

Microwave properties of the generalized Fibonacci quasi-periodic multilayered photonic band gap structure

YOUSSEF TRABELSI, MOUNIR KANZARI*, BAHRI REZIG

Photovoltaic and Semiconductor Materials Laboratory, El-Manar University, Tunis, Tunisia

*Corresponding author: mounir.kanzari@ipeit.rnu.tn

The transmission properties in microwave domains (10 GHz to 40 GHz) of generalized dielectric Fibonacci multilayer generated by the rule $S_{l+1} = S_l^m S_{l-1}^n$; with a pair of positive integers m and n were studied. The initial generations of generalized Fibonacci sequence are taken as follows: $S_0 = L$ and $S_1 = H$, where H and L are two elementary layers with refractive indices $n_L = 1$ (air) and $n_H = 3$ (ceramic). The so-called “trace map method” was used to simulate the transmission spectra of the multilayer structures at normal incidence. Based on the representation of the transmittance spectra in the microwave range an analysis depending on the pair (n, m) is presented. It has been shown that the reflection bands of the proposed quasi-periodic structure could cover the whole spectral range. By comparison, it is impossible to reach this result by using the periodical multilayer structure.

Keywords: generalized Fibonacci multilayer, photonic crystal, Fibonacci dielectric multilayer, microwave band gap structure.

1. Introduction

A great deal of attention has been devoted to photonic crystals (PC), because of their attractive properties and technological application variety, precisely in the microwave domains. They are an artificial material made from periodic arrays of dielectric or metallic building blocks.

Recently, some new periodic structures, such as photonic and electromagnetic band gap structures, have been applied widely to microwave devices [1, 2]. The existence of photonic band gaps (PBG) has brought about an unprecedented power to control and manipulate the propagation of electromagnetic waves [3–5].

This situation is similar to that in semiconductor crystals where the propagation of electron is forbidden in certain energy regions (band gap). The appearance of the band gap can be explained by the concepts of interference and dielectric potential. PBG structures can be one-, two- or three-dimensional periodic structures. The simplest form of a photonic crystal is the one-dimensional periodic structure such as the Bragg mirror [6].

On the other hand, great interest has been observed as regards the properties and applications of one-dimensional spatially periodic, quasi-periodic and random PBG structures [7]. Quasi-periodic systems can be considered as suitable models for describing the transition from the perfect periodic structure to the random structure [8, 9].

Various studies have been based on multilayer systems built recursively according to the Fibonacci sequence, such as those by MACIA [10], who presents an analysis of wave transmission through Fibonacci dielectric multilayer (FDM) structures and demonstrates that they can be used as reflectors. Some of these works were focused on studying the localization of light waves within Fibonacci quasi-periodic multilayer structures in order to create photonic band gaps similar to those existing in periodic structures and developed the omnidirectional band gap [11–14].

The aim of this work is to study, at normal incidence, the transmission properties in microwave domains [10 GHz, 40 GHz] of the one-dimensional multilayer system built according to the generalized Fibonacci sequence. We calculate transmission spectra through these structures using the trace map method. From the numerical results, it has been found that the transmission bands of the quasi-periodic sequence structures can cover the full spectral range by increasing the parameter n and fixing m to 1 or *vice versa*. In addition, extra multi-narrow bands can be obtained and controlled by adjusting the parameter $m = 2n$ or $n = 2m$ from the 3rd generalized Fibonacci sequence. Using the proposed analysis, multi-stop band filters in the microwave spectral domains can be easily designed.

2. Fibonacci model

The generalized Fibonacci sequences are a class of quasi-periodic lattices generated by the substitution rules: $L \rightarrow H^m$ and $H \rightarrow H^m L^n$, where m and n are all positive integers. They can be generated by a recursive relation [15]:

$$S_{l+1} = S_l^m S_{l-1}^n \quad (1)$$

Based on the characteristics of the construction of generalized Fibonacci sequences, we consider the matrices of light propagating through the GF(m, n) multilayer of

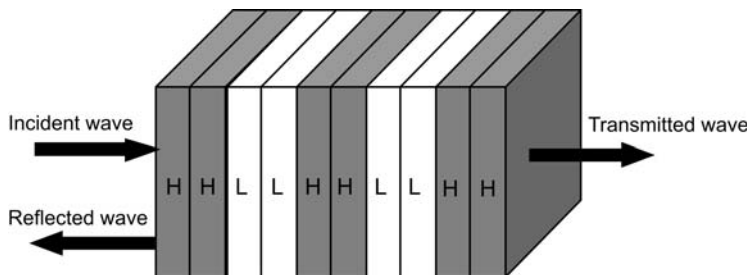


Fig. 1. Fibonacci-class quasi-periodic multilayer stack ($l = 3, m = 2, n = 2$).

the 1-st generation S_l which is sandwiched by two material media types L and H . Figure 1 shows the 3-rd generation of one-dimensional generalized Fibonacci class quasi-periodic multilayer stacks for $m = 2$ and $n = 2$. According to Fibonacci rule, the structure contains 10 layers, as shown in Fig. 1.

3. Transmittance spectra through generalized Fibonacci multilayer

The transmission spectra of electromagnetic radiation through the multilayer periodic and aperiodic systems were widely studied by various methods such as the transfer matrix method [16]. The interesting and representative models of Fibonacci-class (FC(n)) and generalized Fibonacci (GF(m, n)) have been extensively reported by KLAUZER-KRUSZYNA *et al.* [17, 18], who studied the polarized light propagation through optical generalized Fibonacci superlattices.

We use the trace map method to investigate the transmission spectra through the generalized Fibonacci multilayer. The trace-map technique [19] has proven to be a powerful tool to investigate the properties of various aperiodic systems.

The transfer matrices A_l used in the trace-map technique are written as [11]:

$$\begin{aligned} A_1 &= P_{ab} P_b P_{ba} \\ A_2 &= P_a \\ A_{l+1} &= A_l^m A_{l-1}^n \end{aligned} \tag{2}$$

where P_{ab} (P_{ba}) stands for the propagation matrix from layer a (b) to b (a) and P_a (P_b) is the propagation matrix through a single layer a (b). They are given by [19]:

$$P_{ab} = P_{ba}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_a}{n_b} \end{pmatrix} \tag{3}$$

$$P_{a(b)} = \begin{pmatrix} \cos \delta_{a(b)} & -\sin \delta_{a(b)} \\ \sin \delta_{a(b)} & \cos \delta_{a(b)} \end{pmatrix} \tag{4}$$

where $\delta_{a(b)} = kn_{a(b)}d_{a(b)}$, $n_{a(b)}$ is the media refraction index a (b), $d_{a(b)}$ are the layer thicknesses and k the wavenumber in a vacuum. The transmission coefficient is expressed as follows:

$$T_l = \frac{4}{|A_l^2| + 2} \tag{5}$$

where A_l^2 is the sum of four element squares of the A_l . Since the transfer matrix is unimodular, we can express the transmission coefficient as:

$$T_l = \frac{4}{x_l^2 + y_l^2} \quad (6)$$

where x_l and y_l denote respectively the trace and anti-trace of the transfer matrix A_l . The transmission coefficient is completely determined by the trace and anti-trace. Thus, a complete description of the transmission through general aperiodic multilayer requires both trace and anti-trace map.

Given a matrix $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, the anti-trace of A is defined as $y_A = A_{21} - A_{12}$.

In order to study anti-trace maps we need the following identity for two unimodular transfer matrices A and B [20]: $y_{AB} = x_B y_A + x_A y_B - y_{BA}$. In this case, we need to know the n -th power of a unimodular 2×2 matrix A , which can be written as [20, 21]:

$$A^n = U_n(x_A)A - U_{n-1}(x_A)I \quad (7)$$

where I is the unit matrix, and

$$U_n(x_A) = \frac{\lambda_+^n - \lambda_-^n}{\lambda_+ - \lambda_-} \quad (8)$$

$$\lambda_{\pm} = \frac{x_A \pm \sqrt{x_A^2 - 4}}{2}$$

Here, x_A and λ_{\pm} denote the trace and the two eigenvalues of A , respectively. Using Eqs. (2) and (7), we can write the recursion relation of the transfer matrix as:

$$A_{l+1} = \left[U_m^{(l)} A_l - U_{m-1}^{(l)} I \right] \times \left[U_n^{(l-1)} A_{l-1} - U_{n-1}^{(l-1)} I \right] \quad (9)$$

From Equation (6) the trace and anti-trace maps are obtained as:

$$x_{l+1} = U_m^{(l)} U_n^{(l-1)} v_l - U_{m-1}^{(l)} U_{n+1}^{(l-1)} - U_{m+1}^{(l)} U_{n-1}^{(l-1)} \quad (10)$$

$$v_{l+1} = U_m^{(l)} U_{n+1}^{(l-1)} v_l - U_{m-1}^{(l)} U_{n+2}^{(l-1)} - U_{m+1}^{(l)} U_n^{(l-1)} \quad (11)$$

$$y_{l+1} = U_m^{(l)} \left[U_n^{(l-1)} w_l - U_{n-1}^{(l-1)} y_{l-1} \right] - U_{m-1}^{(l)} U_n^{(l-1)} y_l \quad (12)$$

$$w_{l+1} = U_m^{(l)} \left[U_{n-1}^{(l-1)} w_l - U_{n-2}^{(l-1)} y_{l-1} \right] + \left[x_{l+1} - U_{m-1}^{(l)} U_{n-1}^{(l-1)} \right] \quad (13)$$

where $v_l = x_{A_l A_{l-1}}$ and $w_l = y_{A_l A_{l-1}}$.

The roles of v_l and w_l are subsidiary. Equations (10) and (11) represent the trace map whereas Eqs. (12) and (13) give the corresponding anti-trace map. We choose

appropriate layer thicknesses d_a and d_b to make $n_a d_a = n_b d_b$. Then, we have $\delta_a = \delta_b = \delta = (k + 1/2)\pi$, with δ_a and δ_b being the incident angles of light in layers A and B , respectively, where k is a positive integer. The propagation matrices become:

$$P_{a(b)} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \tag{14}$$

Finally, the trace and anti-trace maps are completely determined by Eqs. (10)–(13). So, if we know the initial conditions, the transmission coefficients through general aperiodic multilayer can be determined from the trace and anti-trace maps [22].

4. Results and discussion

4.1. The effect of the n variation with m set to 1

In the following numerical investigation, we have chosen air (L) and ceramic (H) as two elementary layers, with refractive indices $n_L = 1$ and $n_H = 3$, respectively. The thicknesses $d_{L,H}$ of the two materials has been chosen to satisfy the Bragg conditions: $d_L n_L = d_H n_H = \lambda_0/4$, where $\lambda_0 = 12$ mm is the central wavelength. According to these conditions, $d_H = 1$ mm and $d_L = 3$ mm. We use the trace map method to extract the transmission coefficients in the spectral range from 10 GHz to 40 GHz. We show that the corresponding transmission coefficients display interesting properties. As a result, the reflection bands of the multilayer structures cover the entire spectral range by increasing the parameter n and setting m to 1. It is interesting to note that this result is impossible to reach by using the periodical multilayer systems.

The Table gives the width Δf of the pseudo-forbidden gaps and their corresponding central frequencies f_i for the case of $l = 3$, $m = 1$, $n = 20$. It is clear that the width Δf_i increases with an increase of the corresponding central frequency f_i .

T a b l e. Width of the pseudo-forbidden gaps Δf_i and the corresponding central frequency f_i .

f_i [GHz]	10.39	11.16	11.95	13.85	15.19	16.9	18.89	21.27	24.46	28.46	34.97
Δf_i [GHz]	0.48	0.5	0.86	0.95	1.14	1.15	1.71	1.9	2.38	3.61	4.94

Figure 2 shows typical transmission coefficients of the generalized Fibonacci-class multilayer stack. Many pseudo-band gaps appear in the spectral domain [10 GHz, 40 GHz] and the width of these pseudo-band gaps can cover 75% of the whole spectral domain. By varying the parameter m , the total width of the forbidden gaps increases and can reach the value of 20 GHz for $m = 10$ (Fig. 3). Thus, we note a stacking of pseudo-forbidden gap whose sizes increase gradually with the frequency located between 10 GHz and 40 GHz. When we establish the width of each pseudo-band gap and the corresponding central frequency, multi-stop band filter can be easily considered.

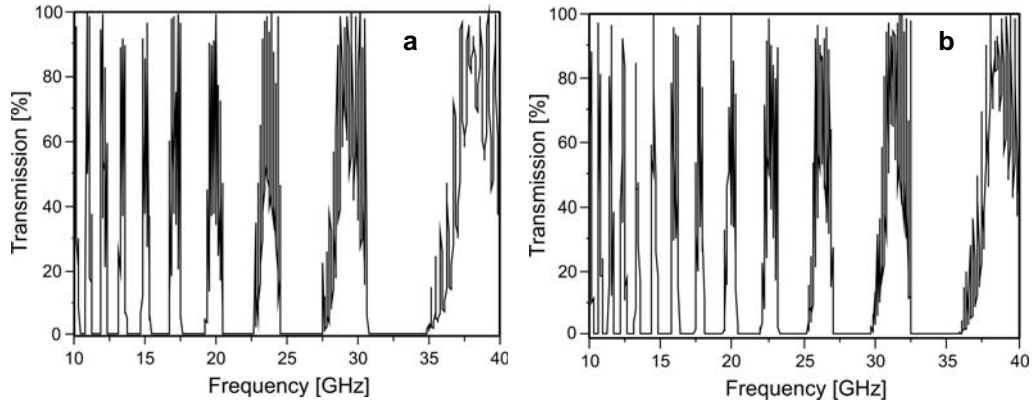


Fig. 2. Transmission coefficient of generalized Fibonacci multilayer where: $l = 3$, $m = 1$, $n = 15$ (a), and $l = 3$, $m = 1$, $n = 20$ (b).

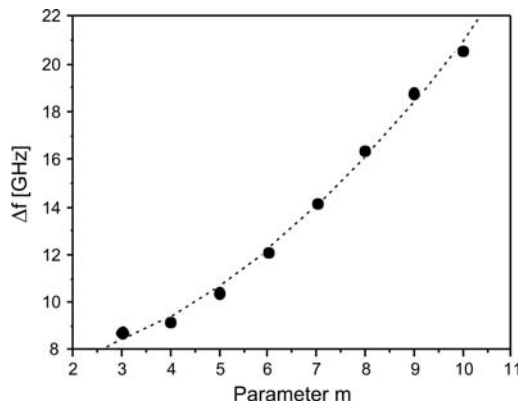


Fig. 3. The total width of forbidden bands versus parameter m .

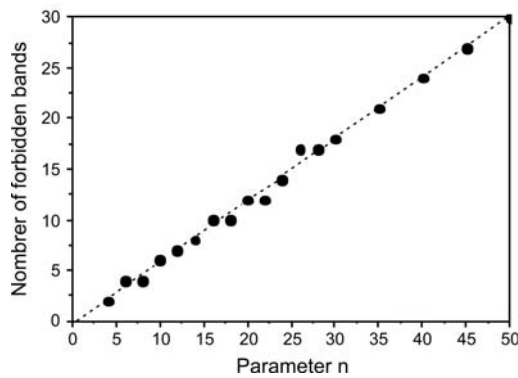


Fig. 4. The number of forbidden bands versus parameter n .

The number of photonic band gaps increases linearly with an increase of the parameter n . We can modulate the number of photonic band gaps, N , according to the parameter n by the following linear variation: $N = 0.6082n - 0.2868$, where n denotes an integer part of real number N . From this approximation we can deduce

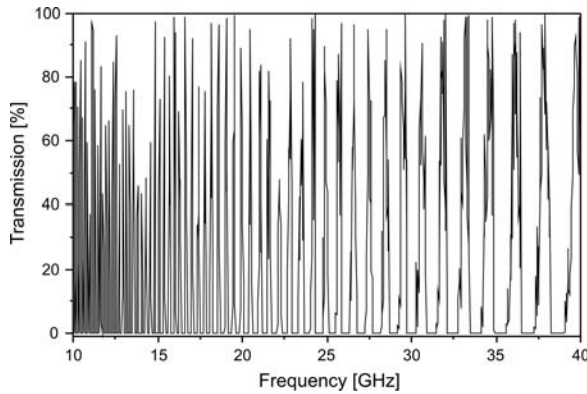


Fig. 5. Transmittance coefficient for $m = 1$, $n = 100$ for third generation of generalized Fibonacci multilayer.

the number of photonic band gaps for any given $n > 3$ (see Fig. 4). Indeed, we verify that for $n = 100$, for example, the number of photonic band gaps is equal to 60 as shown in Fig. 5. This allows us to predict the number of peaks without making calculation which becomes complex for large values of the number n .

4.2. The effect of parameter m variation with n set to 1

In the case where the parameter m varies and n is set to 1, we have found a large zone with 20.54 GHz for m equal 10. Comparing this result with that of the corresponding case with n varying and m set to 1, we have found that n must be taken equal to 20 in order to reach the same result. Figure 6 shows the transmission spectra for many m values for the same iteration $l = 3$. In this case, we can cover more than 75% of the spectral domain.

We note that the behaviour of N relative to the parameter m is not linear as compared with the case where n varies and m is set to 1 (Fig. 7). A good approximation of the N

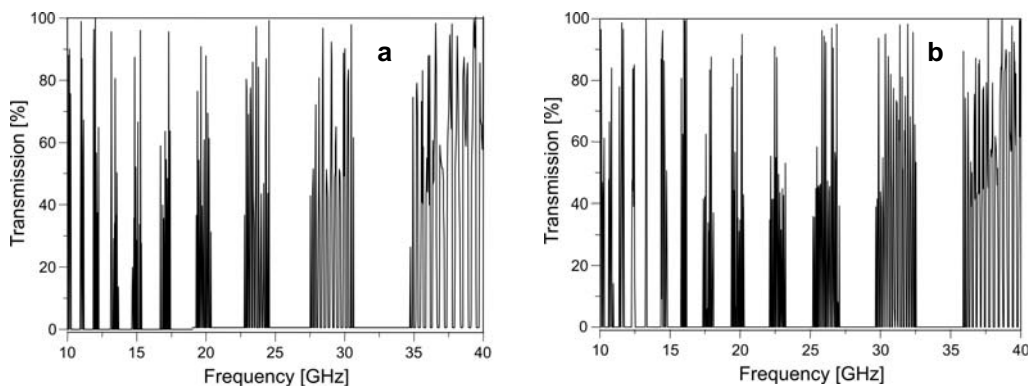


Fig. 6. Transmission coefficient of generalized Fibonacci multilayer, where $l = 3$, $m = 15$, $n = 1$ (a), and $l = 3$, $m = 20$, $n = 1$ (b).

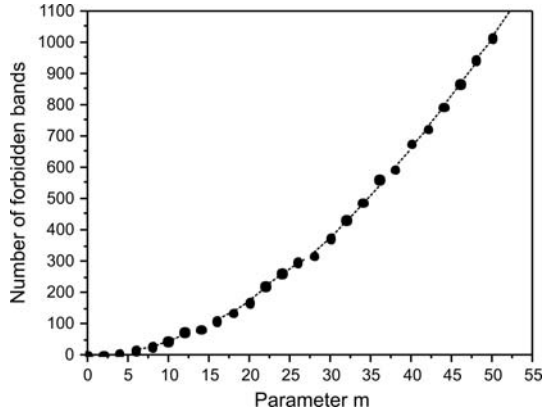


Fig. 7. The number of forbidden bands versus parameter m .

variation versus the parameter m can be made valuable as follows: $N = 0.38685m^2 + 1.11625m - 3.23171$ for $m > 3$ with a coincidence of 99.99%, where m denotes an integer part of the real number N .

4.3. Case of m and n variations

To study the transmission properties, two cases were taken: *i*) $n = 2m$ with $m = 5, 6, 8, 10$ and *ii*) $m = 2n$ with $n = 5, 6, 8, 10$. Hence, the layer numbers of the whole structure increase by varying m and n simultaneously. In all the cases, we note an increase of the forbidden band gaps by increasing the parameters m or n and the transmission spectra show a multitude of bands which increase by increasing the parameters m or n . In addition, we can note that for the case where $m = 2n$ the alternation of higher transmission values (with lower transmission values) is about 40%.

Thus, reducing the lower transmission values to zero can lead to a good mirror for the 10–40 GHz spectral range. The opportunity of this work is to consider the case of $m = 2n$ and trying to reduce the lower transmission values by probably introducing a partial or global defect of the whole system as described elsewhere [11].

With generalized Fibonacci quasi-periodic multilayer structure, we show the existence of several forbidden gaps, all of which increase gradually with the system parameters (Fig. 8). Each forbidden gap represents a multi-narrow stop band. This rejection band is localized around the central frequency. The stacking of the forbidden gap leads to the design of the multi-stop band filters in high frequency. In this case, we can order each filter by knowing the frequency centre and the width of corresponding stop band. We deduce that extra narrow-band filters can be obtained using the multilayer structure studied by the trace map method.

However, this method is applicable only to normal incidence and in the case of not normal incidence we must use, for example, the matrix method (MM) [23]. Indeed in our previous works [23, 24] we show that an omnidirectional high reflector with wide bandwidth was obtained for both S and P polarizations for the all incident angles in

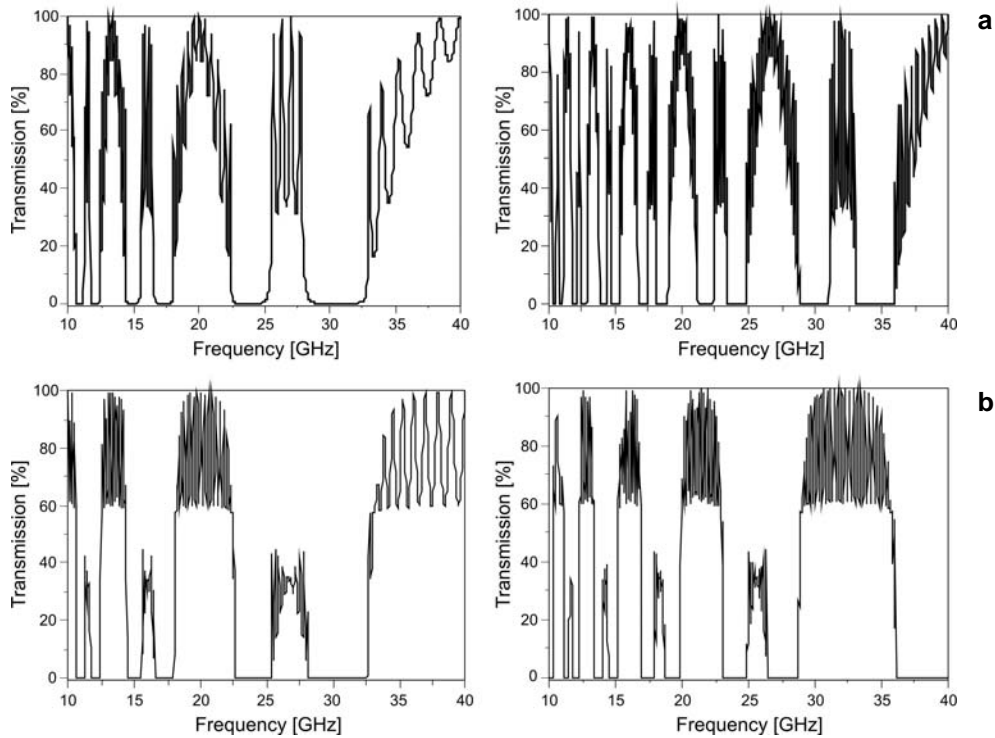


Fig. 8. Transmission spectra of generalized Fibonacci multilayer, where: $l = 3$; $m = 5, 10$; $n = 2m$ (a); $l = 3$; $m = 2n$; $n = 5, 10$ (b).

the range $0\text{--}90^\circ$. So, as an alternative work we expect to use the MM and by varying the incident angle we hope to obtain an omnidirectional mirror which covers the spectral range $10\text{--}40$ GHz.

5. Conclusions

This work is focused firstly on the transmission properties at normal incidence of the multilayer structures built according to the generalized Fibonacci quasi-periodic multilayer $GF(m, n)$ in the microwave spectral domain ($10\text{--}40$ GHz). According to the proposed method (trace and anti-trace), the transmission spectra through the generalized Fibonacci multilayer structure show a stacking multi-narrow stop band. The number of multi-narrow bands can be controlled by varying the parameters m or n . Based on the analysis proposed, multi-stop band filters can be easily designed. In all the cases, by increasing n or m with a fixed Fibonacci iteration, the number of the photonic band gaps increases. By increasing these photonic band gaps, we can obtain high reflector components working in microwave domains, very interesting for

technological applications. As an alternative work we expect to study the effect of the incident angle on the transmission properties both for S and P polarizations in order to obtain an omnidirectional mirror in the microwave spectral domain.

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