 1992, pp. S861–S864.

[2] SIEGMAN A.E., RUFF J., *Effects of spherical aberration on laser beam qualiy*, Proceedings of SPIE **1834**, 1993, pp. 130–139.

[3] ALDA J., *Quality improvement of coherent and aberrated laser beam by using an optimum and smooth pure phase filter*, Optics Communications **192**(3–6), 2001, pp. 199–204.

[4] SIEGMAN A.E., *Analysis of laser beam quality degradation caused by quartic phase aberrations*, Applied Optics **32**(30), 1993, pp. 5893–5901.

[5] MARTÍNEZ-HERRERO R., PIQUERO G., MEJÍAS P.M., *Beam quality changes of radially and azimuthally polarized fields propagating through quartic phase plates*, Optics Communications **281**(4), 2008, pp. 756–759.

[6] HASNAOUI A., BENCHEIKH A., FROMAGER M., CAGNIOT E., AIT-AMEUR K., *Creation of a sharper focus by using a rectified TEM_{p0} beam*, Optics Communications **284**(5), 2011, pp. 1331–1334.

[7] HASNAOUI A., BENCHEIKH A., AIT-AMEUR K., *Tailored TEM_{p0} beams for large size 3-D laser prototyping*, Optics and Lasers in Engineering **49**(2), 2011, pp. 248–251.

[8] GOLUB M.A., SHURMAN V., GROSSINGER I., *Extended focus diffractive optical element for Gaussian laser beams*, Applied Optics **45**(1), 2006, pp. 144–150.

[9] HERMAN R.M., WIGGINS T.A., *Focusing and magnification in Gaussian beams*, Applied Optics **25**(15), 1986, pp. 2473–2474.

[10] JIXIONG PU, *Focusing Gaussian beams by an annular lens with spherical aberration*, Journal of Modern Optics **45**(2), 1998, pp. 239–247.

[11] MAHAJAN V.N., *Axial irradiance of a focused beam*, Journal of the Optical Society of America A **22**(9), 2005, pp. 1814–1823.

[12] ZHONG Y., *Focal shift in focused truncated pulsed-laser beam*, Applied Optics **46**(25), 2007, pp. 6454–6459.

[13] MAHAJAN V.N., *Uniform versus Gaussian beams: A comparison of the effects of diffraction, obscuration, and aberrations*, Journal of the Optical Society of America A **3**(4), 1986, pp. 470–485.

[14] HOPKINS H., *Wave Theory of Aberration*, Oxford University Press, London, 1950.

[15] SELF S.A., *Focusing of spherical Gaussian beams*, Applied Optics **22**(5), 1983, pp. 658–661.

Received December 23, 2010