

Optical vortex sign determination using self-interference methods

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We have proposed a simple method for determining the sign of optical vortex seeded in optical beam. Our method can be applied to any single optical vortex, also the one with topological charge magnitude higher than 1, as well as to the whole vortex lattice. The proposed method has been verified experimentally for all the cases.

Keywords: optical vortex, interference, birefringence.

1. Introduction

Optical vortices are singular lines in a phase distribution of a light field. Usually they are observed in planar cross section (on the screen) as dark points with undefined phase (vortex points) [1–3]. The wavefront takes characteristic helical form in the vicinity of the vortex line (Fig. 1) [1–4].

The twisted helical wavefront results a non-zero angular momentum carried by the beam with the optical vortex [3]. The wavefront's helical geometry allows for vortex classification due to the helix handedness. We say that optical vortex may have

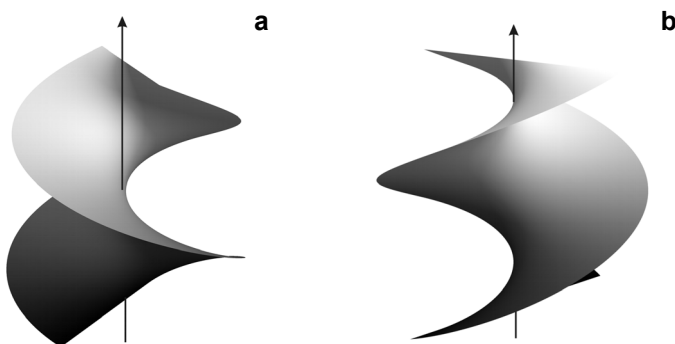


Fig. 1. Left (a) and right (b) oriented helical wavefront. The light wave phase is undetermined along the axis of the helice.

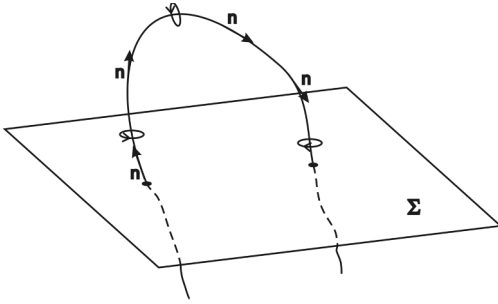


Fig. 2. Generally the vortex lines in a complex scalar field may have complicated geometry. Here, the line intersects the plane Σ twice. The phase circulation determined in plane Σ in the neighborhood of both intersection points circulates in opposite directions. However, the phase circulations, while looking along vectors \mathbf{n} , are constant for the entire line.

positive or negative topological charge. The question of topological charge is more complicated than it is usually referred to in literature. Short discussion on this matter may be found in [5]. Generally, singular lines may have complicated geometry [6–8]. In Fig. 2 versors $\mathbf{n}(s)$ are unit vectors distributed along the singular line. The choice of the vectors \mathbf{n} sense is arbitrary, however, the whole vector field along singular lines must be continuous.

The phase circulation splits vortex lines into two classes: left and right handed. As it was mentioned in many previous papers, the fields containing such singular lines are studied in a given plane, which often represents the plane of detection. In such a plane, vortices are classified according to their phase circulation having either positive or negative topological charge. Figure 2 shows that along the single vortex line the positive and negative vortices can be observed while intersecting with the plane Σ . Here the single well defined singular line can be observed as carrying both the positive and negative topological charges. For the monochromatic waves propagating in a well defined direction the topological charge can be identified without ambiguity. For example, such a situation happens in the case of waves generated by a paraxial superposition of laser beams [9, 10]. The results presented in our paper are valid for such cases.

Optical vortices have found a number of applications: they play an important role in optical manipulators [11], they can be applied as phase markers in classical and speckle interferometry [10, 12–14], or they can be used for determining the light polarization state [15–17]. They are also used as light suppressing elements in fluorescence microscopy [18]. Suppressing the light of the star using the vortex filter possibly will enable direct observation of the terrestrial planet orbiting around the stars [19, 20]. For most of these applications, the exact determination of the optical vortex sign is not necessary and it is enough to classify observed vortices into two classes: positive and negative. However, for some applications, determining the sign of optical vortices [10, 12–17] is important. For example, knowing the sign distribution

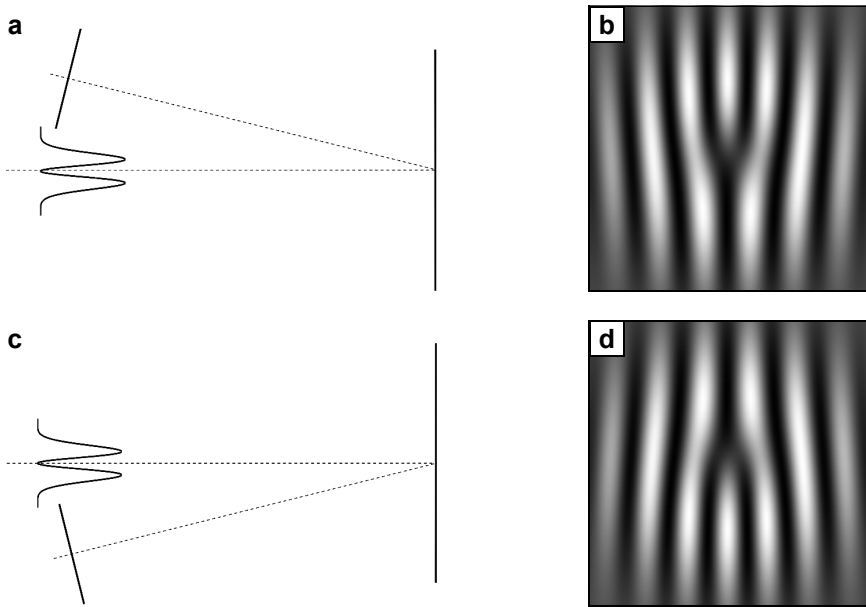


Fig. 3. The interference of paraxial beam carrying optical vortex with unit topological charge and inclined plane wave.

in vortex lattice generated in vortex interferometer allows for continuous phase determination (without phase unwrapping procedures).

In the literature there are several papers devoted to the problem of optical vortex sign determination. In papers [21, 22] the simple interfering methods for determining the sign of a single optical vortex is presented. These methods requires precise adjustment of the interferometric system, otherwise the results are ambiguous. Two methods based on the same concept (Fig. 3) were proposed for determining the vortex sign in the vortex lattice created in optical vortex interferometer [23, 24].

In the present paper the same basic idea is used. Figure 4 explains it in a more detailed way (a bit different figure is shown in paper [24]). The method based on the Young's diffraction experiment is reported in [22, 25, 26]. The method, in which a planar waveguide is used, is reported in [22, 27]. The method using shearing interferometry was reported in [28, 29]. Interferometric technique combined with polarimetry was proposed in paper [30]. This method gives both vortex charge and vortex position but requires the reference beam, so the optical system is more complicated than in case of shearing methods.

This paper reports on a new method for vortex sign determination. The method is based on the interference resulting from splitting incident wave by Wollaston prism or birefringent plate. Depending on the splitting element, the splitting could be angular or lateral. The measurement system is simple and stable – no adjustment and no reference beam is necessary. It can be used for measuring the single vortex sign (also

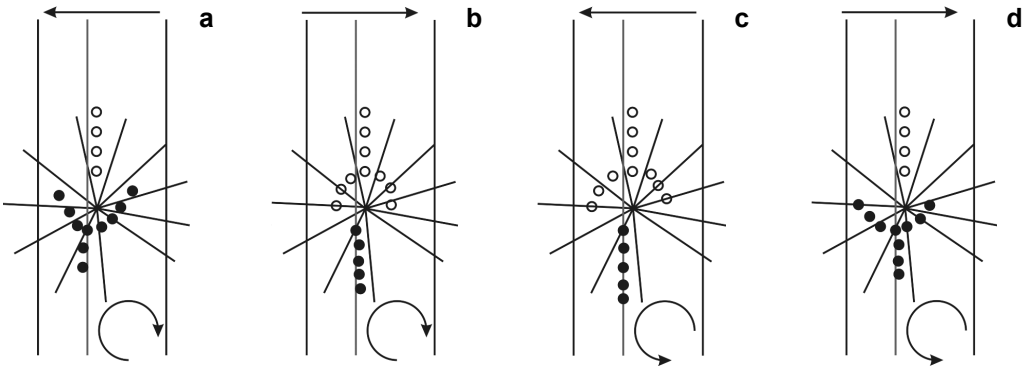


Fig. 4. A negative optical vortex interferes with inclined plane waves (**a** and **b**). The phase increment direction is indicated by arrows. In the case (**a**) the vortex phase increases in the same direction as the plane wave phase in lower part of the figure (below singular point). In the case (**b**) the same happens in upper part of the figure (above singular point). We may find a point of maximum/minimum intensity below singular point (**a**) or above singular point (**b**). The points of maximum/minimum intensity are plotted by dark/light circles. Below/above the singular point phases of both waves increases in the same direction. That means that the dark points (light points) will split to the left and right forming a fork-like fringe. In the opposite part (above/below) the phase increases in opposite directions, so fringes cannot split. Although the vortex sign in both cases (**a**) and (**b**) is the same the fork fringe are in opposite directions. Figures **c** and **d** show the same situation for vortex with positive charge.

with topological charge higher than 1) as well as for determining the vortex charges in the whole vortex lattice as produced, for example, by vortex interferometer. The results are free of ambiguity.

2. Fringe interference – angular sharing setup

The concept of our measurement system originates from works on one beam vortex interferometer [15]. The continuation of these works has led us to the idea of a simple system for measuring the light polarization state [16, 17] as well as to the idea of a system for measuring the vortex sign, which is a subject of this paper. Our optical system consists of a Wollaston prism working between the polarizer and the analyzer (Fig. 5).

The measured light beam enters the system through the polarizer. The Wollaston prism splits the input wave into two: an ordinary and extraordinary one, so the optical field can be considered as two incident wave copies being inclined at small angles. The angle depends on the Wollaston prism angle and the birefringence of the prism's material. These two beams interfere with each other after the analyzer, thus the resulting intensity distribution contains two forks-like fringes as it is shown in Fig. 6.

Both beams are displaced at the observation plane such that the dark point of one beam meets the bright area of the second one. Consider the small neighborhood of the singular point belonging (say) to the left beam at observation plane. The phase of the bright part of the right beam covering this neighborhood will have strong phase

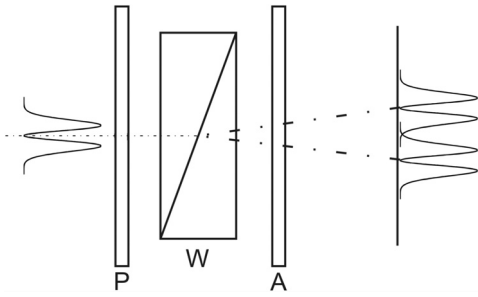


Fig. 5. The measurement setup for optical vortex sign identification: angular sharing case. P – polarizer with the azimuth angle 0° , A – analyzer with the azimuth angle 45° , W – Wollaston prism with the azimuth angle 90° .

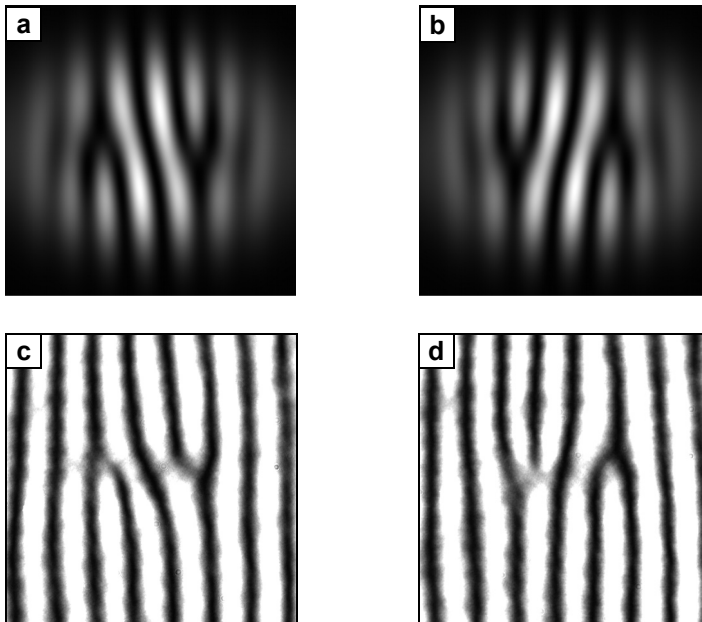


Fig. 6. The interference calculated numerically for positive (a) and negative (b) optical vortex. An optical vortex (positive (c), negative (d)) was introduced into laser beam after reflection from a spatial light modulator and the beam passed through the measurement system shown in Fig. 5.

gradient due to the inclination angle. As a result, in this neighborhood, this small part of the second beam can be treated as a plane wave so one can refer to Fig. 4 when analyzing the experiment. In Figures 3 and 4 one can see that the two copies of the vortex interfere with the non-vortex part of the other beam at opposite inclination. That is why the observed two forks are in opposite orientation. Changing the sign of the vortex seeded into incident beam changes orientation of the forks fringes determining at the same way uniquely the topological charge of the optical vortex. Figure 7 shows the same situation with vortex topological charge of magnitude ± 3 .

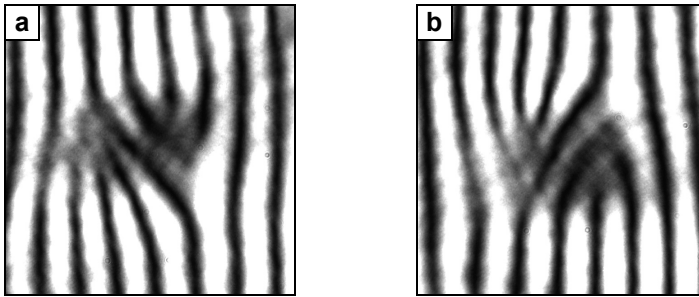


Fig. 7. The same experiment as in Figs. 6c and 6d for the optical vortex with charge magnitude of ± 3 .

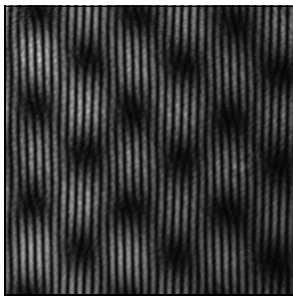


Fig. 8. In this experiment the entire vortex lattice generated by one beam vortex interferometer was passed through the analyzing setup shown in Fig. 5. The opposite oriented fringes identify the sign of vortices.

Figure 8 shows the measurements performed on the vortex lattice generated by an optical vortex interferometer [12] for the Wollaston prism with the wedge angle of 10° .

3. Fringeless interference – lateral sharing setup

A fringeless self-interference idea (FSI) is as follows: the input optical field is split into two fields, next they interfere collinearly with each other at the output of the setup whereas the resulting interference field depends on the phase difference between interfering waves. It can be done in classical interference (Mach–Zehnder or Michelson) or polariscopic setups. There are several papers which conjugated the FSI idea with the vortex fields generation and transformation. For example in [31, 32] self-interference of Laguerre–Gauss beams in Mach–Zehnder setup leading to off-axis vortices appearance is presented; in [33–36] a birefringent plate splitting the vortex field was used and the polarization properties of the resulting optical field were analyzed.

In this part of the paper we present a setup in which the birefringent plane parallel plate is used for splitting the incoming beam (Fig. 9). In such a case both beams propagate parallel to each other with lateral shift Δx proportional to the plate thickness.

Contrary to the setups presented in [33–35] (where split waves were well separated) this shift is relatively small, of order of tens of micrometers, to ensure the strong interference of corresponding central parts of the wave fields. The optical

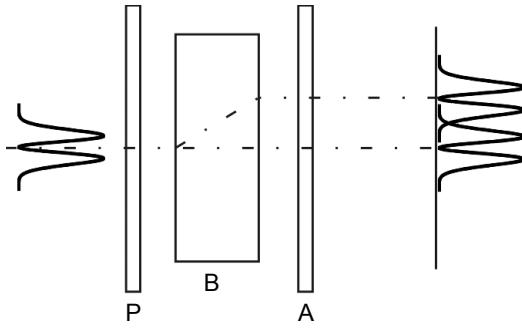


Fig. 9. The measurement setup for optical vortex sign identification: lateral sharing case. P – polarizer with the azimuth angle 0° , A – analyzer with the azimuth angle 45° , B – birefringent plane parallel plate with the azimuth angle 90° . The optical axis of the plate is inclined at the angle of 45° with regard to input and output plate's planes. The plate is rotated around the axis perpendicular to the figure's plane.

axis of the plate is at 45° to its surface. The longitudinal phase retardance δ between the ordinary and extraordinary waves (responsible for an interference result) depends here on the rotation angle of the plate with the rotation axis perpendicular to the input (and output) wave propagation axis. For some initial rotation angle the phase retardance δ is a multiplicity of 2π and, hence, as a result of a constructive interference, the optical vortex appears in the center of the image at the output of the setup (Fig. 10a). The phase shift change $\Delta\delta$ induced by further plate rotation causes that the interference image is no longer symmetrical: for $\Delta\delta = \pm 90^\circ$ one can observe (Figs. 10b, 10d) that the vortex position changes, additionally, relatively well defined maximum in the output intensity distribution appears.

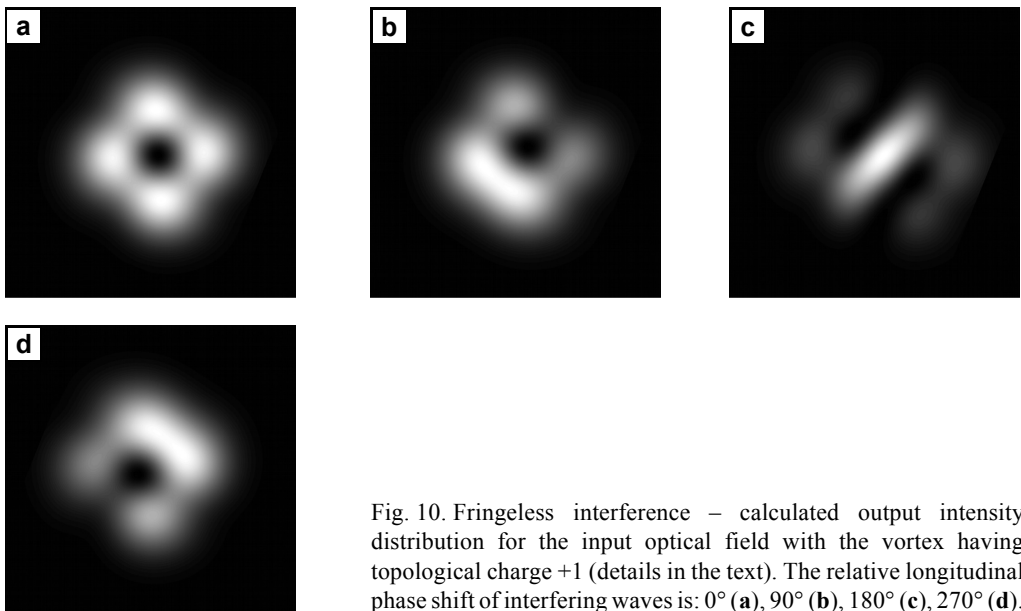


Fig. 10. Fringeless interference – calculated output intensity distribution for the input optical field with the vortex having topological charge +1 (details in the text). The relative longitudinal phase shift of interfering waves is: 0° (a), 90° (b), 180° (c), 270° (d).

The relative orientation of the vortex with regard to the intensity maximum depends both on the sign of the vortex topological charge m and the sign \pm in $\Delta\delta = \pm 90^\circ$. If the sign of m is positive then for $\Delta\delta = +90^\circ$ the vortex moves up with the intensity maximum appearing below it (see Fig. 10b), while for $\Delta\delta = -90^\circ$ (which is equivalent to 270°) the vortex moves down and the intensity maximum appears above it (see Fig. 10d). For the negative vortex sign the phenomena mentioned above go the opposite way. So, by analyzing the movements of the vortex and the intensity maximum affected by the relative phase shift change one can identify the vortex sign. When $\delta = 180^\circ$ both waves interfere constructively in the image center. On both sides of these intensity maximum two vortices are born and the characteristic letter S can be observed in the intensity pattern (Fig. 10c). Changing the vortex sign will reflect this S pattern vertically. All these cases can be illustrated by plotting the phase distribution of the interfering waves as it was done in Fig. 4. In Figure 11 the exemplary interferograms for the described situation are presented. Experiments were carried out for He-Ne laser and 30 mm-thick quartz plane parallel plate.

The same phenomenon occurs for a vortex lattice. Figure 12a shows the initial position of optical vortices ($\Delta\delta = 0^\circ$). The experimental pattern is shown in Fig. 12c.

Here the incidence vortex lattice was generated by one beam vortex interferometer [15]. After passing the birefringent plate B of the measurement system, each vortex is split into two ones of the same charge sign. After the analyzer, due to interference, each vortex pair interferes with each other. As a result one vortex appears. When the relative phase shift between interfering vortex lattices changes from 0° to 90° , these

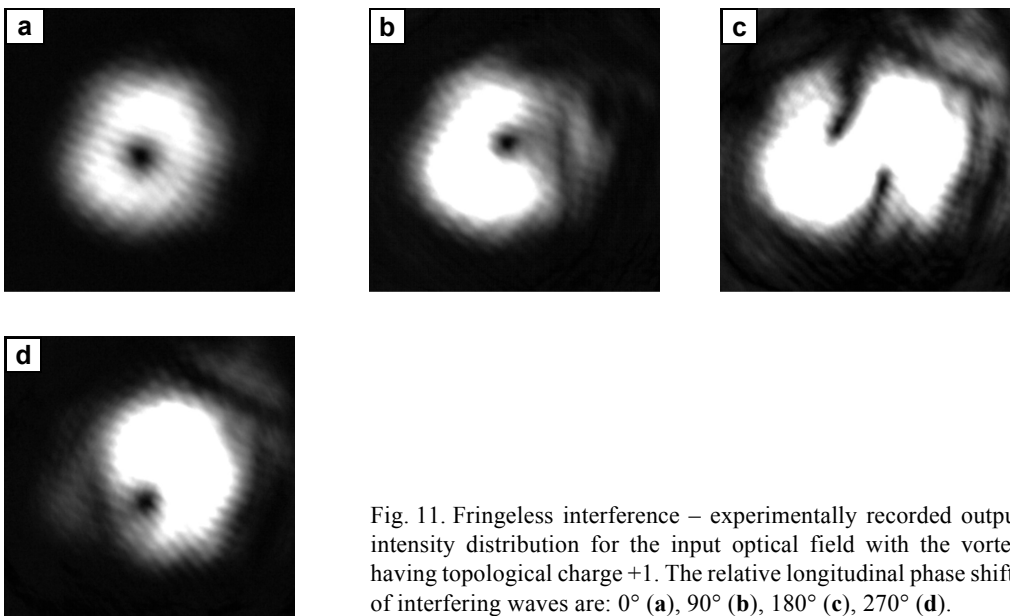


Fig. 11. Fringeless interference – experimentally recorded output intensity distribution for the input optical field with the vortex having topological charge +1. The relative longitudinal phase shifts of interfering waves are: 0° (a), 90° (b), 180° (c), 270° (d).

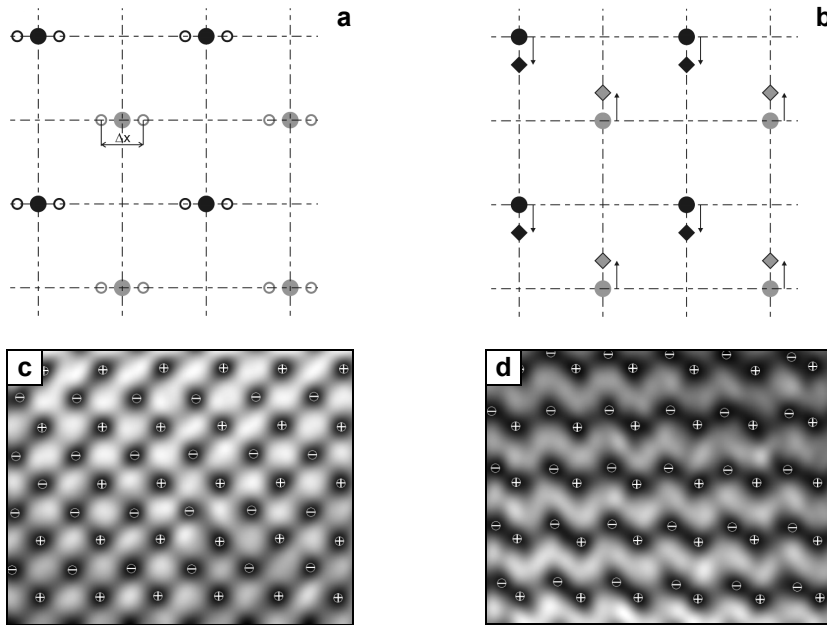


Fig. 12. Fringeless interference–analytical predictions and experimental interferograms. Two vortex lattices after the birefringent plate (open circles) and resultant vortex lattice after the analyzer for the phase shift difference $\delta = 0^\circ$ (filled circles) – **a**. Resultant vortex lattices for $\delta = 0^\circ$ (circles) and $\delta = 90^\circ$ (diamonds) – **b**. Two gray levels correspond to vortices signs. Interferograms taken from the experiment for $\delta = 0^\circ$ (**c**) and $\delta = 90^\circ$ (**d**). Vortices signs have been marked.

single vortices move up or down (dependent on their sign), as was illustrated in Fig. 12**b**. Figure 12**d** shows the vortex field for $\delta = 90^\circ$. Analyzing this movement we can determine the sign of any vortex as it was in the case of single optical vortex.

4. Conclusions

A simple and effective optical system for vortex sign determination was presented. The optical system can be built in a compact and stable way. It can be used both for determining the sign of a single optical vortex and the sign of vortices in vortex lattice. The results can be read directly by inspecting the vortex fringe orientation. The inspection process can be fully computerized just by applying available software for fringe analysis. The system with a birefringent plate requires precise adjustment and the plate must be rotated during measurements so the optical system based on the Wollaston prism seems to be more practical.

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