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The principle of multilayer plane-parallel structure antireflection

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The principle of two- and three-layer transparent system antireflection is based on the method of Fabry–Perot multiple-beam interference spectrum envelopes and it is shown that the presence of half-wave and quarter-wave layer thicknesses or their combination is sufficient but not necessary for its achievement. The necessary condition is the condition of overlapping of interference layer envelopes, therefore the antireflection effect can be observed at arbitrary ratios of optical layer thicknesses, angles of incidence and optical refraction index of media when the condition is satisfied for the minimum of interference contour in the required spectral region.

Keywords: Fabry–Perot interference, antireflection, envelope function.

1. Introduction

It is known that the film antireflection effect for the film at the surface of another body was first observed by TAYLOR [1]. Based on the interference representations, TIMOPHEEVA [2] and STRONG [3] have formulated the conditions of reflected wave intensity decrease by means of changing parameters of deposition of one-layer, and VLASOV [4] two-layer, coatings at the surface. Later on, the principles of light antireflection by a surface with film coating were thoroughly investigated in many works [5–9]. These and other results are generalized in monographs [10–13]. Thus it was established that a system of k-layers does not reflect a wave if optical admittance is $Y = \eta_0$, where $Y = C/B$, the transformation matrix is

$$\begin{bmatrix} B \\ C \end{bmatrix} = \left(\prod_{j=1}^{k} \begin{bmatrix} \cos\delta_j & \dfrac{\sin\delta_j}{\eta_j} \\ i\eta_j \sin\delta_j & \cos\delta_j \end{bmatrix} \right) \begin{bmatrix} 1 \\ \eta_{j+1} \end{bmatrix}, \qquad \eta_j = \begin{cases} \eta_{p,j} = n_j \cos\alpha_j \\ \eta_{s,j} = \dfrac{n_j}{\cos\alpha_j} \end{cases}$$

Here, *s* and *p* correspond to the type of wave polarization; 0 is the index of the medium above the films and $j + 1$ of the substrate.

The conditions of antireflection for two-layer coatings are determined by means of solving the equation $\tilde{r} = (Y - \eta_0)/(Y + \eta) = 0$. At equal optical thicknesses $n_1 d_1 = n_2 d_2$ the refractive indices of the media are inter-related by the following expressions [8–10]:

$$n_0 = n_3, \quad \delta = m\pi, \quad m = 0, 1, 2, 3, ... \tag{1}$$

$$n_1 n_2 = n_0 n_3 \tag{2}$$

$$n_1^3 = n_0^2 n_3 \quad \text{and} \quad n_2^3 = n_0 n_3^2, \quad nd = \frac{\lambda}{4} \tag{3}$$

$$n_2^2 n_0 = n_1^2 n_3, \quad \delta = (2m - 1)\frac{\pi}{2}, \quad m = 1, 2, 3, ... \tag{4}$$

where the subscript 3 corresponds to the substrate. If the relation of optical thicknesses is $2n_1 d_1 = n_2 d_2$, the state of antireflection is achieved under the following conditions:

$$n_0 = n_3, \quad \delta_1 = m\pi, \quad m = 0, 1, 2, 3, ... \tag{5}$$

$$n_1^2 = n_0 n_3, \quad \delta_1 = (2m - 1)\frac{\pi}{2}, \quad m = 1, 2, 3, ... \tag{6}$$

$$n_2^3 - \frac{1}{2}\frac{n_2 n_3}{n_0 n_1}\left(n_0^2 + n_1^2\right)(n_1 + n_2) + n_1 n_3^2 = 0 \tag{7}$$

In work [14], empirical method was used to obtain analytical expressions for the envelopes of the extrema of multiple-beam light interference by two and three films, on the basis of which the principle of antireflection was formulated. The purpose of the present paper was to study the correspondence of a given principle to antireflection conditions (1)–(7). The problem was solved for a two-layer system as an efficient antireflection coating in photoelectronic [15, 16] and solar power devices [17].

2. The basic results and discussion

It is known [14, 18] that in contrast to a one-layer structure, when there are two or more layers in a system the envelopes of the extrema of multiple-beam interference oscillate as a result of optical beating, by analogy with the manifestation of parametric interaction of two oscillators in oscillatory systems. The functions of the extrema envelopes depend on the relation between the geometrical layer thicknesses and have the following form for the minima *m*:

$$R_m = \begin{cases} \left(\dfrac{\sigma_{01} - \sigma_{13}}{1 - \sigma_{01}\sigma_{13}}\right)^2, & d_1 < d_2 \\ \left(\dfrac{\sigma_{02} - \sigma_{23}}{1 - \sigma_{02}\sigma_{23}}\right)^2, & d_1 > d_2 \end{cases} \tag{8}$$

where σ and ϕ are the module and phase of the complex reflection coefficient $\tilde{r}$ in the representation $\tilde{r} = \sigma\exp(-i\phi)$. As a result of optical beating the envelopes also oscillate in the range of values

$$R_{M,\,m_{13,\,02}} = \begin{cases} \left(\dfrac{\sigma_{01} - \sigma_{M,\,m_{13}}}{1 - \sigma_{01}\sigma_{m_{13}}}\right)^2, & d_1 < d_2 \\ \left(\dfrac{\sigma_{M,\,m_{02}} - \sigma_{23}}{1 - \sigma_{23}\sigma_{M,\,m_{02}}}\right)^2, & d_1 > d_2 \end{cases} \tag{9}$$

where

$$\sigma_{M,\,m_{13,\,02}} = \frac{\sigma_{12,\,01} \pm \sigma_{23,\,12}}{1 \pm \sigma_{12,\,01}\sigma_{23,\,12}} \tag{10}$$

and the sign "+" corresponds to the maxima M and the sign "–" corresponds to the minima m in the reflection contour $R(\lambda)$. Full antireflection of a two-layer coating $R(\lambda) \to 0$ occurs at the points of intersection of the optical reflection contours by a single and binary interface

$$\sigma_{01} = \sigma_{13} \text{ and } \sigma_{02} = \sigma_{23} \tag{11}$$

in which $R_{m013,\,023} \to 0$. Let us show that the principle we suggest satisfies the conditions of antireflection in Eqs. (1)–(7).

Condition (1). As can be seen in Fig. 1, in a symmetrical two-layer structure $n_0 = n_3$ at equal optical thicknesses $n_1 d_1 = n_2 d_2$, the transition through a one-layer state $n_1 = n_2$ is accompanied by an inversion of the extrema of multiple-beam interference, which is broken in the interval $n_1 < n_2$ at the refractive index n_1^C, at which the contour $R_{m,\,02}$ has a break.

Under condition (1), the interference minima are antireflected both at fractional values of optical layer thicknesses and at their inequality $n_1 d_1 \neq n_2 d_2$. In this case, at $n_1 d_1 / n_2 d_2 = 1$ every minimum is antireflected, at $n_1 d_1 / n_2 d_2 = 2$ every second one, at $n_1 d_1 / n_2 d_2 = 3$ every third one, and so on.

Condition (2). In asymmetric structures $n_0 \neq n_3$, the contours of the extrema envelopes of every layer coincide. As can be seen in Fig. 2**a**, full antireflection of

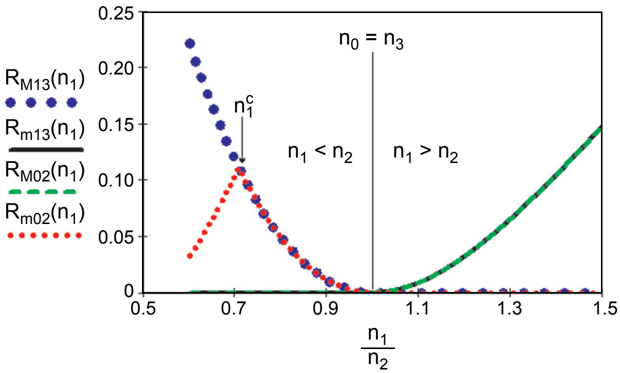

Fig. 1. Dependence of functions (9) on the relation between the refractive indices of layers; $\alpha = 0$, $n_1 = 3$, $n_2 = 2.5$, $n_3 = 1$, $n_1 d_1 = \lambda/2.475$, $n_2 d_2 = \lambda/3.3$.

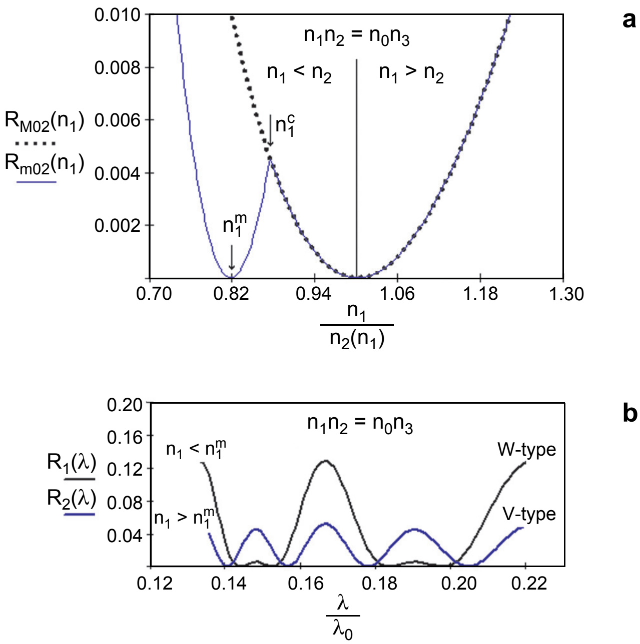

Fig. 2. Dependence of functions (9) on the relation between the layer refractive indices (**a**) and the illustration of the dynamics of *V*- and *W*-type antireflection contour formation (**b**); $\lambda_0 = 1.5$ µm.

a two-layer coating occurs only at the refractive index of the top layer n_1^m as the solution to the equation $\sigma_{m02,\,13} = 0$. For other values of n_1 the antireflection contour has non-zero amplitude. It should be pointed out that contrary to popular opinion [19] antireflection contours of *V*- and *W*-types (Fig. 2**b**) are also characteristic of a system of two plane-parallel layers.

Condition (3). At $n_0 = 1$ and normal light incidence on the interface a given condition is studied [20] for the creation of an antireflection coating of the surface of

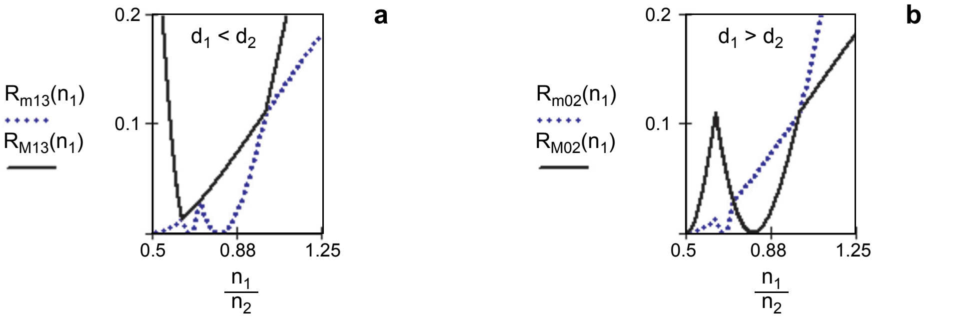

Fig. 3. Illustration of the absence of an antireflection coating with the relation of refractive indices $n_1^3 = n_0^2 n_3$.

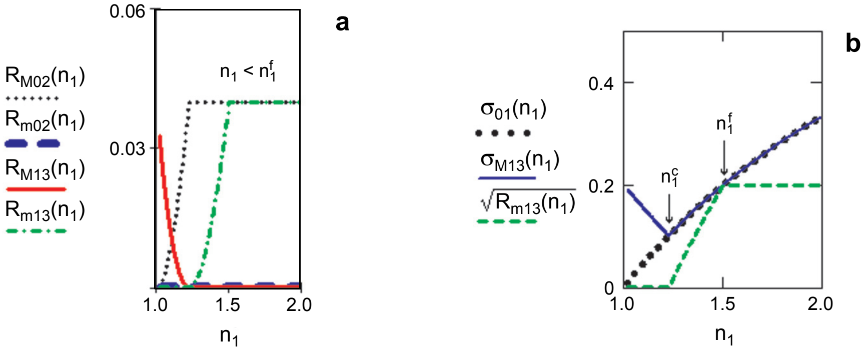

Fig. 4. Illustration of the antireflection regions of a two-layer coating under condition (4).

solar power engineering elements with layer thicknesses $\lambda/4$. However, as can be seen in Fig. 3, the regions of reflection contour corresponding to the contour minima $R_{m13}(n_1)$ and $R_{M02}(n_1)$ are narrow, that is, the corresponding coatings are inefficient for practical use.

Condition (4). As can be seen in Fig. 4**a**, for the refractive index of the top layer $n_1 > n_1^f$ the minima of the reflection contour are antireflected if $nd = n_1 d_1 = n_2 d_2 = m\lambda/2$, $m = 1, 2, 3, \ldots$, where the value n_1^f is determined from the condition $\sigma_{12,M13} = \sqrt{R_{m13}}$ (Fig. 4**b**). If $n_1 d_1 \neq n_2 d_2$ or m is fractional, those minima are antireflected for which the condition of overlapping of the minima of multiple-beam interference envelopes of each layer is satisfied.

Condition (5). A given condition is equivalent to the symmetrical one since the antireflection of the wave reflection minima can occur at integral and fractional values of optical layer thicknesses nd if the minima of multiple-beam interference envelopes of each of the layers are overlapping.

Condition (6). A reflection contour of the structures satisfying the given condition is of V- or W-type with a wide region of antireflection (Fig. 5), with the exception of the transition zone of the changing of spectral form $R(\lambda)$.

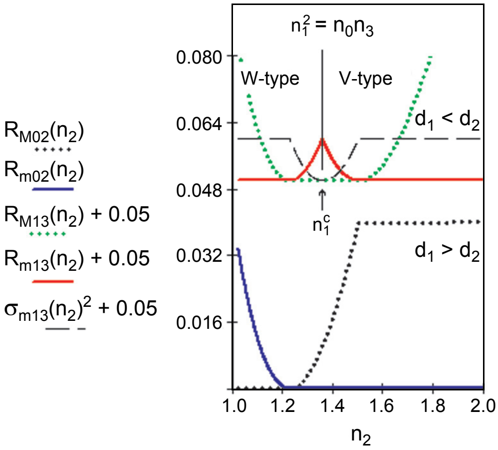

Fig. 5. The dependence of functions (9) under condition (6) on the refractive index of the internal layer for the cases $d_1 < d_2$ or $d_1 > d_2$, where the value n_1^c is determined from the equality $\sigma_{12} = \sigma_{23}$.

Condition (7). $n_3 = \sqrt{\frac{1}{4}\frac{n_2}{n_0 n_1^2}(n_0^2 + n_1^2)(n_1 + n_2) - \frac{n_2^3}{n_1}}$. The given relation between the refractive indices does not ensure the overlapping of the minima of multiple-beam interference envelopes of each layer, therefore it is inefficient from the point of view of antireflection coating.

3. Conclusions

Based on the results presented in the paper, the following may be concluded:

1. Full antireflection is achieved only at the points of intersection of the optical reflection contours by a single and binary interface and can be observed for optical thicknesses which are not necessarily a multiple of $\lambda/4$.

2. Two-layer media with the relation of the refractive indices $n_1^2 = n_0 n_3$, $n_0 n_2^2 = n_3 n_1^2$ have the widest range of light antireflection in the geometry of reflection.

3. Contrary to the opinion established in literature, antireflection contours of *V*- and *W*-types are characteristic of two-layer coatings.

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