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Simulation and design of a wideband *T*-shaped photonic crystal splitter

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In this paper, a high efficiency 2-D *T*-shaped photonic crystal beam splitter is proposed. It consists of a square lattice of GaAs rods ($n = 3.4$) embedded in air. The photonic crystal structure proposed can be used for 1550 nm wavelength, which is an important wavelength for optical fiber data transmission. Finite difference time domain (FDTD) simulation results demonstrate that a conventional *T*-junction can only provide 78% transmission coefficient (39% for each branch) for the incident light, while the proposed *T*-shaped splitter transmits over 90% of the incident light beam (over 45% from each branch) in the single mode region of waveguide. Especially it transmits nearly 98% (49% from each branch) of the input light in the wavelength of 1550 nm. In other words, the proposed devise shows higher beam splitting efficiency and a wider range of flatness of transmission power spectrum in comparison with previous works.

Keywords: photonic crystal, photonic band gap, beam splitter, *T*-junction, finite difference time domain (FDTD).

1. Introduction

During the last two decades many efforts have been made in order to gradually replace electronic integrated circuits with their optical counterparts. Minimizing the dimensions of electronic devices causes unwanted electronic signal interference, noise and cross-talks. So, that bottleneck and scale limitations in classical IC design become more critical. On the other hand, the improvement in semiconductor technology has brought about the possibility of optical devices playing the same role as electronic devices do in human life. In optical devices choosing an appropriate medium for light propagation will minimize the power loss. Photonic crystals are kind of new optical structures which provide suitable foundation for light propagation. These structures have attracted a lot of attention both in academic and industrial areas and they bring on the idea of the development of completely optical integrated circuits in the near future.

Photonic crystals (PCs) consist of two different dielectric materials which are arranged periodically (together). In other words, the refractive index alternates periodically in such medium. Depending on the number of dimensions in which the periodicity exists, PCs are categorized into one, two or three dimensionally periodic. One dimensional photonic crystals have the simplest geometry. They consist of layers of two different dielectric materials and therefore are also called multilayer films [1]. One of the first theoretical analyzes about one dimensional photonic crystals was published by Lord Rayleigh in 1887. For some ranges of frequency these crystals are able to act as a mirror; in this case they are called Bragg mirrors. Additionally, in case of defect being introducted into PC lattice, these crystals can localize the light modes. The unique properties of one dimensional photonic crystals make them beneficial in fabrication of dielectric mirrors or distributed feedback optical filters (DFB) [2]. The second photonic crystal category is the two dimensional types. The mirror reflection symmetry in these crystals causes the modes to be classified into two distinct polarizations, one is transverse electrical (TE) in which the electric field vector resides in the plane of periodicity and the magnetic field vector is perpendicular to the above mentioned plane, and the other one is transverse magnetic (TM) in which the directions of electric and magnetic fields are the opposite of TE condition. However, the actual properties of photonic crystals only exist in three dimensional types. Since the fabrication and analysis of these structures are difficult, therefore usually two dimensional photonic crystals are discussed. In fact, the two dimensional photonic crystals have most of the three dimensional PC properties in addition to simplicity of their fabrication. Thus, in this article, we also focus our attention on two dimensional category of photonic crystals.

The most remarkable specification of photonic crystals is their photonic band gap, which means that light with certain range of frequencies is not permitted to propagate inside the crystal. Many applications of photonic crystals – specially two and three dimensional types – depend on the location and width of their band gap. For instance, a crystal with a band gap can act as a narrow band filter, omitting all frequencies inside the gap or it can be utilized as a reflecting wall forming a resonant cavity for modes inside the gap. It is notable that the more the dielectric constant differs in photonic crystals the larger the band gap is [3].

Two dimensional photonic crystals have two basic topologies. The first one contains a dielectric substrate in which air holes are introduced periodically. The second one consists of dielectric rods embedded in air. Usually, the former topology has band gap for TE modes and the latter provides gaps for TM modes [4]. The distance between two adjacent holes or rods is called the lattice constant.

A pure photonic crystal inhibits light with frequencies in the range of band gap to propagate through the lattice. However, by introducing defects in crystal lattice various photonic devices can be created. For instance, by introducting a point defect in crystal, *i.e.*, eliminating a single hole or rod, a mode with frequency inside the band gap can be localized and as a result a cavity with high quality factor will be formed [1]. Removing a row of rods or holes in PC results in the formation of a linear defect which is utilized for creation of waveguides. The most important advantage of PC waveguides in comparison with the conventional ones is that the modes are able to propagate even through sharp 90 degree bends [5, 6]. Another beneficial property of these waveguides is that they can possess branches splitting the input incident light power into two output waveguides and forming power splitters which are utilized in many applications such as Mach–Zehnder interferometers [7]. This article mainly focuses on photonic crystal power splitters and the configurations which may improve the efficiency of these devices.

Photonic crystal power splitters with different configurations have been studied in the literature. In [8], a *T*-junction splitter is proposed which transmits only 45% of incident light at the wavelength of 1550 nm. A *T*-splitter design is also investigated in [9]. The transmission in the spectral range mentioned is not very flat for the structure in [9].

In this article, we intend to propose a topology with more efficient beam splitting and a wider range flatness of transmission power spectrum. Since this splitter consists of dielectric rods embedded in air, it is suitable for light with TM polarization. The splitter for TE polarized light was studied in our previous works [10]. In [10] a *Y*-branch is proposed which efficiently divides the propagated TE polarized light into two output ports. It is shown that by introducing three additional holes and increasing the radius of another hole the transmission bandwidth can be improved significantly. In comparison to previous topologies this topology is very simple yet it provides a transmission bandwidth equal to 92%.

In Section 2, we will review the numerical methods for analyzing photonic crystals. Specially, we describe plane wave expansion (PWE) and finite difference time domain (FDTD) methods in some detail. We mainly utilize these two methods in our analysis. The *T*-shaped design proposed to obtain more efficient beam splitter will be explained in Section 3, and the band structure for the so-called lattice will also be calculated.

2. Numerical analysis

There are various methods for analyzing and designing a PC, including plane wave expansion (PWE) method, scattering matrix method, finite difference time domain (FDTD) method and finite element method. The PWE method provides a contour map of frequencies called dispersion surface and is used to calculate energy bands. FDTD method is suitable for the analysis and design of the actual photonic crystals. Moreover, it is possible to analyze the time dependence of the optical pulse that propagates through a waveguide. As photonic bands can be calculated with a periodic boundary condition the FDTD method is considered to be one of the principal methods for analyzing photonic crystals along with the PWE method [12]. In this paper, we use PWE method to calculate the band structure of the crystal and then using FDTD method we analyze the transmission spectrum of the *T*-splitter. Here, we briefly describe PWE method.

2.1. The PWE method

The propagation of light in a photonic crystal is governed by four Maxwell equations. Consider an isotropic medium in which there are no sources of light so that ρ and J are equal to zero in Maxwell equations. Also assume that the dielectric constant has no frequency dependence. Finally, consider that the material is transparent so that $\varepsilon(r)$ is purely real and positive. With these considerations the four Maxwell equations will be in the following form:

$$\begin{aligned} \nabla \cdot H(r, t) &= 0 \\ \nabla \cdot \left[\varepsilon(r) E(r, t)\right] &= 0 \\ \nabla \times E(r, t) + \mu_0 \frac{\partial H(r, t)}{\partial t} &= 0 \\ \nabla \times H(r, t) - \varepsilon_0 \varepsilon(r) \frac{\partial E(r, t)}{\partial t} &= 0 \end{aligned} \tag{1}$$

Assuming $H(r, t)$ and $E(r, t)$ to be in the form of complex values as in Eq. (2) and substituting them into Eq. (1) we will finally obtain Eq. (3). Therefore, $H(r)$ and its corresponding frequencies can be found as follows:

$$H(r, t) = H(r)e^{-i\omega t}, \quad E(r, t) = E(r)e^{-i\omega t} \tag{2}$$

$$\nabla \times \left[\frac{1}{\varepsilon(r)} \nabla \times H(r)\right] = \left(\frac{\omega}{c}\right)^2 H(r) \tag{3}$$

According to Bloch's theorem the modes in a periodic structure can be written as:

$$\begin{aligned} \mathbf{H}(r) &= e^{\mathbf{k} \cdot \mathbf{r}} h(\mathbf{r}) \mathbf{e_k} \\ h(\mathbf{r}) &= h(\mathbf{r} + \mathbf{R_l}) \end{aligned} \tag{4}$$

where $\mathbf{R_l}$ is an arbitrary lattice vector and $\mathbf{e_k}$ is the unit vector perpendicular to the vector $\mathbf{k}$ and parallel to $\mathbf{H}$. Since ε and h are periodic functions we can write them as their Fourier expansion:

$$\varepsilon(r) = \sum_{\mathbf{G_i}} \varepsilon(\mathbf{G_i}) e^{i\mathbf{G} \cdot \mathbf{r}}, \quad \frac{1}{\varepsilon(r)} = \sum_{\mathbf{G_i}} \varepsilon^{-1}(\mathbf{G_i}) e^{i\mathbf{G} \cdot \mathbf{r}} \tag{5}$$

$$h(r) = \sum_{\mathbf{G_i}} h(\mathbf{G_i}) e^{i\mathbf{G} \cdot \mathbf{r}} \tag{6}$$

Therefore **H** will be written as:

$$\mathbf{H}(r) = \sum_{\mathbf{G},\lambda} h_{G,\lambda}\,\mathbf{e}_\lambda\, e^{i(\mathbf{k}+\mathbf{G})\mathbf{r}} \tag{7}$$

As Equation (7) demonstrates, **H** is written as the sum of plane waves, where $\lambda = 1, 2, ...$; **k** is the wave vector of the plane wave, **G** is the reciprocal lattice vector, $\mathbf{e}_\lambda$ represents the two unit axis perpendicular to the propagation direction ($\mathbf{k} + \mathbf{G}$); ($\mathbf{e_1}$, $\mathbf{e_2}$, $\mathbf{k} + \mathbf{G}$) are perpendicular to each other; $h_{G,\lambda}$ is the coefficient of the H component along the axes $\mathbf{e}_\lambda$.

Finally, substituting (5) and (7) in to (3) yields:

$$\sum_{\mathbf{G}'} |\mathbf{k}+\mathbf{G}|\,|\mathbf{k}+\mathbf{G}'|\,\varepsilon^{-1}(\mathbf{G}-\mathbf{G}')\begin{bmatrix}\mathbf{e}_2\cdot\mathbf{e}_2' & -\mathbf{e}_2\cdot\mathbf{e}_1' \\ -\mathbf{e}_1\cdot\mathbf{e}_2' & \mathbf{e}_1\cdot\mathbf{e}_1'\end{bmatrix}\begin{bmatrix}h_{1,G'} \\ h_{2,G'}\end{bmatrix} = \frac{\omega^2}{c^2}\begin{bmatrix}h_{1,G} \\ h_{2,G}\end{bmatrix} \tag{8}$$

This is a matrix showing the relation between ω and k. This equation is a standard eigenvalue problem and it can be solved using numerical methods. The number of plane waves required to achieve adequate accuracy depends on structural details of the unit cell. When high accuracy is required for higher frequency ranges or when the atom structure is complicated, the number of plane waves should be increased [13].

3. The proposed beam splitter design

In our previous work [10, 11], we used a structure with a triangular lattice of air holes embedded in GaAs slab. As we have mentioned before, it is the most popular structure for TE polarization. We choose the radius of holes to be $0.3a$, where a is the lattice constant. Figure 1 shows our *Y*-branch topology proposed as well as its transmission power spectrum for the TE polarized incident light.

The input and output powers are usually measured using Poynting vector method, which is defined as follows:

$$S(r, t) = E(r, t) \times H(r, t)^* \tag{9}$$

where E and H are the electric and magnetic fields and r is the space coordinate. If the vector S is numerically calculated over a surface, the real section of the complex Poynting vector, *i.e.*, $\mathrm{Re}(S(t))$, determines the flow of energy through the surface as a function of time. Using discrete-time Fourier transform (DFT), the time-domain data can be mapped into the frequency domain to determine the spectrum of the power flow. In order to do so a sinusoidal pulse with a Gaussian envelope is used as input excitation. This pulse propagates through the waveguide, reaches the end of the waveguide and vanishes in the PML boundary condition section. The real section of the Poynting vector $S(t)$ is calculated somewhere across the waveguide. The ratio of output Fourier

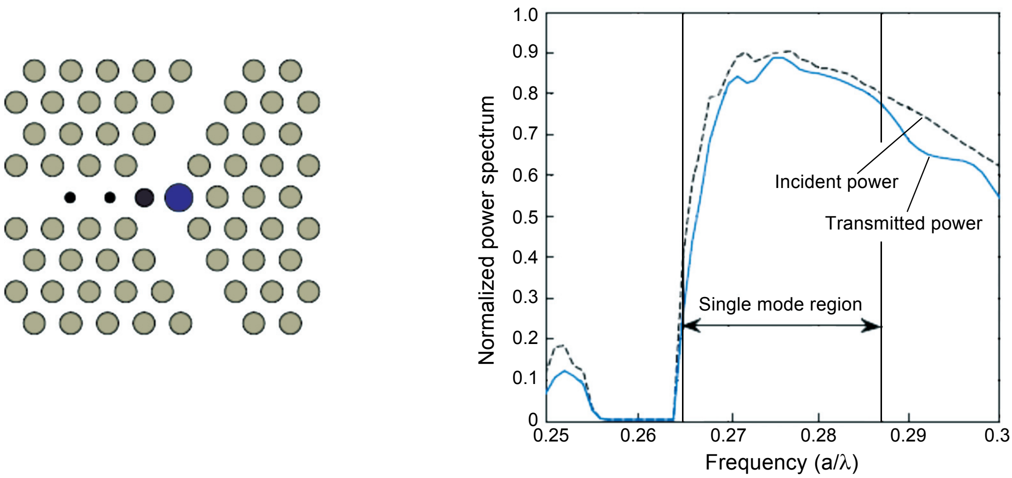

Fig. 1. *Y*-branch topology for TE polarized incident light and its corresponding transmission spectrum [10].

transform to the Fourier transform of the input pulse can determine the transmission power spectrum.

In designing beam splitters for integrated circuits the most important key factor is the quality of optical bend and junctions. High transmission coefficients and low reflection and loss factors are very critical in this case. A PC bend can be obtained by introducing line defects in crystal. In [14, 15], a waveguide bend design is proposed which is a two dimensional square lattice of rods with refractive index of 3.4 embedded in air. GaAs has a refractive index of 3.4 at the wavelength of 1550 nm. This wavelength is one of the most important wavelengths in photonics, since the absorption of light in conventional optical fibers is the least at this wavelength and therefore it is useful for optical fiber data transmission. This article mentions that using rods with radius of $0.18a$ (where a is the lattice constant) in the crystal structure results in the transmission of light to be more than 90% and the reflection to be only about 8%. Therefore, we may also use this ratio for the radius of rod in our splitter.

Figure 2 demonstrates the 2D PC lattice used for designing the splitter. The circles represent the GaAs rods, whose radii are $0.18a$. Using PWE method, the band structure TM modes of the lattice in Fig. 2 is calculated and demonstrated in Fig. 3. As it is illustrated, this lattice provides a wide band gap for TM modes in range of a/λ ratios between 0.3 to 0.44.

Since we are interested in transmitting and splitting light with wavelengths of about 1550 nm, we choose the lattice constant to be $a = 644.8$ nm. Therefore, the a/λ ratio for $\lambda = 1550$ nm would be 0.416 and it resides within the gap. As the radius of rods r is related to a by $r = 0.18a$ its value would be 116.06 nm. An ideal splitter should divide the incident light beam into two distinct sections with 50% of the incident power travelling in each direction (100% transmission) and zero reflection for all the frequency range of the corresponding waveguide [10].

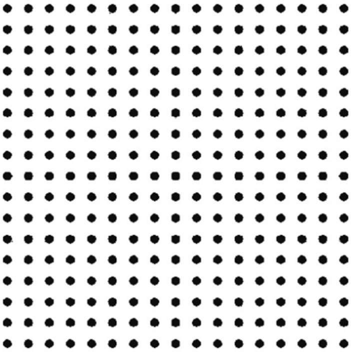

Fig. 2. Schematic of 2D PC with square lattice of GaAs rods in air.

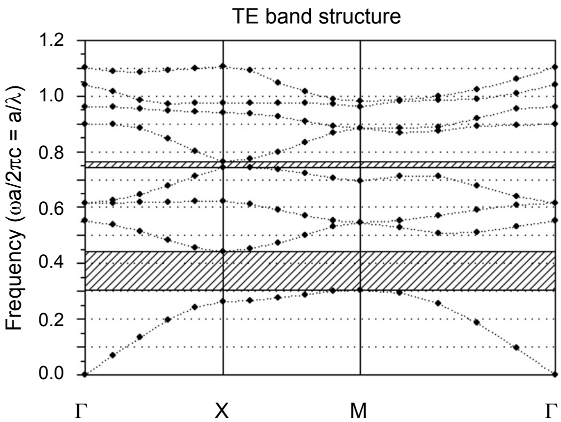

Fig. 3. Band structure for a PC comprising a square lattice of GaAs rods with radius equal to 0.18*a* which are embedded in air.

The simplest geometry which can be used as a *T*-junction is shown in Fig. 4. FDTD simulation results show that the junction depicted in Fig. 4 can only provide 78% transmission (39% for each branch) for the incident light beam and the rest of the light is reflected. Therefore, this simple *T*-junction may not be suitable enough for efficient integrated optics applications.

The structure proposed in this article is depicted in Fig. 5. In comparison with the simple *T*-junction (Fig. 4) the radius of rods in the vicinity of the input waveguide is reduced gradually from left to right. An optimization method is used for determining the radius of the rods. The method mentioned takes the radius of the rods as input variables and calculates the bandwidth for each structure. The bandwidth is considered to be the region where the transmission coefficient exceeds 0.9 (0.45 for each output port).

The optimization algorithm for this particular problem acts as follows: first, an initial value for each of the variables is guessed. It can be either based on previous observations or the designer insight. Each of the variables is in turn swept in its permissible range while the others are kept constant. The bandwidth is calculated for

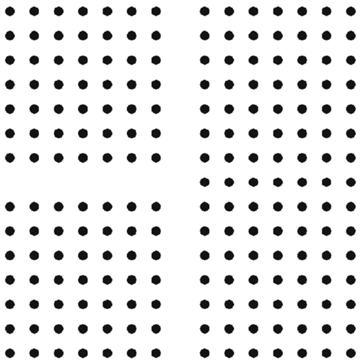

Fig. 4. A simple *T*-junction.

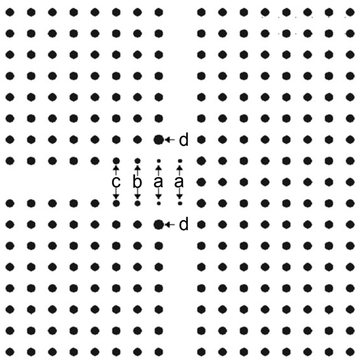

Fig. 5. The proposed *T*-shaped photonic crystal topology for TM polarized incident light.

each of these states to determine the optimum value for the variable (which maximizes the bandwidth). After the optimum value is chosen for the first variable, this process is repeated for the next one. After a couple of iterations an optimum result will be obtained. The radius of rods labeled *a* in Fig. 5 is 50 nm, the radius of the rods labeled *b* is 78 nm and the rods labeled *c* have the radius of 92 nm. Two additional rods with radius of 50 nm are introduced to the structure; one at the beginning of each branch at the *T*-junction. In addition, the radius of two rods labeled *d* is increased to 145 nm. This modification of the radius of rods ensures the least reflection of incident light back into the input port.

The splitter can be considered to be comprised of three waveguides and a junction section. The quality of transmission depends on the strength of coupling between the waveguides and the junction section. A fine modeling method has been proposed in [16] which can provide a physical insight on how to improve the transmission efficiency. In a typical photonic crystal waveguide obtained by removing a column of rods in a square lattice of dielectric cylinders, introduction of additional rods in the waveguides results in the reduction of group velocity of the propagation mode. Simulation results show that adding extra rods in the junction section can enhance the coupling strength. The radius of these rods is determined from the optimization process.

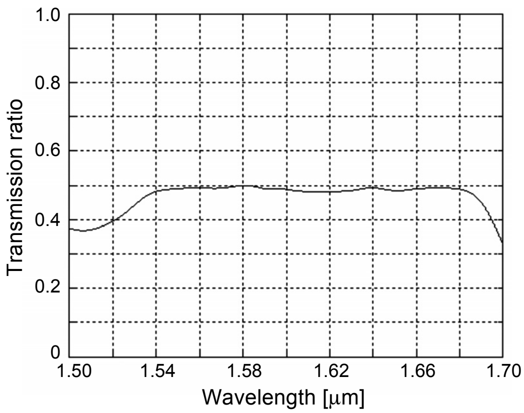

Fig. 6. Transmission power spectrum for the *T*-splitter proposed.

Figure 6 shows the transmission power spectrum of the *T*-splitter proposed. The transmission ratio for each branch in the wavelength region from 1530 to 1685 nm is not less than 45%. Especially this splitter transmits nearly 98% (49% from each branch) of the input light in the wavelength of 1550 nm.

4. Conclusions

In this paper, we proposed an improved *T*-junction beam splitter for TM polarizations. It was shown that by gradually reducing the radius of rods near the junction and introducing two additional rods into the junction the transmission ratio of the incident light will be more than 45% for wavelengths of 1530 to 1685 nm. Especially this beam splitter provides nearly 50% splitting for the most functional wavelength of 1550 nm. Also the light reflection of this structure in the aforementioned wavelength range is not above 8%. In comparison with previous works this structure shows more efficient beam splitting and a wider range of flatness of transmission power spectrum.

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