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Investigation and analysis of time response in Geiger mode avalanche photodiode

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Statistical properties of the impulse response of avalanche photodiode (APDs) are determined. The model is based on recurrence equations. These equations are solved numerically to calculate the mean current impulse response and standard deviation as a function of time. In this paper, we investigate the effects of parameters such as ionization coefficient-multiplication thickness product (αw), dead space, excess noise factor, mole fraction, temperature on the mean current impulse response of APD in the Geiger mode.

Keywords: avalanche photodiode, Geiger mode, time response.

1. Introduction

Avalanche photodiodes (APDs) are known as detectors in long-haul fiber optic systems and Geiger mode applications due to their advantage of high internal gain generated by avalanche multiplication [1, 2]. According to the local-field avalanche theory, both the multiplication noise and the gain-bandwidth product of APDs are determined by the ratio of the electron and hole ionization coefficients of the semiconductor in the multiplication region. Since this ratio is a material property, for a given electric field, efforts to improve the APD performance have focused on optimizing the electric field profile and characterizing new materials. APDs can be operated in Geiger mode to count single photons. In this mode the APD is biased above its breakdown voltage. When the reverse bias voltage of a *p*-*n* junction is raised above the breakdown voltage, even a single carrier can trigger an avalanche process, leading to a measurable current. The absorption of photon in the depletion layer initiates the avalanche breakdown, which can be easily detected. After breakdown, the current is quenched and the diode is recharged to allow the detection of new photon. Silicon *p-n* junctions reverse biased above the breakdown voltage are usually called single photon avalanche diodes (SPADs).

Geiger mode APDs (GM-APDs) are excellent devices for detecting weak optical signals. Because of their excellent time resolution, they are often used for photon timing measurements. Recent advances in GM-APDs have made these devices promising candidates for detectors in photon-counting receivers. Today, SPADs are profitably used in various applications such as time-resolved spectroscopy, chemistry, physics, and biology [3], fluid velocimetry [4], laser ranging [5], optical time-domain reflectometry [6], single molecule detection [7, 8]. Several works have been done as regarding calculation and analysis of impulse response and quantum detection efficiency of GM-APD [9, 10], but to the best of our knowledge the effects of ionization coefficient-multiplication thickness product (αw), temperature, mole fraction, and dead space have not been demonstrated yet. In this paper, we study these characteristics of GM-APD. This paper is organized as follows. In Section 2, a modified model to calculate the mean impulse response and standard deviation by solving the recurrence equations is presented. In Section 3, the model is applied to SPADs and effects of αw, dead space, velocity and ionization coefficients on the mean impulse response are discussed. Finally, conclusions are presented in Section 4.

2. Theory of model

We consider an APD with a multiplication region of width w. A parent photo-electron is injected into the multiplication region at $x = 0$ with a fixed velocity v_e under the effect of an electric field. After traveling a fixed dead space d_e, in the x-direction, the electron becomes capable of impact ionizing with an ionization coefficient α. Upon ionization, an electron−hole pair is generated, so that the parent electron is replaced by two electrons and a hole. The hole travels in the ($-x$)-direction and becomes capable of impact ionizing with an impact ionization coefficient β only after traveling a dead space d_h. This avalanche of ionization events continues until all carriers exit the multiplication region. In the case of multiplication with a fixed dead space d_e, the probability density function (pdf) of carriers *vs*. time τ and distance ξ is given by

$$h_e(\xi, \tau) = \begin{cases} 0, & \xi \le d_e \\ \alpha \exp\left[-\alpha(\xi - d_e)\right]\delta\left(\tau - \dfrac{\xi}{v_e}\right), & \xi > d_e \end{cases} \tag{1}$$

$$h_h(\xi, \tau) = \begin{cases} 0, & \xi \le d_h \\ \beta \exp\left[-\beta(\xi - d_h)\right]\delta\left(\tau - \dfrac{\xi}{v_h}\right), & \xi > d_h \end{cases} \tag{2}$$

where d_e and d_h are the electron and hole dead spaces, respectively, v_e and v_h are the velocity of the electrons and holes, respectively; α and β are the ionization rates for electrons and holes, respectively, that are often modeled by standard equation [11, 12]

$$\alpha(E), \beta(E) = A\exp\left[-\left(\frac{E_c}{E}\right)^m\right] \tag{3}$$

were A, E_c and m are the parameters taken from [13, 14]. With integration of this distribution function over the total time, the position dependent ionization pdf is given as

$$h_{e(h)}(\xi) = \int_0^\infty h_{e(h)}(\xi, \tau)\mathrm{d}\tau \tag{4}$$

The recurrence equation for electron and hole mean current impulse response are given by [15]

$$\langle I_e(z, t)\rangle = P_e(z, t)\langle I_e(z, t)\rangle + \\ + \int_0^{\min(w-z,\, v_e t)} \left[2\left\langle I_e\left(z+\xi, t-\frac{\xi}{v_e}\right)\right\rangle + \left\langle I_h\left(z+\xi, t-\frac{\xi}{v_e}\right)\right\rangle\right] h_e(\xi)\mathrm{d}\xi \tag{5}$$

$$\langle I_h(z, t)\rangle = P_h(z, t)\langle I_h(z, t)\rangle + \\ + \int_0^{\min(w-z,\, v_h t)} \left[2\left\langle I_h\left(z+\xi, t-\frac{\xi}{v_h}\right)\right\rangle + \left\langle I_e\left(z+\xi, t-\frac{\xi}{v_h}\right)\right\rangle\right] h_h(\xi)\mathrm{d}\xi \tag{6}$$

where the first terms on the right-hand side of these equations represent the contributions from the injected, primary currents $\langle I_{e(h)}(z, t)\rangle$. The probabilities that the injected carriers avoid ionizing before exiting the multiplication region before time t is given by

$$P_e(z, t) = 1 - \int_0^{\min(w-z,\, v_e t)} h_e(\xi)\mathrm{d}\xi \tag{7}$$

$$P_h(z, t) = 1 - \int_0^{\min(w-z,\, v_h t)} h_h(\xi)\mathrm{d}\xi \tag{8}$$

The initial current from electrons and holes can be calculated as

$$I_{e0}(z,t) = \begin{cases} 0, & t > \dfrac{w-z}{v_e} \\ \dfrac{qv_e}{w}, & t \le \dfrac{w-z}{v_e} \end{cases} \tag{9}$$

$$I_{h0}(z,t) = \begin{cases} 0, & t > \dfrac{w-z}{v_h} \\ \dfrac{qv_h}{w}, & t \le \dfrac{w-z}{v_h} \end{cases} \tag{10}$$

Standard deviation of the impulse response can be determined by developing recurrent expressions for the second order statistics of $I_e(z, t)$, $I_h(z, t)$ using the same technique as that for the mean currents. The second moment of the impulse response $i_2(z, t) = \langle I^2(z, t)\rangle$ can be computed by

$$\begin{aligned}\langle I_e^2(z,t)\rangle = {} & P_e(z,t)\langle I_{e0}^2(z,t)\rangle + \int_0^{w-z} \mathrm{d}\xi \int_0^t \Big[2\langle I_e^2(z+\xi, t-\tau)\rangle + \\ & + 2\langle I_e(z+\xi, t-\tau)\rangle^2 + \langle I_h^2(z+\xi, t-\tau)\rangle + \\ & + 4\langle I_h(z+\xi, t-\tau)\rangle \times \langle I_e(z+\xi, t-\tau)\rangle\Big] \times h_e(\xi,\tau)\mathrm{d}\tau\end{aligned} \tag{11}$$

$$\begin{aligned}\langle I_h^2(z,t)\rangle = {} & P_h(z,t)\langle I_{h0}^2(z,t)\rangle + \int_0^{z} \mathrm{d}\xi \int_0^t \Big[2\langle I_h^2(z+\xi, t-\tau)\rangle + \\ & + 2\langle I_h(z+\xi, t-\tau)\rangle^2 + \langle I_e^2(z+\xi, t-\tau)\rangle + \\ & + 4\langle I_h(z+\xi, t-\tau)\rangle \times \langle I_e(z+\xi, t-\tau)\rangle\Big] \times h_h(\xi,\tau)\mathrm{d}\tau\end{aligned} \tag{12}$$

And the standard deviation of $I(z, t)$ can then be obtained using [16]

$$\sigma(z,t) = \sqrt{i_2(z,t) - i^2(z,t)} \tag{13}$$

3. Results and discussion

One of the important parameters in the APDs is the value of ionization coefficient--multiplication thickness product (αw), where α is the electron ionization coefficient

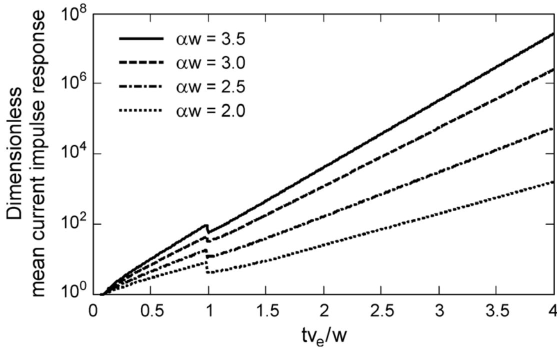

Fig. 1. Dimensionless mean current impulse response for different values of αw with $v_h = v_e = 10^5$ m/s, $w = 100$ nm, $d/w = 0.1$, $k = 1$.

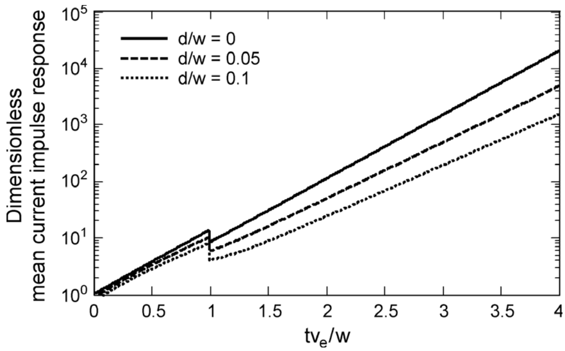

Fig. 2. Dimensionless mean impulse response for different values of d/w and with $\alpha w = 2$, $v_h = v_e = 10^5$ m/s, $w = 100$ nm, $k = 1$.

and w is the thickness of multiplication region. With changing the value of αw, APD can operate in the Geiger or analogue mode. For smaller values of αw, the APD operates in analogue mode. In Figure 1, the effect of αw on the mean current impulse response in Geiger mode, considering the dead space effect is shown which is normalized to the injected primary current qv_e/w. According to this figure we find that with an increase of αw, the value of impulse response increases with higher rate.

In Figure 2, the effect of dead space on impulse response is studied. In this figure, the impulse response without dead space effect ($d/w = 0$) and with dead space ($d/w = 0.05$ and 0.1) are shown. We find that the presence of dead space results in a reduction of the impulse response for all the times. According to this figure, the rate of response in APD decreases for higher values of dead space.

In Figure 3, the mean current impulse responses for different values of k ($k = \beta/\alpha$) are shown. We find that the rate of response in APD increases with k.

Figure 4 shows the mean current impulse response for different ratios of carrier velocities. With an increase of the ratio v_h/v_e, the peak of current and therefore, the rate of response in APD are increased.

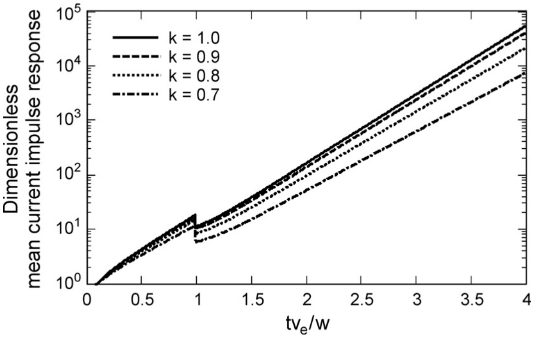

Fig. 3. Dimensionless mean impulse response for different values of k with $\alpha w = 2.5$, $v_h = v_e = 10^5$ m/s, $w = 100$ nm, $d/w = 0.1$.

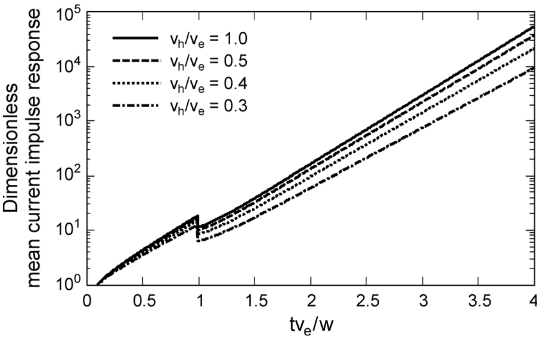

Fig. 4. Dimensionless mean impulse response for different values of v_h/v_e with $\alpha w = 2.5$, $w = 100$ nm, $d/w = 0.1$, $k = 1$.

Multiplication noise strongly depends on the carrier injection into the multiplication region. The noise becomes small when photogenerated electrons are selectively injected into the multiplication region which has a large electron ionization rate α in comparison with that of holes. The photoabsorption in the multiplication region causes contamination by the hole injection which accompanies an increase of the multiplication noise. Transparency of the multiplication region is therefore essential to get low noise performance. In Figure 5, dimensionless standard deviation for different values of k is shown. According to this figure, the excess noise increases with k.

It is well known that the fluctuation of temperature changes the ionization coefficient parameters. With an increase of temperature, the value of ionization coefficient is decreased. In Figure 6, we compare the mean current impulse response for different values of temperature with $w = 100$ nm, $d/w = 0.1$, $k = 1$, $v_h/v_e = 1$. According to this figure, we find that with an increase of temperature the peak of mean current impulse response and the rate of response decrease. Meanwhile, we have better Geiger mode characteristics at lower temperature.

In Figure 7, we study the effect of changing the mole fraction on the mean current impulse response. For each of the four materials ($Al_{0.6}Ga_{0.4}As$, $Al_{0.3}Ga_{0.7}As$,

$Al_{0.15}Ga_{0.85}As$ and GaAs) we are able to find a single set of parameters (A, E_c and m) that satisfy the exponential model presented in Eq. (3) independent of the multiplication region width. With a constant value of multiplication width, we have a larger value of ionization coefficient-multiplication thickness product (αw) with an increase of Al

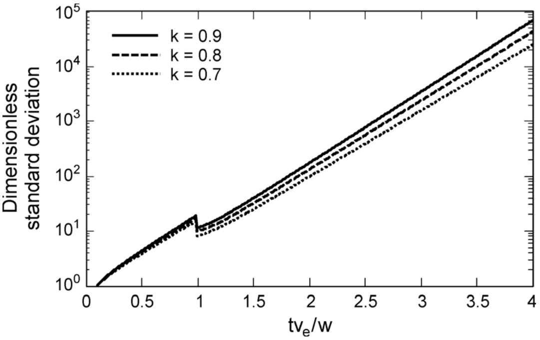

Fig. 5. Dimensionless standard deviation for different values of k with $\alpha w = 2.5$, $w = 100$ nm, $d/w = 0.1$, $v_h/v_e = 1$.

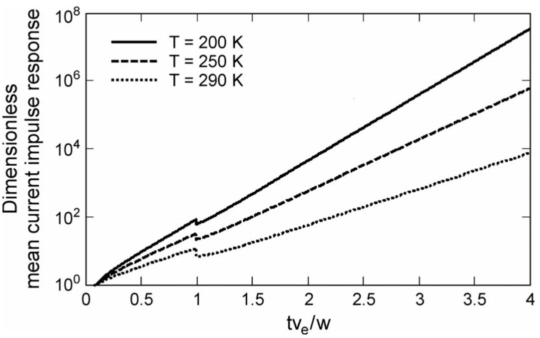

Fig. 6. Mean current impulse response for $w = 100$ nm with $d/w = 0.1$, $k = 1$, $v_h/v_e = 1$ at $T = 200$ K, 250 K and 290 K.

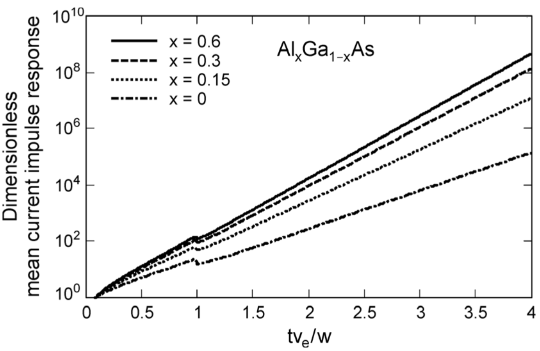

Fig. 7. Effect of mole fraction variation on the mean current impulse response for $w = 100$ nm, $d/w = 0.1$, $k = 1$, $v_h/v_e = 1$.

mole fraction and finally the peak of mean current impulse response increases and APD has a good performance in the Geiger mode.

4. Conclusions

In this paper, using a model based on recurrence equations, we investigated the effects of several parameters such as αw, dead space, excess noise factor, velocity on the mean and standard deviation of impulse response time of avalanche photodiode in Geiger mode. We found that with an increase of αw, ratio v_h/v_e and k, the peak of the mean current impulse response in the Geiger mode is increased. We also studied the effect of temperature and mole fraction on the Geiger mode characteristics of APD, and we showed that for better operation, lower temperature and higher value of Al mole fraction in $Al_xGa_{1-x}As$-APD must be chosen.

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