Propagation of a four-beamlets laser array through an apertured optical system

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By expanding the hard-aperture function into a finite sum of complex Gaussian functions, an analytical formula for a four-beamlets laser array propagating through an apertured ABCD optical system is derived based on the generalized Collins formula. As a numerical example, the on-axis irradiance of a four-beamlets laser array focused by a squarely apertured bifocal thin lens is studied, and it is found that the focused irradiance is closely related to the parameters of the optical system and the laser array. Our formula provides a convenient way for studying the paraxial propagation of a four-beamlets laser array through an apertured ABCD optical system.

Keywords: four-beamlets laser array, propagation, optical system.

1. Introduction

In the past decade, laser array beams have been widely investigated due to their wide applications in high-power systems, inertial confinement fusion and high-energy weapons, *etc*. [1–5]. Up to now, a variety of linear, rectangular, and radial laser array beams have been developed to achieve high system powers, and the propagation properties of such laser array beams in free space or through a paraxial optical system have been studied in detail [5–11]. More recently, CAI *et al*. [12, 13] studied the propagation of coherent and partially coherent laser array beams in turbulent atmosphere. EYYUBOĞLU *et al*. [14] studied the scintillation properties of laser array beams in turbulent atmosphere, and found that the scintillation index of laser array beams can be smaller that of Gaussian beams for suitable beam parameters, which means that laser array beams have an advantage over Gaussian beams for free-space optical communications.

In theory, the scientists usually constructed the laser arrays using the combination of decentered beams. In 2005, CAI and LIN [15] proposed a simple but convenient mathematical model to describe a laser array with four beamlets (*i.e.*, four-beamlets laser array). The paraxial propagation of a four-beamlets laser array through an unapertured ABCD optical system has been studied in Ref. [15]. On the other hand, aperture confinement is commonly encountered in a practical optical system. Some interesting phenomena in laser beams, such as focal shift, focal switch, spectral shift and spectral switch, have been found to be related to the aperture [16, 17]. So it is necessary to study propagation of laser beams through an apertured ABCD optical system. Propagation of various coherent and partially coherent laser beams through an apertured optical system has been studied in the past for several years [18–29]. In this paper, we study the propagation of a four-beamlets laser array through an apertured ABCD optical system. Analytical propagation formula is derived. As a numerical example, the focusing properties of a four-beamlets laser array passing through an apertured bifocal thin lens have been studied.

2. Paraxial propagation of a four-beamlets laser array through an apertured ABCD optical system

The electric field of a four-beamlets laser array at the source plane $z = 0$ is expressed as follows [15]

$$E(x_0, y_0, 0) = \left(\frac{x_0^2}{w_{0x}^2}\right)^m \left(\frac{y_0^2}{w_{0y}^2}\right)^n \exp\left[-\frac{x_0^2}{w_{0x}^2} - \frac{y_0^2}{w_{0y}^2}\right] \tag{1}$$

where m and n are the beam orders of the four-beamlets laser array, w_{0x}, w_{0y} are the beam waist sizes in x and y direction, respectively. When $m = 0$, $n = 0$, Eq. (1) reduces to the electric field of an elliptical Gaussian beam [10, 30]. When $m \neq 0$, $n \neq 0$, as the value of m (or n) increases, the distance between the peak points of beamlets in x (or y) direction increases as shown by Fig. 1 of Ref. [15].

Within the framework of paraxial approximation, the paraxial propagation of a four-beamlets laser array through a rectangularly apertured ABCD optical system can be treated by the following generalized Collins formula [31],

$$E(x, y, z) = \frac{i}{\lambda}\sqrt{\frac{i}{B_x B_y}} \int_{-a}^{a}\int_{-b}^{b} E(x_0, y_0, 0) \exp\left[-\frac{ik}{2B_x}\left(A_x x_0^2 - 2x_0 x + D_x x^2\right)\right] \times$$
$$\times \exp\left[-\frac{ik}{2B_y}\left(A_y y_0^2 - 2y_0 y + D_y y^2\right)\right] \mathrm{d}x_0 \mathrm{d}y_0 \tag{2}$$

where x and y are the transverse coordinates at the output plane z, $k = 2\pi/\lambda$ is the wave number with λ being the wavelength, $2a$ and $2b$ denote the aperture widths in x and y directions, respectively, A_x, B_x, C_x, D_x and A_y, B_y, C_y, D_y are the transfer matrix elements of the optical system in x and y directions, respectively.

By introducing the following hard aperture function for a rectangular aperture

$$H(x_0, y_0) = \begin{cases} 1, & |x_0| \le a, |y_0| \le b \\ 0, & |x_0| > a, |y_0| > b \end{cases} \tag{3}$$

equation (2) can be expressed as follows

$$E(x, y, z) = \frac{i}{\lambda}\sqrt{\frac{i}{B_x B_y}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(x_0, y_0, 0) H(x_0, y_0) \times$$
$$\times \exp\left[-\frac{ik}{2B_x}\left(A_x x_0^2 - 2x_0 x + D_x x^2\right)\right] \times$$
$$\times \exp\left[-\frac{ik}{2B_y}\left(A_y y_0^2 - 2y_0 y + D_y y^2\right)\right] \mathrm{d}x_0 \mathrm{d}y_0 \tag{4}$$

The hard aperture function of a rectangular aperture can be expanded as the following sum (finite terms) of complex Gaussian functions [20, 32, 33]

$$H(x_0, y_0) = \sum_{l=1}^{L} A_l \exp\left(-\frac{B_l x_0^2}{a^2}\right) \sum_{j=1}^{L} A_j \exp\left(-\frac{B_j y_0^2}{b^2}\right) \tag{5}$$

where A_l, B_l, A_j and B_j are the expansion and Gaussian coefficients, which can be obtained by optimization computation directly. This expansion method has been proved to be reliable and efficient for $L \ge 10$ [20, 32, 33].

Substituting Eqs. (1) and (5) into Eq. (4), we obtain (after tedious integration) the following analytical propagation formula for a four-beamlets laser array through a rectangularly apertured ABCD optical system

$$E(x, y, z) = \frac{i}{\lambda}\sqrt{\frac{1}{B_x B_y}}\left(\frac{1}{w_{0x}^2}\right)^m \left(\frac{1}{w_{0y}^2}\right)^n \exp\left(-\frac{ikD_x}{2B_x}x^2 - \frac{ikD_y}{2B_y}y^2\right) \times$$

$$\times \left(-\frac{1}{4}\right)^{(m+n)} \pi \sum_{l=1}^{L} A_l \left(\frac{1}{w_{0x}^2} + \frac{B_l}{a^2} + \frac{ikA_x}{2B_x}\right)^{-\frac{2m+1}{2}} \times$$

$$\times \exp\left[-\frac{\dfrac{k^2x^2}{B_x^2}}{4\left(\dfrac{1}{w_{0x}^2} + \dfrac{B_l}{a^2} + \dfrac{ikA_x}{2B_x}\right)}\right] H_{2m}\left[\frac{\dfrac{kx}{B_x}}{2\sqrt{\dfrac{1}{w_{0x}^2} + \dfrac{B_l}{a^2} + \dfrac{ikA_x}{2B_x}}}\right] \times$$

$$\times \sum_{j=1}^{L} A_j \left(\frac{1}{w_{0y}^2} + \frac{B_j}{b^2} + \frac{ikA_y}{2B_y} \right)^{-\frac{2n+1}{2}} \times$$

$$\times \exp\left[- \frac{\frac{k^2 y^2}{B_y^2}}{4\left(\frac{1}{w_{0y}^2} + \frac{B_j}{b^2} + \frac{ikA_y}{2B_y} \right)} \right] H_{2n}\left[\frac{\frac{ky}{B_y}}{2\sqrt{\frac{1}{w_{0y}^2} + \frac{B_j}{b^2} + \frac{ikA_y}{2B_y}}} \right] \tag{6}$$

In above derivation, we have used the following integral formula [34]

$$\int_0^{\infty} x^{2n} \exp(-\alpha^2 x^2) \cos(xy) \mathrm{d}x = (-1)^n \pi^{\frac{1}{2}} 2^{-(2n+1)} \alpha^{-(2n+1)} \exp\left(-\frac{y^2}{4\alpha^2} \right) H_{2n}\left(\frac{2}{2\alpha} \right) \tag{7}$$

If the hard-edged aperture is circular, the hard aperture function can be expressed as follows

$$H(x_0, y_0) = H(r_0) = \begin{cases} 1, & |r_0| \le a \\ 0, & |r_0| > a \end{cases} \tag{8}$$

where a denotes the radius of the aperture. In a similar way, the hard-edged aperture function can be expanded as the sum of complex Gaussian functions with finite numbers as follows

$$H(x_0, y_0) = \sum_{l=1}^{L} A_l \exp\left[-\frac{B_l}{a^2} \left(x_0^2 + y_0^2 \right) \right] \tag{9}$$

with A_l and B_l being the expansion and Gaussian coefficients.

Substituting Eqs. (1) and (9) into Eq. (4), we obtain the following analytical propagation formula for a four-beamlets laser array through a circularly apertured ABCD optical system

$$E(x, y, z) = \frac{i}{\lambda} \sqrt{\frac{1}{B_x B_y}} \left(\frac{1}{w_{0x}^2} \right)^m \left(\frac{1}{w_{0y}^2} \right)^n \exp\left(-\frac{ikD_x}{2B_x} x^2 - \frac{ikD_y}{2B_y} y^2 \right) \left(-\frac{1}{4} \right)^{(m+n)} \pi \times$$

$$\times \sum_{l=1}^{L} A_l \left(\frac{1}{w_{0x}^2} + \frac{B_l}{a^2} + \frac{ikA_x}{2B_x} \right)^{-\frac{2m+1}{2}} \left(\frac{1}{w_{0y}^2} + \frac{B_l}{a^2} + \frac{ikA_y}{2B_y} \right)^{-\frac{2n+1}{2}} \times$$

$$\times \exp\left[-\frac{\dfrac{k^2x^2}{B_x^2}}{4\left(\dfrac{1}{w_{0x}^2}+\dfrac{B_l}{a^2}+\dfrac{ikA_x}{2B_x}\right)}\right]H_{2m}\left[\frac{\dfrac{kx}{B_x}}{2\sqrt{\dfrac{1}{w_{0x}^2}+\dfrac{B_l}{a^2}+\dfrac{ikA_x}{2B_x}}}\right]\times$$

$$\times \exp\left[-\frac{\dfrac{k^2y^2}{B_y^2}}{4\left(\dfrac{1}{w_{0y}^2}+\dfrac{B_l}{a^2}+\dfrac{ikA_y}{2B_y}\right)}\right]H_{2n}\left[\frac{\dfrac{ky}{B_y}}{2\sqrt{\dfrac{1}{w_{0y}^2}+\dfrac{B_l}{a^2}+\dfrac{ikA_y}{2B_y}}}\right] \tag{10}$$

Under the condition of $a \to \infty$ and $b \to \infty$, Eq. (6) or Eq. (10) reduces to the propagation formula for a four-beamlets laser array through an unapertured ABCD optical system (Eq. (6) of Ref. [15]).

3. Numerical example

As a numerical example, in this section, we study the on-axis irradiance properties of a four-beamlets laser array passing through a squarely ($a = b$) apertured bifocal thin lens. The focusing geometry is shown in Fig. 1.

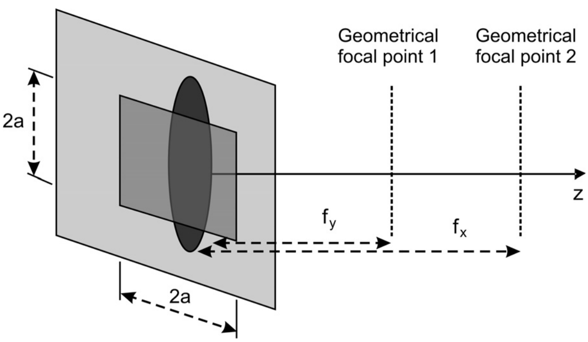

Fig. 1. Focusing geometry.

The transfer matrix of the optical system in Fig. 1 between the input plane ($z = 0$) and the output plane (z) in x or y direction reads as

$$\begin{bmatrix} A_j & B_j \\ C_j & D_j \end{bmatrix} = \begin{bmatrix} 1-\dfrac{z}{f_j} & z \\ -\dfrac{1}{f_j} & 1 \end{bmatrix}, \quad j = x, y \tag{11}$$

where f_x and f_y denote the focal lengths of the bifocal lens in the x and y directions, respectively.

By applying Eqs. (6) and (11), we can obtain the following on-axis irradiance for a four-beamlets laser array after passing through the squarely apertured bifocal lens

$$I_{00}(z) = |E(0, 0, z)|^2 = I_0(z) I_x(z) I_y(z) \tag{12}$$

with

$$I_0(z) = \frac{\pi^2}{\lambda^2 z^2} \tag{13}$$

$$I_x(z) = \left| \left(-\frac{1}{4}\right)^m \left(\frac{1}{F_{wx}\lambda f_x}\right)^m \sum_{l=1}^{L} A_l \left[\frac{1}{F_{wx}\lambda f_x} + \frac{B_l}{F\lambda f_x} + \frac{ik}{2z}\left(1 - \frac{z}{f_x}\right)\right]^{-m-\frac{1}{2}} H_{2m}(0) \right|^2 \tag{14}$$

$$I_y(z) = \left| \left(-\frac{1}{4}\right)^n \left(\frac{1}{F_{wy}\lambda f_y}\right)^n \sum_{l'=1}^{L'} A_{l'} \left[\frac{1}{F_{wy}\lambda f_y} + \frac{B_{l'}}{F\lambda f_y} + \frac{ik}{2z}\left(1 - \frac{z}{f_y}\right)\right]^{-n-\frac{1}{2}} H_{2n}(0) \right|^2 \tag{15}$$

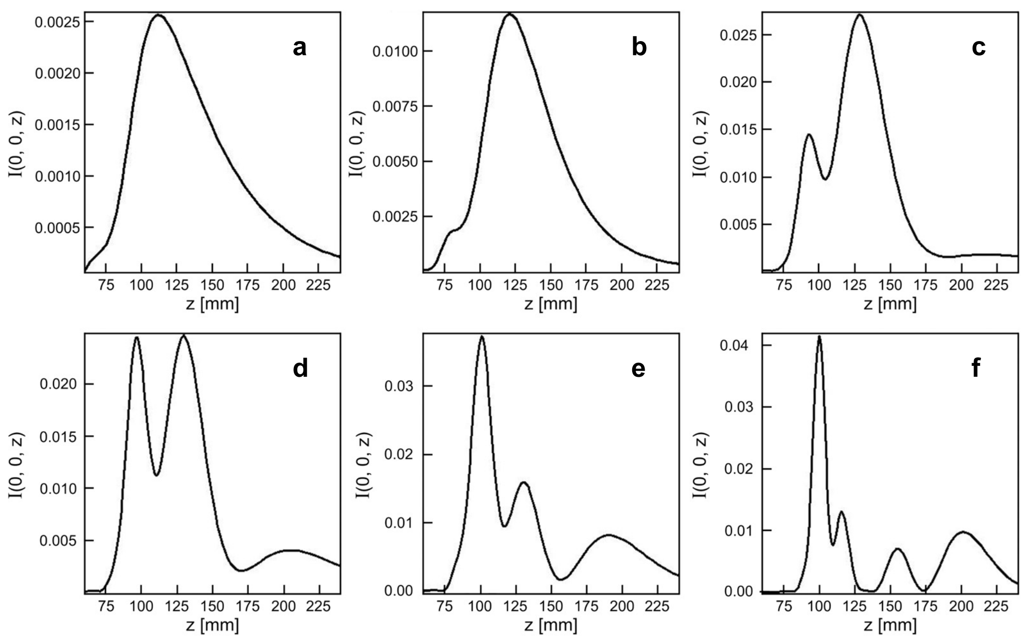

Fig. 2. On-axis irradiance distribution of a four-beamlets laser array focused by a squarely apertured bifocal lens along the propagation distance as the Fresnel number of the aperture F_a increases with $m = n = 2$: $F = 1.5$ (**a**), $F = 2$ (**b**), $F = 2.8$ (**c**), $F = 3.12$ (**d**), $F = 3.5$ (**e**), $F = 5.5$ (**f**).

For a square aperture (with width a), the Fresnel number of the aperture is defined as $F = a^2/\lambda f_x$. The Fresnel number of the beam in x or y direction is defined as $F_{wj} = w_{0j}^2/\lambda f_j$, ($j = x$ or y). For all the figures in this paper, the following parameters of the beam and of the optical system will be chosen $\lambda = 1.06\times10^{-3}$ mm, $F_{wx} = 2$, $F_{wy} = 3$, $f_x = 200$ mm, $f_y = 100$ mm.

In Figure 2 we calculate the on-axis irradiance distribution of a four-beamlets laser array focused by a squarely apertured bifocal lens along the propagation distance as the Fresnel number of the aperture F increases with $m = n = 2$. It can be seen in Figs. 2**a** and 2**b** that when F is small, there is only one focal point on the axis, which is located between the two geometrical focal points. So focal shift exists in our case. As the value of F increases, two main irradiance peaks on the axis appear (see Fig. 2**c**), and the relative value of the maximal peak decreases, while the relative value of the other main peak increases. When F increases to 3.12, the two main peaks reach the same height, which means that the critical irradiance has arrived at the point where a focal switch takes place (see Fig. 2**d**). Then the second main peak becomes the maximal peak (the focal shift experiences a rapid transition). With a further increase of F, the relative value of the maximal peak increases further while the relative value of the other main peak decreases (see Figs. 2**e** and 2**f**). Figure 3 shows the on-axis irradiance distribution of a four-beamlets laser array focused by a squarely apertured bifocal lens along the propagation distance as the Fresnel number of the aperture F increases with $m = n = 6$. One sees from Fig. 3 that the focused on-axis irradiance

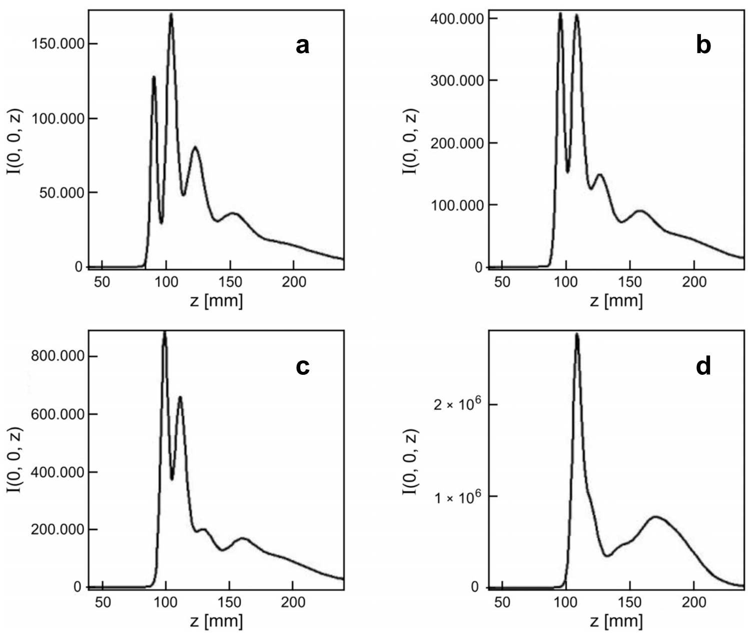

Fig. 3. On-axis irradiance distribution of a four-beamlets laser array focused by a squarely apertured bifocal lens along the propagation distance as the Fresnel number of the aperture F_a increases with $m = n = 6$: $F = 5.5$ (**a**), $F = 6.06$ (**b**), $F = 6.5$ (**c**), $F = 8$ (**d**).

distribution is also closely related to the beam orders of the four-beamlets laser array. By varying the values of F, a focal shift and focal switch also can be observed in this case. The focal switch takes place when $F = 6.06$ in the case of $m = n = 6$, which is much different from the case of $m = n = 2$. So we come to the conclusion that the focusing properties of a four-beamlets laser array beam are closely related to the Fresnel number of the aperture F and the beam orders of the four-beamlets laser array beam.

4. Conclusions

We have studied the propagation of a four-beamlets laser array through an apertured ABCD optical system. An analytical propagation formula has been derived based on the generalized Collins formula by expanding the hard-aperture function into a finite sum of complex Gaussian functions. Numerical results show that the focusing properties of a four-beamlets laser array are closely related to the parameters of the optical system and the laser array. Our formula provides a convenient way for investigating the paraxial propagation of a four-beamlets laser array through an apertured ABCD optical system.

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