Optica Applicata, Vol. XXXIX, No. 2, 2009

Coherence characterization of partially coherent flat-topped beam propagating through atmospheric turbulence

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We study the change in the degree of coherence of a partially coherent flat-topped (PCFT) beam propagating through atmospheric turbulence. It is shown analytically that with a fixed set of source parameters and under a particular atmospheric turbulence model, a PCFT beam propagating through atmospheric turbulence reaches its maximum value of coherence after propagating a particular distance, and the effective width of the spectral degree of coherence also has its maximum value. This phenomenon is independent of the turbulence model used. We also study the effects of beam width values, the structure constant of turbulent media and the degree of coherence on effective width of spectral degree of coherence. The results are illustrated by numerically calculated curves.

Keywords: partially coherent flat-topped beams (PCFT), atmospheric turbulence, degree of coherence, effective width of the spectral degree of coherence.

1. Introduction

Optical beams have been a subject of great interest since the advent of the laser in 1960 [1]. They find useful applications in laser radar, optical communication systems and some optical information processing techniques; for example, it is frequently asserted that high directionality of optical beams is a consequence of high spatial coherence of the sources. However, it was predicted theoretically [2–4] in the late 1970's and confirmed experimentally [5, 6] soon afterwards that high spatial coherence of the source is not necessary to produce very directional beams. In fact, some partially coherent sources may generate beams which have the same far-zone intensity distribution as a fully coherent single-mode laser beam.

It is well known that partially coherent beams are less influenced by turbulent atmosphere than completely coherent beams [7–10]. Therefore, a considerable number of investigations have paid attention to the characterization of partially coherent light propagation through turbulent atmosphere.

On the other hand, the spectral density, the spectral degree of the polarization and the spectral degree of coherence may change on propagation, even in free space [11].

Many interesting results have been presented in numerous publications on the subject of changes in the spectral degree of coherence of partially coherent beam in any transverse cross-section, see MANDEL and WOLF [12]. Fewer papers have appeared focusing on the change in degree of coherence of beam on propagation in free space or through some random media [13].

In recent years, much more attention has been paid to the propagation properties of partially coherent beams concerning their application in wireless optical communication involving both free space and atmosphere, which are always carried out in the scalar approximation [13–16]. Therefore, in addition to the research on changes in the spectral density and the degree of polarization, it is necessary for partially coherent beams to study the change in the degree of coherence on propagation, especially through some random medium.

Spectral density of the PCFT beam has been investigated in some papers [8, 9]. But, to the best of our knowledge, there are no investigations on the spectral degree of coherence of PCFT beam.

The aim of this paper is to study the behavior of change in the degree of coherence of PCFT beam with circular symmetry when it propagates through atmospheric turbulence.

Treatments using scalar theory cannot provide any information about the coherence and polarization properties of beam [13]. Based on the extended Huygens–Fresnel principle [17] and according to a unified theory of coherence and polarization [18], we analyze the evolution of the cross-spectral density matrix that describes the second-order coherence and polarization properties of beam on propagation in atmospheric turbulence and then we give expressions for the cross-spectral density matrix and the spectral degree of coherence of PCFT beam in turbulent media.

We employ the effective width of the spectral degree of coherence of beam to characterize the coherence property of beam on propagation. We investigate the effect of correlation length and order of flatness on it. Finally, we study the influence of turbulent atmosphere and different turbulence models on spectral degree of coherence and its effective width.

2. Cross-spectral density matrix of partially coherent flat-topped beams in atmospheric turbulence and its degree of coherence

We consider a field propagation from the plane $z = 0$, *i.e.*, the partially coherent source plane, into the half-space $z > 0$ where the turbulent atmosphere exists. The electric field vector $\mathbf{E}(\rho, z; \omega)$ propagates in a linear medium, in which variations of the refractive index are much smaller than the average value of the refractive index. Based on the extended Huygens–Fresnel principle, we give the expression for the field $E_j(\rho, z; \omega)$ at any point in the half-space $z > 0$:

$$E_j(\rho, z;\, \omega) = \frac{-ik\exp(ikz)}{2\pi z}\iint E_j^{(0)}(\rho', 0;\, \omega)\exp\left[\frac{ik(\rho-\rho')^2}{2z}\right]\exp\left[\psi(\rho, \rho', z;\, \omega)\right]\mathrm{d}^2\rho' \tag{1}$$

$$(j = x, y)$$

where $E_j(\rho, 0;\, \omega)$ is the electric field vector at the point $(\rho, 0)$ in the source plane; ψ is a random phase factor that characterizes the effect of atmospheric turbulence on propagating spherical wave. Here, we assume that the incident field propagates close to the z-axis and further neglect the contribution of the z component [15]. The subscript j denotes the Cartesian components x and y across the transverse plane perpendicular to the propagation direction. The subscripts l and j appearing in the remainder of this paper have the same sense as the elements of cross-spectral density matrix that describe the second-order coherence and polarization properties of beam on propagation, which can be given as [13]:

$$\begin{aligned} W_{lj}(\rho_1, \rho_2, z;\, \omega) = {} & \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}^2\rho_1' \times \iint \mathrm{d}^2\rho_2' W_{lj}^{(0)}(\rho_1', \rho_2', 0;\, \omega)\times \\ & \times \exp\left\{\frac{-ik\left[(\rho_1-\rho_1')^2-(\rho_2-\rho_2')^2\right]}{2z}\right\}\times \\ & \times \left\langle \exp\left[\psi^*(\rho_1, \rho_1', z;\, \omega) + \psi(\rho_2, \rho_2', z;\, \omega)\right]\right\rangle_m \end{aligned} \tag{2}$$

where $W_{lj}^{(0)}(\rho_1', \rho_2', 0;\, \omega) = \langle E_l^{(0)*}(\rho_1';\, \omega)\, E_j^{(0)}(\rho_2';\, \omega)\rangle$ is the electric cross-spectral density matrix in the source plane $z = 0$. The term $\langle \ldots \rangle_m$ denotes averaging over the ensemble of statistical realizations of the turbulent medium.

In Equation (2), the angular bracket that describes the turbulence effect can be approximated as follows:

$$\begin{aligned} & \left\langle \exp\left[\psi^*(\rho_1, \rho_1', z;\, \omega) + \psi(\rho_2, \rho_2', z;\, \omega)\right]\right\rangle_m \cong \\ & \cong \exp\left\{\frac{-\pi^2 k^2 z}{3}\left[(\rho_1-\rho_2)^2 + (\rho_1-\rho_2)\cdot(\rho_1'-\rho_2') + (\rho_1'-\rho_2')\int_0^\infty \kappa^3 \phi_n(\kappa)\mathrm{d}\kappa\right]\right\} \end{aligned} \tag{3}$$

where the quantity $\int_0^\infty \kappa^3 \phi_n(\kappa)\mathrm{d}\kappa$ describes the effect of turbulence, $\phi_n(\kappa)$ being the spectrum of the refractive-index fluctuations that can be characterized by the Tatarskii model or the Kolmogorov model [20]. As for the PCFT source, $W_{lj}^{(0)}(\rho_1', \rho_2', 0; \omega)$ can be written:

$$E_{IN}(\rho_1'; \omega) = \sum_{n=1}^{N} A_l \frac{(-1)^{n-1}}{N} \binom{N}{n} \exp\left(\frac{-n\rho_1'^2}{2\sigma_s^2}\right) \tag{4a}$$

$$g_{lj}^{(0)}(\rho_1' - \rho_2'; \omega) = \sum_{n=1}^{N} B_{lj} \exp\left(\frac{-c(\rho_1' - \rho_2')^2}{2\sigma_{glj}^2}\right) \tag{4b}$$

$$W_{lj}^{(0)}(\rho_1', \rho_2', 0; \omega) = \sum_{c=1}^{N} \sum_{m=1}^{N} \sum_{n=1}^{N} A_l A_j B_{lj} \frac{(-1)^{n+m}}{N^2} \binom{N}{n} \binom{N}{m} \times$$
$$\times \exp\left(\frac{-(n\rho_1'^2 + m\rho_2'^2)}{2\sigma_s^2}\right) \exp\left(\frac{-c(\rho_1' - \rho_2')^2}{2\sigma_{glj}^2}\right) \tag{4c}$$

where the coefficients A_l, B_{lj}, σ_s and σ_g are positive quantities, which are independent of position. The coefficients B_{lj} satisfy the relations

$$B_{lj} = 1 \quad \text{when} \quad l = j$$
$$|B_{lj}| \le 1 \quad \text{when} \quad l \ne j \tag{5a}$$

and

$$B_{jl} = B_{lj}^* \tag{5b}$$

The parameters σ_s and σ_g and characterize the effective source size and the effective width of the spectral degree of coherence of source, respectively.

Parameters characterizing an electromagnetic Gaussian source cannot be chosen arbitrarily due to the sufficient conditions they must satisfy [13]. The restrictions on the choice of parameters of an electromagnetic Gaussian source are provided in [20–22]. For isotropic source, *i.e.*, for the source with $\sigma_{sl} = \sigma_{sj} = \sigma_s$ $(l, j = x, y)$, the following constraints on the parameters of the source have been derived in [20, 21]:

$$\frac{A_x^2 \sigma_{gxx}^2}{\sigma_{gxx}^2 + 4\sigma_s^2} - \frac{2A_x A_y |B_{xy}| \sigma_{gxy}^2}{\sigma_{gxy}^2 + 4\sigma_s^2} + \frac{A_y^2 \sigma_{gyy}^2}{\sigma_{gyy}^2 + 4\sigma_s^2} \ge 0 \tag{6a}$$

$$\frac{\sigma_{gxx}^2}{\sigma_{gxx}^2 + 4\sigma_s^2} - \frac{2\sigma_{gxy}^2}{\sigma_{gxy}^2 + 4\sigma_s^2} + \frac{\sigma_{gyy}^2}{\sigma_{gyy}^2 + 4\sigma_s^2} \le 0 \tag{6b}$$

and

$$\frac{1}{4\sigma_s^2} + \frac{1}{\sigma_{gxx}^2} \ll \frac{2\pi}{\lambda^2}, \qquad \frac{1}{4\sigma_s^2} + \frac{1}{\sigma_{gyy}^2} \ll \frac{2\pi}{\lambda^2} \tag{6c}$$

where the choice of σ_{gxy}^2 should be in the range:

$$\max\left\{\sigma_{gxx}^2, \sigma_{gyy}^2\right\} \le \sigma_{gxy}^2 \le \min\left\{\frac{\sigma_{gxx}^2}{\sqrt{|B_{xy}|}}, \frac{\sigma_{gyy}^2}{\sqrt{|B_{xy}|}}\right\} \tag{6d}$$

The inequalities (6) are the necessary and sufficient conditions the parameters of the source must satisfy in order to generate a physically realizable electromagnetic Gaussian source [20].

To evaluate the integration in Eq. (2), we define two arguments **u** and ***v*** such that

$$\mathbf{u} = \frac{\boldsymbol{\rho}_1' + \boldsymbol{\rho}_2'}{2}, \qquad \boldsymbol{v} = \boldsymbol{\rho}_1' - \boldsymbol{\rho}_2' \tag{7}$$

Substituting Equation (4c) into Equation (2) and calculating, the related integral $W_{lj}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', 0; \omega)$ is obtained as follows:

$$W_{lj}^{(0)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', z; \omega) = \sum_{c=1}^{N}\sum_{m=1}^{N}\sum_{n=1}^{N} A_l A_j B_{lj}\eta \frac{(-1)^{n+m}}{N^3}\binom{N}{n}\binom{N}{m}\frac{1}{4\alpha_1\alpha_2 - \alpha_5^2} \times$$

$$\times \exp\left[\frac{\alpha_1\left(\beta_2^2 + \alpha_3^2\right) - \alpha_5^2(\beta_1\beta_2 + \alpha_3\alpha_4) + \alpha_2\left(\beta_1^2 + \alpha_4^2\right)}{4\alpha_1\alpha_2 - \alpha_5^2}\right] \tag{8}$$

where:

$$\alpha_1 = \frac{n+m}{4\sigma_s^2}$$

$$\alpha_2 = \frac{n+m}{16\sigma_s^2} + \frac{c}{2\sigma_{glj}^2} + M = \frac{\alpha_1}{4} + \frac{c}{2\sigma_{glj}^2} + M$$

$$\alpha_3 = \frac{-ik}{2z}\left(\rho_{1x} + \rho_{2x}\right) + M\left(\rho_{1x} - \rho_{2x}\right)$$

$$\alpha_4 = \frac{-ik}{z}\left(\rho_{1x} - \rho_{2x}\right)$$

$$\alpha_5 = \frac{ik}{z} + \frac{n-m}{4\sigma_s^2}$$

$$\beta_1 = \frac{-ik}{z}\left(\rho_{1y} - \rho_{2y}\right)$$

$$\beta_2 = \frac{-ik}{2z}\left(\rho_{1y} + \rho_{2y}\right) + M\left(\rho_{1y} - \rho_{2y}\right)$$

$$\eta = \exp\left[\frac{-ik}{2z}\left(\rho_1^2 - \rho_2^2\right) - M\left(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2\right)^2\right]$$

where the quantity M is given as $0.5465 C_n^2\, l_0^{-1/3}\, k^2 z$ for the Tatarskii spectrum and as $0.49(C_n^2)^{6/5} k^{12/5} z^{6/5}$ for the Kolmogorov spectrum [20], with C_n^2 being the refractive index structure parameter and l_0 being the inner scale of turbulence.

Equation (8) is the main result of this paper, by which one can study the change in degree of coherence for the partially coherent beam propagating through the atmospheric turbulence. According to the unified theory of coherence and polarization [18], the degree of coherence can be given as:

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) = \frac{\mathrm{Tr}W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega)}{\sqrt{\mathrm{Tr}W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, z; \omega)}\sqrt{\mathrm{Tr}W(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2, z; \omega)}} =$$

$$= \frac{W_{xx}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega) + W_{yy}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z; \omega)}{\sqrt{W_{xx}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, z; \omega) + W_{yy}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, z; \omega)} \times \sqrt{W_{xx}(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2, z; \omega) + W_{yy}(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2, z; \omega)}} \quad (9)$$

where Tr denotes the trace of matrix. This equation is the base of the results presented in the next section.

3. Discussion

It can be seen from Eqs. (8) and (9) that the spectral degree of coherence for the PCFT beam is determined by both source parameters and turbulence parameters, simultaneously. We may study the change in the spectral degree of the coherence of beam on propagation in turbulence by choosing different condition parameters and further calculating the absolute value of the spectral degree of coherence.

Considering that only σ_{gjj} ($j = x, y$) may be used in the calculation, we along with it analyze the special case, *i.e.*, $\sigma_{gxx} = \sigma_{gyy} = \sigma_g$. To represent the partial coherence, the spectral degree of correlation σ_{gjj} should be smaller than the effective source size [13].

At first, we analyze the change in the degree of coherence of PCFT beam only the increasing propagation distance. It is shown in Fig. 1**a** that when the length of the beam path increases, the central peak of spectral degree of coherence generally degrades, which is as expected [13]. However, one can find that the degree of coherence improved after the beam initiates from the source, plane $z = 0$, and the spectral degree of coherence reaches its maximum value when the beam travels a distance about 2.7 km, then it begins degrading and keeps decreasing along with the distance. After sufficiently long propagation distance the degree of coherence almost fades away and the beam can be considered as almost incoherent one. This interesting phenomenon is evidently shown in Fig. 1**b**. In this figure, we define the width of the spectral degree of coherence $\overline{\rho}_{\mu}(z)$ of the beam in turbulence as the separation $|\rho_1 - \rho_2|$ of points in a transverse cross-section at which $|\mu(\rho_1, \rho_2, z, \omega)|$ drops from its maximum value of unity (for $|\rho_1 - \rho_2| = 0$) to the value $1/e$.

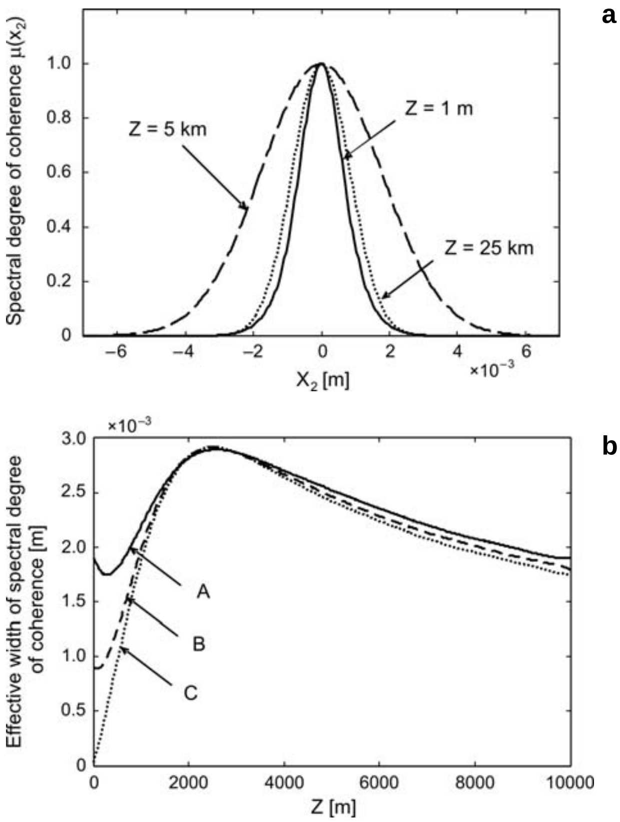

Fig. 1. **a** – The change in the central peak of spectral degree of coherence of PCFT beam. The turbulence is described by Tatarskii spectrum. The curves correspond to different distances in turbulence labeled in the figure. The parameters are taken as: $\lambda = 632.8$ nm, $C_n^2 = 10^{-13}$ m$^{-2/3}$ and $l_0 = 5$ mm, $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_{sx} = \sigma_{sy} = \sigma_s = 5$ cm, $\sigma_{gxx} = \sigma_{gyy} = \sigma_g = 1$ mm, $N = 4$. **b** – The effective widths of the spectral degree of coherence for beams generated from three partially coherent sources with different coherence properties, which propagate through atmospheric turbulence. Three sets of source parameters are taken as: $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_{sx} = \sigma_{sy} = \sigma_s = 5$ cm, $N = 4$, and $\sigma_{gxx} = \sigma_{gyy} = 2$ mm (case A), $\sigma_{gxx} = \sigma_{gyy} = 1$ mm (case B), $\sigma_{gxx} = \sigma_{gyy} = 0.1$ mm (case C).

The behavior of $\bar{\rho}_\mu(z)$ with increasing distance z is shown in Fig. 1**b**. One can readily see that the effective width of the spectral degree of coherence increases firstly and then decreases after reaching a maximum value. This can be explained as follows [13]. There are two sets of parameters used to determine the behavior of the beam coherence in the beam propagation, *i.e.*, source parameters and parameters of turbulence spectrum model. When the beam travels in turbulence only over a small distance from the source, the strength of the turbulence is negligible. Therefore, it cannot overcome the effect of coherence properties of the PCFT source. The source parameter determines dominantly the evaluation of the coherence area of the beam in a transverse cross-section of the beam, which increases with z, as shown in Fig. 1**b** (rising part of the curve). At the same time, atmospheric turbulence also plays its role with increasing propagation distance, which as is well known, degrades the beam coherence properties. As the beam propagates a particular distance (about 2.7 km in our example), the strength of atmospheric turbulence matches that of source coherence property and then the width of the spectral degree of coherence reaches its maximum value. After that the effect of atmospheric turbulence dominates the behavior of the width. This causes the value of the effective width of the spectral degree of coherence to decrease gradually (see dropped part of the curve).

Using three different sets of the values of σ_s and σ_g, we study the influence of source parameters on the behavior of the central peak of spectral degree of coherence of beam propagating in atmospheric turbulence characterized by Tatarskii spectrum, as shown in Fig. 1**b**. It can be seen from this figure that decreasing the correlation length (σ_g) causes a decrease of the effective width of spectral degree of coherence.

Figure 2 shows the width of spectral degree of coherence of PCFT beam propagating in atmospheric turbulence versus propagation distance z. It is evident that increasing the effective size of source causes the effective width of spectral degree of coherence to decrease.

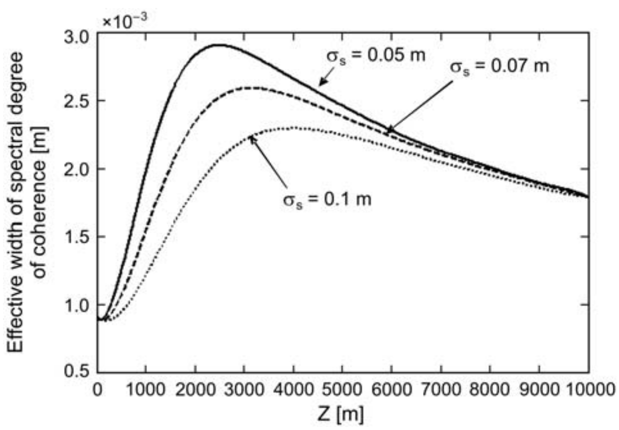

Fig. 2. The effective widths of the spectral degree of coherence for beams generated from three PCFT beams sources with different effective size labeled in the figure. The parameters are taken as: λ = 632.8 nm, $C_n^2 = 10^{-13}$ m$^{-2/3}$ and l_0 = 5 mm, $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_{sx} = \sigma_{gxx} = \sigma_{gyy}$ = 1 mm, $N = 4$.

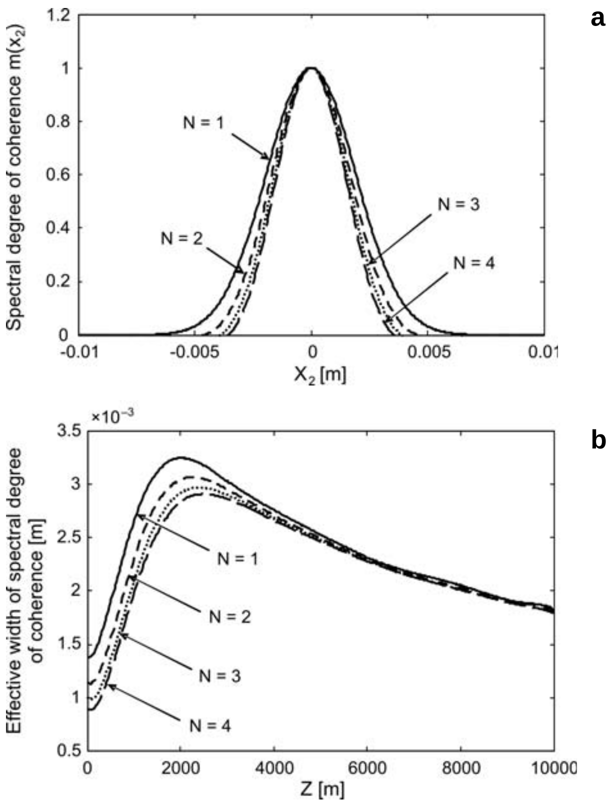

Fig. 3. **a** – The change in the central peak of spectral degree of coherence of PCFT beam. The turbulence is described by Tatarskii spectrum. The curves correspond to different order of flatness in turbulence labeled in the figure. The parameters are taken as: $\lambda = 632.8$ nm, $C_n^2 = 10^{-13}$ m$^{-2/3}$ and $l_0 = 5$ mm, $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_{sx} = \sigma_{sy} = \sigma_s = 5$ cm, $\sigma_{gxx} = \sigma_{gyy} = \sigma_g = 1$ mm, $N = 1, 2, 3, 4$. **b** – The effective widths of the spectral degree of coherence for beams generated from PCFT sources with different order of flatness, which propagate through atmospheric turbulence. The source parameters are taken as: $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_{sx} = \sigma_{sy} = \sigma_s = 5$ cm, $\sigma_{gxx} = \sigma_{gyy} = 1$ mm, $N = 1, 2, 3, 4$.

We also study the influence of an order of flatness on the behavior of the degree of coherence as well as its effective spectral width.

Figure 3**a** shows the behaviors of degree of coherence of the PCFT beam propagation through the turbulences characterized by Tatarskii spectrum. It can be seen that increasing the order of flatness causes its effective spectral width to decrease. Figure 3**b** shows the effective width of spectral degree of coherence. It can approve the result of Fig. 3**a**.

Another important parameter that affects the effective spectral width is turbulence. As is shown in Fig. 4, the effective spectral width decreases as C_n^2 increases.

Finally, we study the behaviors of the degree of coherence of the PCFT beam propagating through the turbulences characterized by Kolmogorov spectrum and Tatarskii spectrum, respectively. It can be seen that there is not much difference

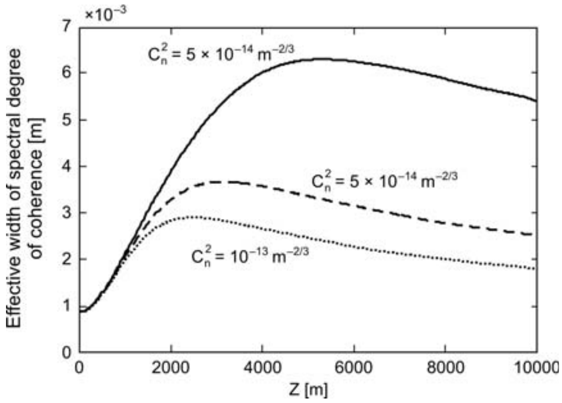

Fig. 4. The effective widths of the spectral degree of coherence for beams generated from PCFT source, which propagate through atmospheric turbulence with different structure constants as labeled in the figure. The source parameters may be chosen as: $A_x^2 = A_y^2 = 1$, $B_{xy} = 0$, $\sigma_{sx} = \sigma_{sy} = \sigma_s = 5$ cm, $\sigma_{gxx} = \sigma_{gyy} = 1$ mm, $N = 4$.

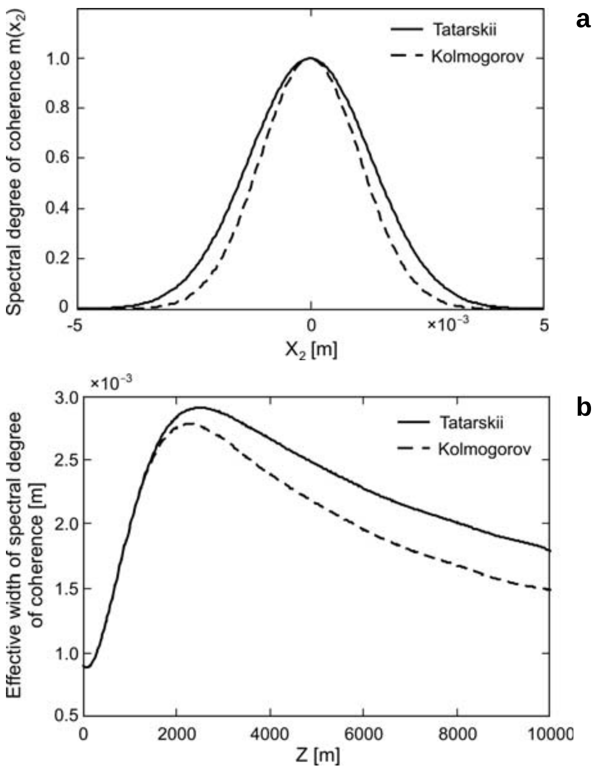

Fig. 5. Behaviors of the degree of coherence (**a**) as well as the effective width of spectral degree of coherence (**b**) for a PCFT beam propagating through two different turbulences characterized by Tatarskii spectrum and Kolmogorov spectrum, respectively. The beam source parameters take the same values as those used in Fig. 1. The propagation distance used in (**a**) is 10 km.

between the behaviors of the degree of coherence when using the different turbulence models.

Figure 5**b** shows the effective widths of the spectral degree of coherence for both cases. The position that maximum value of the width happens under the Kolmogorov spectrum is closing to its counterpart under the Tatarskii spectrum. Both the results also show that choosing different turbulence models does not bring any appreciable difference in either the behavior of the degree of coherence or the effective width of the spectral degree of coherence.

4. Conclusions

In this paper, we have studied the change in the spectral degree of coherence of PCFT beam propagating through atmospheric turbulence. Both source parameters and atmospheric turbulence models influence the behavior of a beam. Here, we employ the effective width of the spectral degree of coherence to characterize the beam coherence properties on propagation in turbulent atmosphere. In free space, the effective width of the spectral degree of coherence keeps increasing along with the increasing distance, which is determined only with the source parameters, but after a special distance, the effective spectral width becomes constant. However, in the turbulent atmosphere, the change in the effective width of the spectral degree of coherence can be affected by two parts; free space part and turbulence part. The former causes the width to increase and the latter leads to the width decreasing. With the presence of the two mechanisms, the effective width of the spectral degree of coherence has its maximum value when the beam propagates a particular distance. We investigate the flat-topped order effect which makes it clear that the higher order the PCFT beams, the smaller the effective width of spectral degree of coherence. We have also analyzed the effect of correlation length; increasing correlation length causes a decrease in the spectral effective width.

The results of our analysis may find applications in problems involving optical imaging as well as atmospheric optical communication system.

References

[1] Shirai T., Wolf E., *Spatial coherence properties of the far field of a class of partially coherent beams which have the same directionality as a fully coherent laser beam*, Optics Communications **204**(1–6), 2002, pp. 25–31.

[2] Collett E., Wolf E., *Is complete spatial coherence necessary for the generation of highly directional light beams?*, Optics Letters **2**(2), 1978, pp. 27–29.

[3] Wolf E., Collett E., *Partially coherent sources which produce the same far-field intensity distribution as a laser*, Optics Communications **25**(3), 1978, pp. 293–296.

[4] Collett E., Wolf E., *New equivalence theorems for planar sources that generate the same distributions of radiant intensity*, Journal of the Optical Society of America **69**(7), 1979, pp. 942–950.

[5] De Santis P., Gori F., Guattari G., Palma C., *An example of a Collett–Wolf source*, Optics Communications **29**(3), 1979, pp. 256–260.

[6] FARINA J.D., NARDUCCI L.M., COLLETT E., *Generation of highly directional beams from a globally incoherent source*, Optics Communications **32**(2), 1980, pp. 203–208.
[7] SALEM M., SHIRAI T., DOGARIU A., WOLF E., *Long-distance propagation of partially coherent beams through atmospheric turbulence*, Optics Communications **216**(4–6), 2003, pp. 261–265.
[8] KASHANI F.D., ALAVINEJAD M., GHAFARY S., *Propagation properties of a non-circular partially coherent flat-topped beam in turbulent atmosphere*, Optics and Laser Technology **41**(5), 2009, pp. 659–664.
[9] ALAVINEJAD M., GHAFARY B., *Turbulence-induced degradation properties of partially coherent flat-topped beams*, Optics and Lasers in Engineering **46**(5), 2008, pp. 357–362.
[10] WU JIAN, *Propagation of Gaussian–Shell beam through turbulent media*, Journal of Modern Optics **37**(4), 1990, pp. 671–684.
[11] WOLF E., *Correlation-induced changes in the degree of polarization, the degree of coherence, and the spectrum of random electromagnetic beams on propagation*, Optics Letters **28**(13), 2003, pp. 1078–1080.
[12] MANDEL L., WOLF E., *Optical Coherence and Quantum Optics*, Cambridge University Press, Cambridge, 1995.
[13] LU W., LIU L., SUN J., YANG Q., ZHU Y., *Change in degree of coherence of partially coherent electromagnetic beams propagating through atmospheric turbulence*, Optics Communications **271**(1), 2007, pp. 1–8.
[14] ROYCHOWDHURY H., PONOMARENKO S.A., WOLF E., *Change in the polarization of partially coherent electromagnetic beams propagating through the turbulent atmosphere*, Journal of Modern Optics **52**(11), 2005, pp. 1611–1618.
[15] RICKLIN J.C., DAVIDSON F.M., *Atmospheric turbulence effects on a partially coherent Gaussian beam: implications for free-space laser communication*, Journal of the Optical Society of America A **19**(9), 1794–1802.
[16] KOROTKOVA O., ANDREWS L.C., PHILIPS R.L., *Model for a partially coherent Gaussian beam in atmospheric turbulence with application in Lasercom*, Optical Engineering **43**(2), 2004, pp. 330–341.
[17] ANDREWS L.C., PHILIPS R.L., *Laser Beam Propagation through Random Media*, SPIE Optical Engineering Press, Bellingham, WA, 1998.
[18] WOLF E., *Unified theory of coherence and polarization of random electromagnetic beams*, Physics Letters A **312**(5–6), 2003, pp. 263–267.
[19] KOROTKOVA O., SALEM M., WOLF E., *The far-zone behavior of the degree of polarization of electromagnetic beams propagating through atmospheric turbulence*, Optics Communications **233**(4–6), 2004, pp. 225–230.
[20] ROYCHOWDHURY H., KOROTKOVA O., *Realizability conditions for electromagnetic Gaussian Schell model sources*, Optics Communications **249**(4–6), 2005, pp. 379–385.
[21] KOROTKOVA O., SALEM M., WOLF E., *Beam conditions for radiation generated by an electromagnetic Gaussian Schell-model source*, Optics Letters **29**(11), 2004, pp. 1173–1175.
[22] GORI F., SANTARSIERO M., PIQUERO G., BORGHI R., MONDELLO A., SIMON R., *Partially polarized Gaussian Schell-model beams*, Journal of Optics A: Pure and Applied Optics **3**(1), 2001, pp. 1–9.

Received August 4, 2008