Optica Applicata, Vol. XXXVIII, No. 4, 2008

Detection of the vortices signs in the scalar fields

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The technique for determining the vortex sign in the scalar fields (including the statistical ones) under conditions when the use of the regular reference beam is impossible is described. The elaborated approach is based on the shift-interferometry technique. The conditions of the optimal vortices identification are formulated. The results of the computer simulation and experimental confirmation are presented.

Keywords: vortex, shift-interferometry, interference forklets, topological charge.

1. Introduction

Restoration of the field phase distribution from the analysis of its intensity distribution (the so-called inverse source problems in optics) is an old optical problem. The theoretical and practical aspects of this problem have been comprehensively reasearched (see, for example, [1]). However, new approaches to solving this problem are formed on the basis of understanding the special role of amplitude zeroes in a scalar field [2, 3]. As it follows from [2, 4], amplitude zeroes are wavefront dislocations (optical vortices) which are combined into some systems – vortex networks. Elements of these networks are connected to each other. Their characteristics define the behavior of a field phase in each point [3]. As a result, the information about characteristics of the networks and their vortices provides the feasibilities for restoring the field parameters with high reliability. At the same time, such approaches to solving the inverse source problems, being now at the initial stage of development, are not widely applied.

One of the aspects of the problem is the impossibility of the field zeroes "identification". The main characteristic of an optical vortex is its topological charge (sign), which defines the direction (clockwise or counter-clockwise) of the phase increasing under the bypass of the vortex centre (see, for example, [4, 5]). As it is known, the complete information on the vortex characteristics may be obtained only from the analysis of the data of the interferometric experiment [6–9]. At the same time, as a rule, formation of the regular reference beam is impossible due to the lack of knowledge of the pre-history of the analyzed field.

Thus, the elaboration of the technique, which allows us to obtain the information on the vortices signs at the field with some intensity distribution, is necessary. Note, that in general case this field is a statistical one or in other words – a speckle-field. On the other hand, we can state the following facts:

(i) Any field under its amplitude dividing enables us to form the coherent superposition (see, for example, [16]). Such components may be easily shifted to one another in the transversal direction.

(ii) Phase is practically constant in the speckle centre due to the fact that the phase saddle is positioned in this region [13].

(iii) The mean speckle dimension is comparable with the field intensity correlation length [2, 14].

(iv) Field intensity and phase correlation lengths coincide if the field is analyzed in far field. Moreover, their magnitude coincides with the mean space between absolute intensity minima (or phase vortices) [2, 14].

(v) Intensity networks and phase ones (at least, in statistical sense) are connected. First of all, positions of absolute intensity minima coincide with the localization of the centers of phase vortices. The intensity saddle points are preferentially positioned in the areas with relatively large phase gradient [2, 10–12]. This allows us:

- to define the field correlation length as the magnitude which is an inverse ratio to the spatial density of the field intensity minima (mean space between them);
- to indicate the areas where phase saddles are positioned.

In accordance with the said above, if the field is divided by amplitude on transversely shifted components it can be stated that the areas, where vortices of one of the component are located, coincide with the regions, where the other component has its phase saddle points (i.e., regions where the phase is practically constant). In this case the process of interference of the vortices related to the first component with the other ("shifted") component may be interpreted as a "classical" interference of vortex field structure with a reference plane wave. As a result of this superposition, the corresponding interference forklets appear in the positions of vortices localized in the both interfering components. Naturally, the shift must be less than mean speckle dimension.

It follows from these facts that one of the solutions of the problem of interest may be the employment of the shift-interferometry techniques (see, for example, [15]).

This paper is devoted to the results of the shift-interferometry method of the vortices signs identification.

2. Method of the vortices signs identification

The base of the elaborated method is as follows. Obviously, each field may be divided into the identical components and one can obtain (see, for example, [15]) the interference of the field with itself. The angle between components, the shift between them, and their intensity ratio can be easily controlled.

Let the field $U(\omega, \upsilon)$ be divided into the components $\tilde{U}_1(\omega, \upsilon)$ and $\tilde{U}_2(\omega, \upsilon)$. Assume that the component $\tilde{U}_2(\omega, \upsilon)$ is shifted relatively to the field $\tilde{U}_1(\omega, \upsilon)$ by Δ in the plane ω, υ in arbitrary direction. Naturally, the choice of the component $\tilde{U}_2(\omega, \upsilon)$ to be shifted is quite arbitrary, so both components are equivalent in this sense. Moreover, as it will follow from further considerations, only a relative component shift is important.

Let us assume (without loss of generality) that the directions of the predominant propagation of these fields are different and that they are described by the following equations:

$$\begin{aligned} \tilde{U}_1(\omega, \upsilon) &= U_1(\omega, \upsilon)\exp[j(\alpha\omega + \beta\upsilon)] \\ \tilde{U}_2(\omega, \upsilon) &= U_2(\omega - \Delta_\omega, \upsilon - \Delta_\upsilon)\exp[-j(\alpha\omega + \beta\upsilon)] \end{aligned} \tag{1}$$

where U_1 and U_2 are the "exact replicas" of the field U with corresponding amplitude coefficients determining the mean intensity ratio of the components, $\alpha = k\sin\alpha'$, $\beta = k\sin\beta'$ determine the "incline" of the fields U_1 and U_2 relative to the z-axis, determining the predominant propagation of the resulting field, $\Delta^2 = \sqrt{\Delta_\omega^2 + \Delta_\upsilon^2}$ is a shift of the field U_2 relative to the field U_1.

The following relations describe the intensity of such superposition:

$$\begin{aligned} J(\omega, \upsilon) = J_1 + J_2 + 2\{&[R_1R_2 + I_1I_2]\cos[2(\alpha\omega + \beta\upsilon)] \\ &+ [R_1I_2 - I_1R_2]\sin[2(\alpha\omega + \beta\upsilon)]\} \end{aligned} \tag{2}$$

where R_i, I_i (i = 1, 2) – the real and imaginary parts of the fields U_1 and U_2.

As it follows from eq. (2), the third term is zero, when $\Delta = 0$. Correspondingly, the intensity of the resulting field is the interference pattern containing the set of straight fringes without any discontinuities. Such pattern is modulated by the intensity of the field $U(\omega, \upsilon)$.

The shift Δ (less than $l_{cor}/2$ and more than $l_{cor}/10 \div l_{cor}/5$) between the fields $\tilde{U}_1(\omega, \upsilon)$ and $\tilde{U}_2(\omega, \upsilon)$ leads to the situation, when the areas of the fields containing the vortices superpose with practically plane waves, corresponding to the areas with the phase saddle points. Thus, "classical" interference forklets appear in the positions of vortices of the fields U_1 and U_2 when Δ is nonzero.

Note:

(i) Vortices of different signs, associated with one of the fields (U_1 or U_2) form the forklets with different directions.

(ii) The directions of the forklets are also different when vortices of the same signs are associated with different components.

(iii) The direction of the forklets obtained as the result of the interference of the vortex field with a plane reference wave is determined by two factors [9]:

• topological charge of the vortex;

• mutual orientation of the vortex and reference wave relatively to the observation plane.

(iv) The normal to the wavefront in the phase saddle points coincides with the predominant direction of wave propagation. Thus, one can state that information on the predominant directions of propagation of the waves U_1 and U_2 unambiguously determines the mutual orientation of the "vortex" and "reference" waves (the field part with a saddle point) relatively to the observation plane.

As it follows from these facts, the control of the parameters of the interfering components U_1 and U_2 (their propagation directions, relative shift) allows us to determine unambiguously the true value of topological charge of any field vortex.

As it is known, the maximal visibility of the interference pattern is achieved when the intensities of the superposing fields are equal. The uniform visibility is impossible when superposing fields are statistical and the shift Δ is nonzero. Visibility in the areas with vortices (for example, associated with the field U_1) tends to the maximal one, whereas the intensity J_1 of one of the fields (in our case the field U_1) is significantly higher than the intensity J_2 of the other component (the field U_2). In other words, the favorable conditions for visualizing the vortices associated with the field U_1 appear due to the fact that low intensity parts of the field U_1 superpose with the parts of the field U_2, which have similar intensity. At the same time, the interference conditions for the areas with vortices associated with the component U_2 are extremely unfavorable, even in comparison with the other parts of the field U_2. Only the interference forklets corresponding to the non-shifted component must be well observed when the intensity J_1 is significantly higher than J_2. And vice versa, when the intensity J_2 is significantly higher than J_1, the vortices of the field U_2 are visualized.

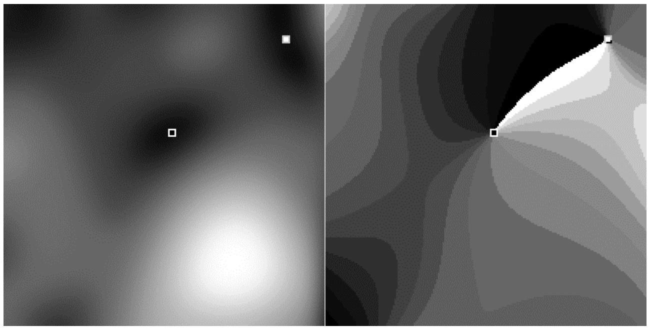
Thus, one can determine the positions and signs of the vortices of practically arbitrary field under the control of the components shift Δ and their intensities J_1 and J_2.

3. Results of the computer simulation of the speckle field shift-interferometry

In this section, we present the results of the computer simulation of the vortices visualization by the speckle field shift-interferometry.

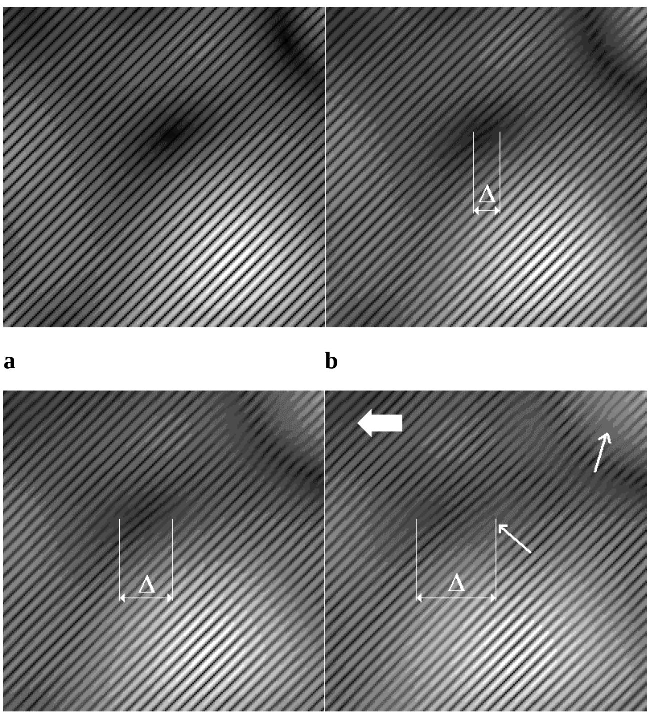
These results are illustrated in Figs. 1–3.

The intensity and the "phase map" for certain field area are presented in Fig. 1. As it is seen, two adjacent vortices with opposite topological charges are present within the analyzed region.



a **b**

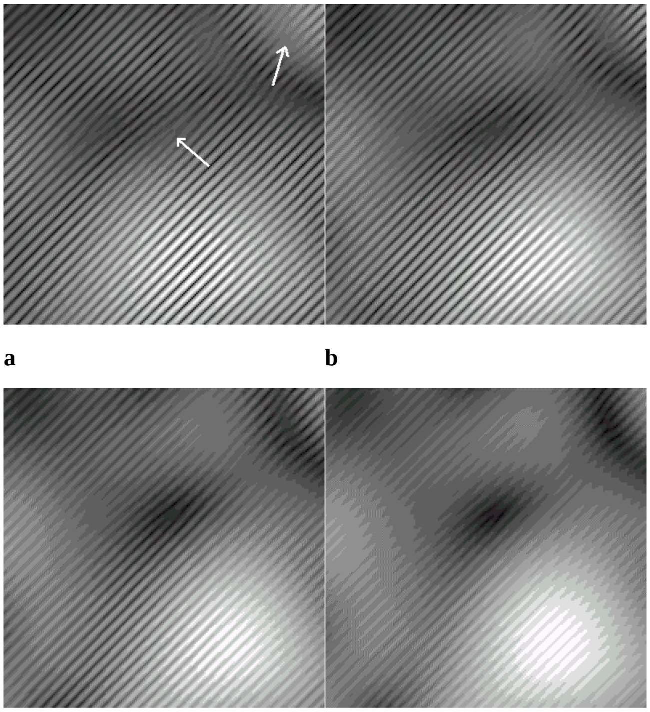
Fig. 1. Space distributions of the intensity (**a**) and phase (**b**) of the speckle field fragment. The phase changes from 0 (white) to 2π (black). Gray *x*-like areas in Figure (**b**) are the areas where phase saddles are positioned. Two adjacent vortices are depicted by squares. A dark square with a white fringe effect is a negative vortex, and a light square with a black fringe effect is a positive one.



a **b**

c **d**

Fig. 2. Shift-interferograms of the speckle field for different magnitudes of the shift Δ. Interference forklets corresponding to the steady field (field U_1) are indicated in Fig. **d** by white arrows. The direction of the shift of the field U_2 is shown in the same figure by the thick white arrow. Additionally, the shift magnitudes of the field U_2 are denoted in Figures (**b**)–(**d**): **a** – $\Delta = 0$, **b** – $\Delta = 0.1 l_{cor}$, **c** – $\Delta = 0.2 l_{cor}$, **d** – $\Delta = 0.3 l_{cor}$.



a b c d

Fig. 3. Shift-interferograms of the speckle field for magnitudes of the shift $\Delta = 0.3l_{cor}$ and different intensity ratios, corresponding to the components U_1 and U_2. Interference forklets corresponding to the steady field (field U_1) are indicated in Figure **a** by white arrows. The shift direction of the field U_2 is the same as in Figure 2. **a** – the mean intensity of the field U_1 is equal to the intensity of the component U_2, **b** – it exceeds the intensity of the field U_2 4 times, **c** – the intensity of the field U_1 exceeds the intensity of the field U_2 16 times, **d** – the intensity of the component U_1 exceeds the intensity of the field U_2 100 times.

The shift-interferograms for this field area and the shifts Δ of different magnitudes are presented in Fig. 2. The interference forklets, corresponding to the non-shifted field, are indicated in Fig. 2**d** by white arrows. The direction of the shift of the field U_2 is shown in the same figure by a thick white arrow. Additionally, shift magnitudes of the field U_2 are denoted in Figs. 2**b**–**d**. As it follows from Fig. 2**a**, the interference fringes are straight and continuous, when $\Delta = 0$, and as a result, the interference forklets corresponding to the vortices are absent. The pairs of the oppositely directed interference forklets corresponding to the fields U_1 and U_2 appear for non-zero shift (Figs. 2**b**–**d**) due to the interference of the vortices with smooth parts of these fields. The interference forklets associated with the component U_2 shift relatively to the ones corresponding to the field U_1 and they are in proportion to the increasing shift Δ.

The results of the shift-interference for different intensity ratios of the fields U_1 and U_2 are illustrated in Fig. 3. As it follows from Figs. 3**c**, **d**, only the interference forklets corresponding the the field U_1 are reliably identified for the case when the intensity of the component U_1 exceeds the intensity of the field U_2 by 20 to 100 times.

4. Experimental disclosure of the vortices signs on the basis of the shift-interferometry

The experimental arrangement for detection of the vortices signs in the statistical scalar field is presented in Fig. 4.

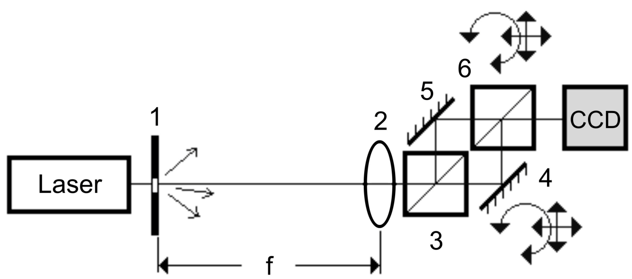


Fig. 4. Experimental arrangement: 1 – object; 2 – Fourier-transform objective; 3, 6 – beam-splitters; 4, 5 – mirrors.

A non-collimated beam from the He-Ne-laser illuminates the scattering object 1 – a ground glass screen, limited by a square hole. The magnitude of the phase variance of the boundary field is chosen in such a manner that the regular component of the scattered field behind the object is absent. The Fourier-transform objective 2 is placed at the focal length from the object. Due to that, the Fourier-transform of the boundary field (far field) has been observed in the back focal plane of this objective. The Mach-Zehnder interferometer 3–6 is placed just behind the objective 2. One of its mirrors, 4, and an output beam-splitter 6 are mounted on precise mechanical devices providing fine control of the shift and convergence angle of the field components. The resulting interference patterns and component intensities are fixed by a CCD-camera.

The results of the shift-interference are presented in Fig. 5. Fig. 5**a** corresponds to the zero shift between the field components. Fig. 5**b** illustrates the interference for the case when a shift between the fields U_1 and U_2 is about a half of the correlation length (a half of a mean dimension of a speckle). The intensity of the steady component exceeds the intensity of the shifted one by ~10 times.

As it follows from Fig. 5**b**, the vortices of the steady component are easily identified in comparison with the vortices of the shifted one.

Naturally, determining the vortex characteristics from an interference pattern is not a simple problem, and it is always subjective. Nevertheless, we state that the signs of all (practically, without any exclusions) vortices of the analyzed field may be identified with high reliability due to the operative modifying of the intensity ratio of the components and the shift magnitude between them, even in the case when the use of the regular reference beam is impossible.

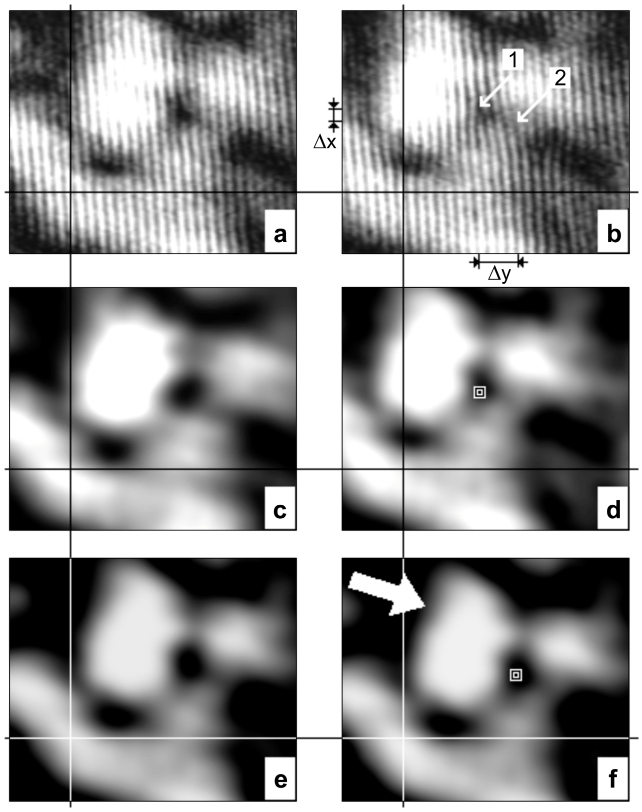


Fig. 5. The results of the experimental detection of the vortices signs on the basis of the shift-interferometry. **a** – interference pattern of the resulting field for zero shift between the components, **c**, **d** and **e**, **f** – intensity distributions of the steady and shifted field components, respectively, **b** – interference pattern of the resulting field for the shift between components, which is equal to about a half of correlation length. Distances Δx, Δy illustrate the magnitudes of the shifts in the corresponding directions of the coordinate plane. White arrows, numbers 1 and 2 in **b**, indicate the positions of the vortices (interference forklets) corresponding to different field components. 1 – vortex of the steady component, 2 – vortex of the shifted component. Squares in **d**, **f** illustrate the positions of the vortices. (**d**) – vortex of the steady component, (**f**) – vortex of the shifted component. Shift direction of the field U_2 is denoted in **f** by the white thick arrow. Black and white lines denote the "coordinate map", to make the observation and perception of the fields shifts easier.

5. Conclusions

1. The shift-interferometry technique can be used for determining the vortex characteristics of an arbitrary field, including statistical ones.
2. The optimal conditions of the interferogram formation are the following:

(i) shift between the field components is about 0.3–0.5 of the field correlation length;

(ii) intensity ratio of the components must be chosen in such a manner that the intensity of the steady component exceeds the intensity of the shifted component by 20–50 times.

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Received March 30, 2007
in revised form August 28, 2007