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Light propagation in thermally expanded core fibers with graded-index

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Light propagation is analyzed in thermally expanded core (TEC) fibers with graded-index profile. Used as power mixers among others, their core structure at the boundary between the heated and non-heated regions is represented by linear taper. Ray optics is used as the transverse taper dimensions are large relative to the wavelength of propagating light. Trajectories of meridian rays are derived analytically. Numerical results presented show ray trajectories as functions of the position within the taper and taper slope. These are modulated sinusoidal functions whose amplitude and period rise with the taper radius. Both, bound and leaky rays have been examined.

Keywords: optical fibers, geometric optics approach; thermally diffused expanded core fibers; ray trajectories; linearly taper model.

1. Introduction

Because of the complexity of Maxwell's and derivative equations describing light propagation through optical waveguides, simpler but approximate solutions are often sought. Asymptotic theory, with ray optics as its main component, has been found to be adequate for most aspects of light propagation in step- and graded-index optical waveguides (*e.g.*, [1]). A comparison of wave and ray techniques for solving graded-index (GI) optical waveguide problems is presented in [2]. As a wide range of problems involving GI waveguides cannot be solved analytically, techniques of ray optics elaborated in [2] are often attempted. Ray optics has attracted the attention of researchers because its applications for various types of optical fibers have made it possible to calculate ray-path parameters and analyze the ray temporal dispersion. Application of geometric (ray) optics to investigate the ray dispersion, light power acceptance properties of multi-step fibers, and coupling losses for multi-step index plastic optical fibers have been reported [3].

Tapered dielectric structures maximize the coupling of light into optical fibers and integrated-optic devices or waveguides in general. With the advancement of optical communication networks and optical switching and fiber sensing systems, the thermally diffused expanded core (TEC) fibers [4–6] have gained in prominence. TEC fibers have an expanded mode field diameter obtained by heating a step-index single-mode fiber locally at high temperature (of approximately 1300 °C to 1650 °C) and diffusing the germanium dopant into the core. The expansion rate of the core depends on the heating temperature and duration and dopant intensity in the fiber core. When a TEC fiber is applied to low-connection loss connectors and laser diode modules, the enlarged mode field diameter is an important factor which is related to fabrication time and cost. Theoretical and experimental analysis of TEC fiber characteristics has been reported [7]. Methods for analyzing the propagation, modal and coupling characteristics of same fibers have been elaborated [8, 9].

In this paper, the propagation is described of meridian rays in TEC fibers using a geometric optics approach. Ray trajectories in the expanded core region of the fiber are estimated. Using the model of linear taper with small angles, the analytical solution for trajectories of meridian rays is obtained. This solution describes the ray path in terms of sinusoidal functions with amplitudes proportional to the taper core radius. Attention is given to rays that remain bound to the core region for a specified length of taper.

In terms of the structure of the paper, we first present briefly the refractive-index profile of a TEC fiber under isotropic thermal diffusion in order to explain the linear taper model. We then use the general equation for the ray path in the medium of refractive index n in order to obtain an equation of ray trajectories. Finally, the propagation of merdian rays in TEC fibers is described and numerical simulation of the ray trajectories is presented.

2. Linearly taper model for TEC fibers

Longitudinal view of a graded-index tapered core of thermally diffused expanded core fibers is shown in Fig. 1 in references [8] and [9]. Following the formalism used in ref. [9], the variation of the refractive index profile under conditions of isotropic thermal diffusion has the form

$$n^2(r) = n_1^2 \left[1 - 2\varDelta f(r) \right] \tag{1}$$

where $\varDelta = (n_1^2 - n_2^2)/2n_1^2$ is the index profile height, n_1 is the maximum core refractive index (on the taper axis), n_2 is the refractive index of the cladding layer, $f(r) = 1 - (a^2/A^2)\exp[-(r^2/A^2)]$ while a is the radius of the input end of taper, and A is radius of the larger end of taper. A feature of TEC fibers is that the dopants in the core are thermally diffused into the radial direction as the core expands. Variation of the refractive index profile of a TEC fiber as a function of heating time is given in Fig. 2 in ref. [8].

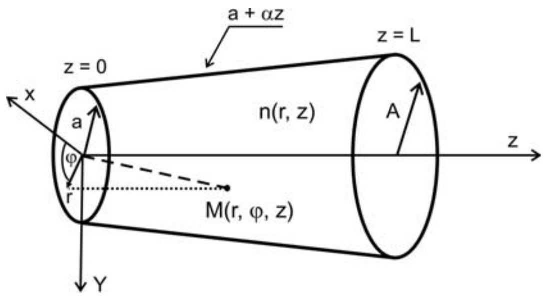

Fig. 1. Geometry and dimensions of the linear taper.

We assume that the thermally diffused expanded core fibers have a slow change in non-uniformity along the fiber. If α is the slope of the taper, the variable taper's radius is defined as $A(z) = a + \alpha z$. In this manner, the refractive profile could be approximated by

$$n^2(r, z) \cong \begin{cases} n_1^2 \left[1 - 2\Delta \dfrac{a^2 r^2}{(a + \alpha z)^4} \right] & r < a + \alpha z \\ n_1^2 \left[1 - 2\Delta \right] = n_2^2 & r > a + \alpha z \end{cases} \tag{2}$$

where the approximation $\exp(-nx) \cong (1 - nx)$ was used. In this manner, the TEC fiber considered is modeled as linearly tapered graded-index fiber as shown in Fig. 1. For tapered graded-index fibers, the cladding in the region $r > a + \alpha z$ is a homogeneous medium with refractive index $n_2 = n(a, 0) = n(a + \alpha z, z)$. For the taper of length L, the core radius increases linearly from a at $z = 0$ to $A = (a + \alpha L)$ at the larger end of the taper. It is assumed that both Δ and α are much smaller than unity. It is also assumed that A is much larger than the wavelength of light propagating in the fiber, allowing us to use ray-optics in studying the propagation of light trough the tapered region of the fiber.

3. Rays in graded-index linearly tapered fiber: analytical solution

The general vector form of equation for the ray path in the medium of refractive index n is [10]

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n \frac{\mathrm{d}\mathbf{R}}{\mathrm{d}s} \right) = \nabla n \tag{3}$$

where s is the distance measured along the ray path and $\mathbf{R}$ is the position vector for a point on the ray path as shown in Fig. 2. This equation can be regarded as a generalization of Snell's law. It can be derived in more than one way [11]. The ray equation describes the beam trajectory in terms of the position vector $\mathbf{R}$ along the ray measured from some starting point. In this analysis, propagation properties of multimode

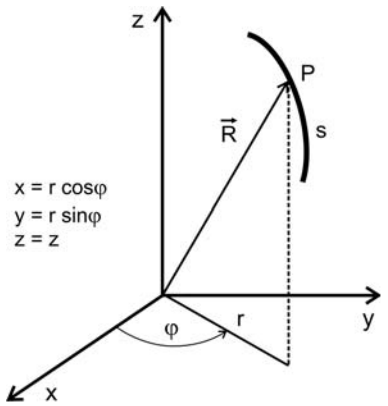

Fig. 2. Ray trajectory; s is distance measured along the ray trajectory, r, φ, z are cylindrical coordinates, $P(\mathbf{R}) = P(x, y, z) = P(r\cos\varphi, r\sin\varphi, z)$.

Gaussian index profile linear taper are investigated using geometric optics on which the Eq. (3) is based.

Since the transverse dimensions of the tapers are assumed to be large compared with the wavelength of light, the ray optics approach is sufficiently accurate, yet simple for the description of light propagation within the taper. Consider a tapered dielectric waveguide with geometry shown in Fig. 1. This waveguide serves as a simplified model for TC fibers with graded-index refractive profile. The parameter $\Delta \ll 1$ is the index profile height, n_1 is the maximum core index and r, z represent the cylindrical radial and longitudinal coordinates, respectively. This waveguide is tapered in such way that the core radius increases linearly in the z direction from its initial value of a to a final value of A over a length L. The core-cladding boundaries of the taper form a plane defined by $A(z) = a + \alpha z$ where $\alpha = (A - a)/L$ is the slope of the taper. It is assumed that the taper angle is small so that $\tan^{-1}\alpha \approx \alpha \ll 1$. Using this model in the ray equation, ray trajectories of merdian rays are investigated. In the vector form (3), ray equation is independent of any particular choice of coordinate system. In Cartesian coordinates, it can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}x}{\mathrm{d}s}\right) = \frac{\partial n}{\partial x} \tag{4a}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}y}{\mathrm{d}s}\right) = \frac{\partial n}{\partial y} \tag{4b}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}z}{\mathrm{d}s}\right) = \frac{\partial n}{\partial z} \tag{4c}$$

For applications involving optical fibers, ray equation in cylindrical coordinates must be known. The transformation from Cartesian coordinates (x, y, z) to cylindrical coordinates (r, φ, z) is accomplished by the following transformations

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

$$z = z \tag{5}$$

$$r = (x^2 + y^2)^{1/2}$$

$$\varphi = \arctan(y/x)$$

The partial derivatives of n with respect to x and y may be expressed as

$$\frac{\partial n}{\partial x} = \frac{\partial n}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial n}{\partial \varphi}\frac{\partial \varphi}{\partial x} = \frac{\partial n}{\partial r}\cos\varphi - \frac{\partial n}{\partial \varphi}\frac{\sin\varphi}{r} \tag{6}$$

$$\frac{\partial n}{\partial y} = \frac{\partial n}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial n}{\partial \varphi}\frac{\partial \varphi}{\partial y} = \frac{\partial n}{\partial r}\sin\varphi + \frac{\partial n}{\partial \varphi}\frac{\cos\varphi}{r} \tag{7}$$

and the derivatives of x and y with respect to s become

$$\frac{\mathrm{d}x}{\mathrm{d}r} = \frac{\partial r}{\partial s}\cos\varphi - r\frac{\partial \varphi}{\partial s}\sin\varphi \tag{8}$$

$$\frac{\mathrm{d}y}{\mathrm{d}r} = \frac{\partial r}{\partial s}\sin\varphi + r\frac{\partial \varphi}{\partial s}\cos\varphi \tag{9}$$

Using Eqs. (5) to (9), the ray equation can be derived in cylindrical coordinates. Listed below are r, φ and z in cylindrical coordinates of the ray equation:

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}r}{\mathrm{d}s}\right) - nr\left(\frac{\mathrm{d}\varphi}{\mathrm{d}s}\right)^2 = \frac{\partial n}{\partial r} \tag{10a}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(nr^2\frac{\mathrm{d}\varphi}{\mathrm{d}s}\right) = \frac{\partial n}{\partial \varphi} \tag{10b}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}z}{\mathrm{d}s}\right) = \frac{\partial n}{\partial z} \tag{10c}$$

For the meridian rays ($\mathrm{d}/(\mathrm{d}\varphi) = 0$) of the TEC fibers with refractive index profile given by (2), the Eqs. (10) are summarized as follows:

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n\frac{\mathrm{d}r}{\mathrm{d}s}\right) \approx -\frac{2n_1a^2\Delta r}{(a+\alpha z)^4} \tag{11a}$$

$$\frac{d\varphi}{ds} = 0 \tag{11b}$$

$$\frac{d}{ds}\left(n\frac{dz}{ds}\right) = \frac{4n_1\Delta\alpha a^2 r^2}{(a+\alpha z)^5} \approx 0 \tag{11c}$$

It is noted that the z component of ∇n is of order $\alpha\Delta$, and its r component is of the order Δ, which is much larger than the z component for the tapers with small slope ($\alpha \ll 1$). Hence, the right-hand side of Eq. (11c) is set to zero in order to determine the solution for the ray trajectories. After this assumption is made, this equation becomes

$$\frac{d}{ds}\left(n\frac{dz}{ds}\right) = 0 \tag{12}$$

Integrating both sides of Eq. (6), it is obtained that

$$n\frac{dz}{ds} = \text{const} = \gamma \tag{13}$$

Equation (7) is Snell's law derived from ray optics. For the case of $\Delta \ll 1$ the assumption $\gamma \approx n_1$ can be used. Using the result (13) and noting that $d/ds =$ $= (d/dz)(dz/ds) = (\gamma/n)(d/dz)$, the first equation in the set (11) becomes

$$\gamma^2\frac{d^2r}{dz^2} - \frac{1}{2n}\frac{\partial n^2}{\partial r} = 0 \tag{14}$$

Substituting for $n(r, z)$ from Eq. (2), Equation (14) reduces to

$$(a+\alpha z)^4\frac{d^2r}{dz^2} + \frac{2n_1^2 a^2\Delta}{\gamma^2}r = 0 \tag{15}$$

In order to solve (15), a new variable ρ, such that $\rho = (a + \alpha z)/a$ is introduced. Then, in terms of ρ, Eq. (9) becomes

$$\rho^4\frac{d^2r}{d\rho^2} + \omega r = 0 \tag{16}$$

where $\omega = \dfrac{2n_1^2\Delta}{\alpha^2\gamma^2}$. For solving Eq. (16), we used the transformation $\rho = 1/t$. Equation (16), in terms of t, is then expressed as

$$\frac{d^2r}{dt^2} + \frac{2}{t}\frac{dr}{dt} + \omega r = 0 \tag{17}$$

Next, we state that $r = u/t$, where u is a new function of t. Then, Eq. (17) becomes

$$u'' + \omega u = 0 \tag{18}$$

The solution of (18) is readily available as $u = c_1 \sin(\sqrt{\omega}\, t) + c_2 \cos(\sqrt{\omega}\, t)$. After some arithmetic, the solution of Eq. (15) is obtained as

$$r(z) = C_0 \frac{a + \alpha z}{a} \sin\left[\sqrt{\frac{2 n_1^2 \Delta}{\alpha^2 \gamma^2}} \frac{a}{a + \alpha z} + \theta_0\right] \tag{19}$$

where C_0 and θ_0 are constants of integration. These constants can be determined by using the initial ray conditions at $z = 0$ for r and $\mathrm{d}r/\mathrm{d}z$. If we assume that at $z = 0$, $r = 0$ and $\mathrm{d}r/\mathrm{d}z = \alpha_0$, the constant C_0 and θ_0 are obtained in the following form:

$$\theta_0 = -\sqrt{\omega} \equiv -b$$

$$C_0 = \pm \frac{a \alpha_0 \left[- b \cos\left(b^2\right) + b \sin(b) \sqrt{\sin\left(b^2\right)}\right]\left[\sin(b) - \sqrt{\sin\left(b^2\right)}\right]}{b \alpha \left[b \cos(2b) - \sin(2b)\right]}$$

The solution (19) together with $\varphi = \varphi_0 = \mathrm{const}$ completely describe the ray trajectories of meridian rays in the tapered region of TEC fibers.

The solution (19) indicates that the ray trajectories of meridian rays are described by sinusoidal functions. Their amplitude and period increase as rays propagate from smaller to larger end of the taper. It is emphasized that (19) describes ray trajectories of meridian rays in the core of the fibers only. If a ray reaches the core-cladding boundary of the taper, it will enter and remain in the cladding region travelling straight along and away from the boundary. Such rays are considered leaky. They contribute to the radiation loss of the fiber. In order for rays to remain bound to the core over the entire length of the tapper, the condition $|r| < (a + \alpha z)$, $z \le L$ must be satisfied. This condition is met if $L \le z_0$ where the z_0 is the smallest positive solution of $|r(z_0)| = = a + \alpha z_0$.

Figure 3 shows trajectories of meridian rays with the slope of 0.0004, $a = 4$ μm, $A = 8$ μm, $n_1 = 1.50$, $n_2 = 1.48$, $\Delta = 0.27$ and $L = 10$ mm for $\varphi = 0$. It should be noted that the ray in Fig. 3 is fully bound to the core region throughout the entire length of the taper.

Figure 4 shows trajectories of a meridian ray with the slope of 0.0004, $a = 4$ μm, $A = 15$ μm, $n_1 = 1.492$, $n_2 = 1.48$, $\Delta = 0.00804$ and $L = 10$ mm for the $\varphi = 0$. For rays to be bound to the core throughout the taper, it is required that Eq. (19) satisfy the condition $|r| < a + \alpha z$. This condition is met if $L \le z_0$, where z_0 is the smallest positive solution of $|r(z_0)| = a + \alpha z_0$. Figure 4 shows also that the ray is bound for a portion of the taper. It leaves the core and enters into the cladding region at $z_0 \approx 500$ μm, it is thus considered to be a leaky ray.

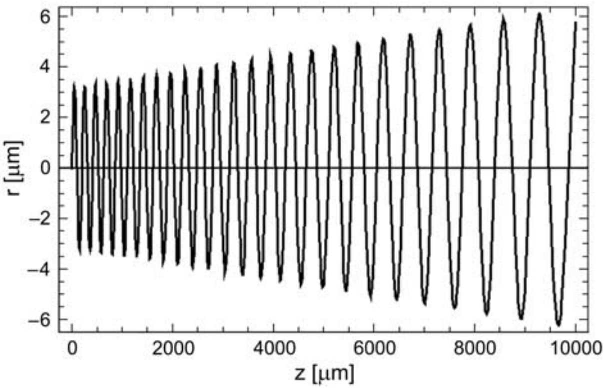

Fig. 3. Trajectory of meridian rays in graded-index TEC fibers (slope 0.0004, $a = 4$ μm, $A = 8$ μm, $n_1 = 1.50$, $\Delta = 0.27$ and taper length $L = 10$ mm).

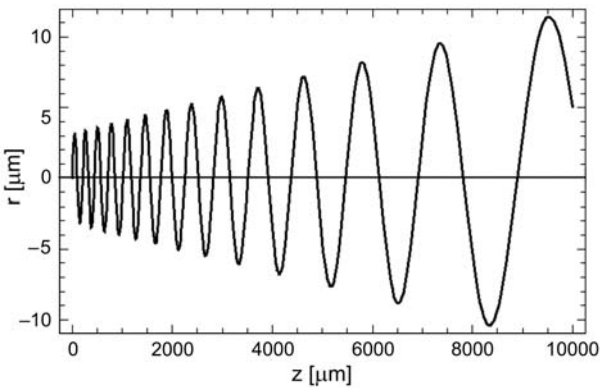

Fig. 4. Trajectory of meridian rays in graded-index TEC fibers (slope 0.0004, $a = 4$ μm, $A = 15$ μm, $n_1 = 1.492$, $\Delta = 0.00804$ and taper length of $L = 10$ mm).

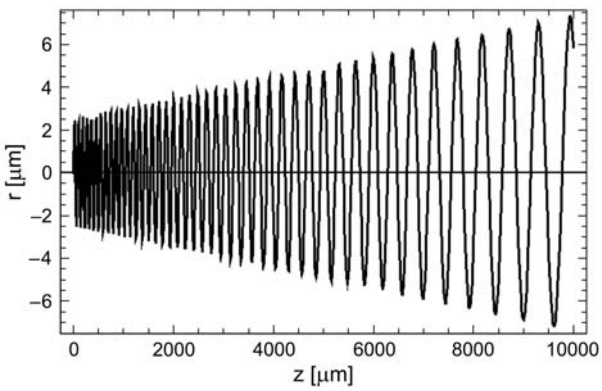

Fig. 5. Trajectory of meridian rays after heat treatment of 10 h (TEC fiber with $D = 3.9\times10^{-16}$ m^2/s, $n_2 = 1.46$ and $\Delta = 1.25\%$).

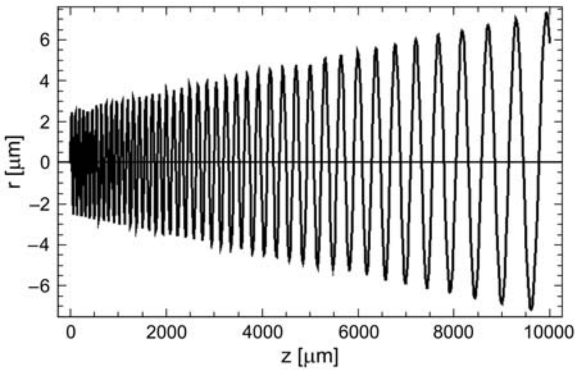

Fig. 6. Trajectory of meridian rays after heat treatment of 6 h for a TEC fiber with $D = 3.9\times10^{-16}$ m^2/s, $n_2 = 1.46$ and $\Delta = 1.25\%$.

Following the formalism used in the study [8, 9], we studied the ray trajectory in the TEC fiber as a function of heating time. After the heat treatment of 10 h for $D = 3.9\times10^{-16}$ m^2/s, for the fiber with $a = 2$ μm, $n_2 = 1.46$ and $\Delta = 1.25\%$, the ray trajectory of meridian rays is shown in Fig. 5. After the heat treatment of 10 h, the maximum value of A is around 7.494 μm. It can be observed in Fig. 5 that the rays are fully bound to the core region throughout the entire length of the taper.

For a TEC fiber with $D = 3.9\times10^{-16}$ m^2/s, $a = 2$ μm, $n_2 = 1.46$ and $\Delta = 1.25\%$, Fig. 6 shows the calculated trajectory of meridian rays after a 6 h heat treatment. It is obvious that the ray in Fig. 6 is not fully bound to the core region throughout the entire length of the taper. These results indicate that, for specific initial conditions, the taper length depends on the duration of the heat treatment. As the duration is increased, larger length of the taper becomes possible. It should be mentioned that rays analyzed entered the taper region at small angle with respect to the axis as the approximation of weakly guiding fiber was used.

4. Conclusions

A graded-index thermally expanded core (TEC) fiber is analyzed. Ray optics is used as the transverse taper dimensions are large relative to the wavelength of propagating light. A linear taper model for analysis of TEC fiber is proposed. This model describes the behavior of meridian rays in tapered region of TEC fibers. For small angles consistent with the approximation that the fiber is weakly guiding, an analytical solution for the trajectory of meridian rays is obtained. The solution describes the ray trajectory by a sinusoidal function whose amplitude and period rise with the taper-core radius. Both bound and leaky rays have been examined. Ray trajectories of meridian rays have been calculated for two sample cases of TEC fibers. The proposed model may be used to determine conditions for rays to remain bound to the core region throughout the taper length. The function of TEC fiber suggests a possible application of the taper as a power mixer.

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References

[1] ANKIEWICZ A., PASK C., *Geometric optics approach to light acceptance and propagation in graded index fibres*, Optical and Quantum Electronics **9**(2), 1977, pp. 87–109.

[2] ANKIEWICZ A., *Comparison of wave and ray techniques for solution of graded index optical waveguide problems*, Optica Acta **25**(5), 1978, pp. 361–373.

[3] ZUBIA J., ALDABALDETREKU A., DURANA G., ARRUE A., BUNGE C.-A., POISEL H., *Geometric optics analysis of multi-step index optical fibers*, Fiber and Integrated Optics **23**(2), 2004, pp. 121–156.

[4] HANAFUSA H., HORIGUCHI M., NODA J., *Thermally-diffused expanded core fibres for low-loss and inexpensive photonic components*, Electronics Letters **27**(21), 1991, pp. 1968–1969.

[5] HAIBARA T., NAKASHIMA T., MATSUMOTO M., HANAFUSA H., *Connection loss reduction by thermally-diffused expanded core fiber*, IEEE Photonics Technology Letters **3**(4), 1991, pp. 348–350.

[6] STONE J., STULZ L.W., MARCUSE D., BURRUS C.A., CENTANNI J.C., *Narrow-band FiEnd etalon filters using expanded-core fibers*, Journal of Lightwave Technology **10**(12), 1992, pp. 1841–1854.

[7] KIHARA M., MATSUMOTO M., HAIBARA T., TOMITA S., *Characteristics of thermally expanded core fiber*, Journal of Lightwave Technology **14**(10), 1996, pp. 2209–2214.

[8] KLIROS G. S., TSIRONIKOS N., *Variational analysis of propagation characteristics in thermally diffused expanded core fibers*, Optik: International Journal for Light and Electron Optics **116**(8), 2005, pp. 365–374.

[9] DIVARI P.C., KLIROS G.S., *Modal and coupling characteristics of low-order modes in thermally diffused expanded core fibers*, Optik: International Journal for Light and Electron Optics **120**(5), 2009, pp. 222–230.

[10] BORN M., WOLF E., *Principles of Optics*, 4th Edition, Pergamon Press, London 1970.

[11] KRUEGER D.A., *Spatial varying index of refraction: an open ended undergraduate topic*, American Journal of Physics **48**(3), 1980, pp. 183–188.

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