Optica Applicata, Vol. XXXVI, No. 2–3, 2006

Cascaded Fresnel digital hologram and its application to watermarking

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A cascaded Fresnel digital hologram (CFDH) is proposed, together with its mathematical derivation. Its application to watermarking has been demonstrated by a simulation procedure, in which the watermark image to be hidden is encoded into the phase of the host image. The watermark image can be deciphered by the CFDH setup, the reconstructed image shows good quality and the error is almost close to zero. Compared with previous technique, this is a lensless architecture which minimizes the hardware requirement, and it is used for the encryption of digital image.

Keywords: Fresnel digital holography, encryption and decryption, watermarking, image processing.

1. Introduction

Information security or encryption technique has recently received increasing attention. Information watermarking or encryption is a method for protecting data from unauthorized distribution, in which the secret imperceptible signal (or watermark) is encoded into the original data (or host) so that it always remains present against the attacks by a third party. Various methods of hiding information have been studied by many research groups [1–15]. However, in the most widely known techniques, when the host file is printed out as a picture or text, the digital watermark does not survive. The recently invented concealogram solves the problem of maintaining watermarks in printed pictures [11–15], in which method the data or information is concealed in a picture by manipulating the binary representation of continuous-tone images. The binary representation termed halftone coding is commonly used by most printers. The concealed data are not hidden in the paper, ink, or other material used for printing the host picture. The watermark is coded globally by the distances between the dots that compose the complete halftone picture, which is deciphered by cross correlation between the halftone picture and a reference function. The information is hidden in such a way that it prevents the hidden message from being read by unauthorized persons [14].

In this article we propose an alternative approach with an inherently simpler system architecture, which may offer a more useful approach to a practical optical encryption or watermarking (*i.e.*, hiding a picture, optical security verification or product authenticity verification), it compared with that reported in [11–15]. It can be called cascaded Fresnel digital hologram (CFDH), which means the reconstructed image is yielded by the Fresnel diffraction of two cascaded sub-holograms located at different distances from the imaging plane.

2. Cascaded Fresnel digital hologram

Compared with that in ref. [11–15], CFDH is a method without lenses, which minimizes the hardware requirement and is easier to implement. The optical setup of the system, shown in Fig. 1, is a cascaded Fresnel hologram optical architecture with three planes: the input plane P_1 in which the input mask $h_1(x_1, y_1)$ of a phase-only function $\exp[j2\pi\phi(x_1, y_1)]$, is displayed; the filter plane P_2 in which the filter mask $H_2(x_2, y_2)$, which is another random phase-only function statistically independent of $h_1(x_1, y_1)$, is displayed; and the imaging plane P_3 in which the camera should record the reconstructed output image. The distances between the adjacent planes are z_1 and z_2, which satisfy the Fresnel approximation according to the size of the aperture.

Using Fresnel diffraction theory, we obtain the electric field arriving at the filter plane P_2 located at a distance z_1 from the input plane P_1:

$$u(x_2, y_2) = \frac{\exp(j2\pi z_1/\lambda)}{j\lambda z_1} \iint \exp\left[j\phi(x_1, y_1)\right] \times$$

$$\times \exp\left\{\frac{j\pi}{\lambda z_1}\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]\right\} \mathrm{d}x\mathrm{d}y \tag{1}$$

where λ represents the illumination wavelength. For simplicity, we rewrite Eq. (1) in the form

$$u(x_2, y_2) = \mathrm{FrT}\left\{h_1(x_1, y_1);\, z_1\right\} \tag{2}$$

where FrT represents the Fresnel transform, z_1 denotes the distance of the Fresnel diffraction. Then $u(x_2, y_2)$ is multiplied by H_2 and totally Fresnel transformed to the output, the electric field in the imaging plane P_3 can be written as,

$$c(x_3, y_3) = \mathrm{FrT}\left\{\mathrm{FrT}\left\{h_1(x_1, y_1);\, z_1\right\} H_2(x_2, y_2);\, z_2\right\} \tag{3}$$

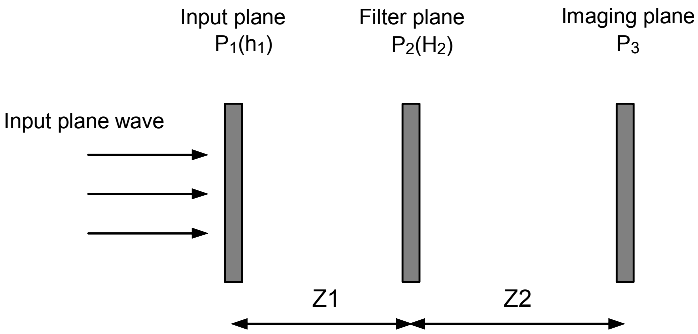

Fig. 1. Optical setup of the cascaded Fresnel hologram system.

while now the image in the imaging plane P_3 is not the expected one, so the projection onto convex sets (POCS) algorithm is employed to adjust the phase-only function $h_1(x_1, y_1)$ in the Fresnel diffraction procedure just as in [8], in which the Fourier transforms are replaced by the Fresnel transforms. The system's output expected is

$$c(x_3, y_3) = A(x_3, y_3)\exp\left[j\psi(x_3, y_3)\right] \tag{4}$$

where $A(x_3, y_3)$ denotes the amplitude of the expected output image and $\psi(x_3, y_3)$ denotes the phase of $c(x_3, y_3)$. From Eq. (3) the input function $h_1(x_1, y_1)$ is given by

$$h_1(x_1, y_1) = \mathrm{IFrT}\left\{\frac{\mathrm{IFrT}\left\{c(x_3, y_3);\, z_1\right\}}{H_2(x_2, y_2)};\, z_2\right\} \tag{5}$$

where IFrT is the inverse Fresnel operator (which is implemented by a virtual procedure, because there exists no the inverse Fresnel transform in the real world).

The POCS-based iterative algorithm, shown in Fig. 2, starts with a random-phase only function for the first $h_1(x_1, y_1)$. Then the function $h_1(x_1, y_1)$ is transformed by the cascaded Fresnel diffraction, defined in Eq. (3), into the output function $c(x_3, y_3)$ and then retransformed back through the inverse Fresnel diffraction defined by Eq. (5). At every iteration performed, in each of the two domains (x_1, y_1) and (x_3, y_3), the obtained functions are projected onto the constraint sets. The one in the (x_3, y_3) domain means the expectations to get the predefined image. The constraint set in the (x_1, y_1) domain manifests the limitation of the input function to be a phase-only function. The algorithm continues to circulate between two domains until the error between the actual and the desired output functions is meaningfully reduced.

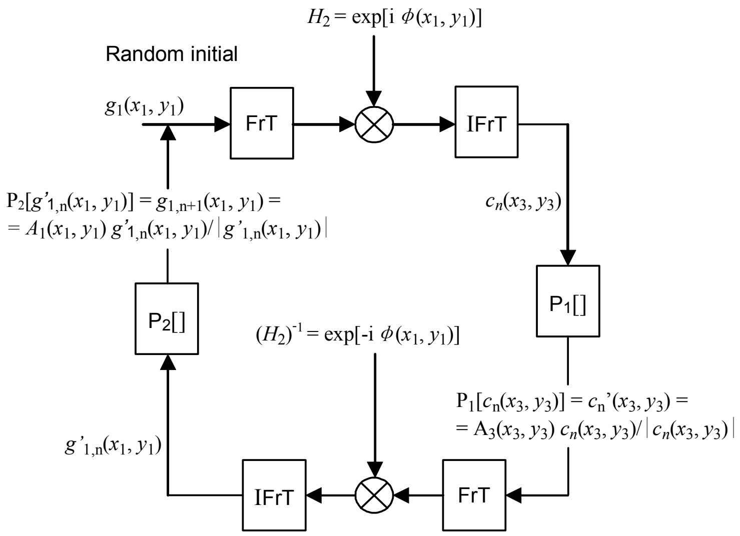

Fig. 2. Block diagram of the main POCS algorithm used to compute the phase function $h_1(x_1, y_1)$.

The mentioned above projection $P_1[...]$ on the constraint set in the output plane is

$$P_1\Big[c(x_3, y_3)\Big] = A(x_3, y_3)\exp\Big[j\psi(x_3, y_3)\Big] \tag{6}$$

where $A(x_3, y_3)$ is a real positive function representing the output image. In the input plane we recall that $h_1(x_1, y_1)$ should be a phase-only function, and therefore the projection $P_2[...]$ on the constraint set is

$$P_2[h_1(x_1, y_1)] = \begin{cases} \exp\Big[j\phi(x_1, y_1)\Big] & \text{if } (x_1, y_1) \in W \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $\phi(x_1, y_1)$ denotes the phase of $h_1(x_1, y_1)$, that is, $\exp\Big[j\phi(x_1, y_1)\Big] = \dfrac{h_1(x_1, y_1)}{|h_1(x_1, y_1)|}$, and W is a window function. Note that $H_2(x_2, y_2)$ chosen only once before the beginning of the iterations. $H_2(x_2, y_2)$ after having been defined, becomes part of the cascaded Fresnel diffraction setup and should never be changed during the circulating process.

The average mean-square error $e_{1,n}$ between the intensity of the output function before and after the projection, used to evaluate the convergence of the algorithm to the desired image in the n-th iteration, is defined as

$$e_{1,n} = \frac{1}{M}\iint\left|\left|P_1\Big[c_n(x_3, y_3)\Big]\right|^2 - \left|c_n(x_3, y_3)\right|^2\right|^2 \mathrm{d}x\mathrm{d}y \tag{8}$$

where M is the entire area of the output plane.

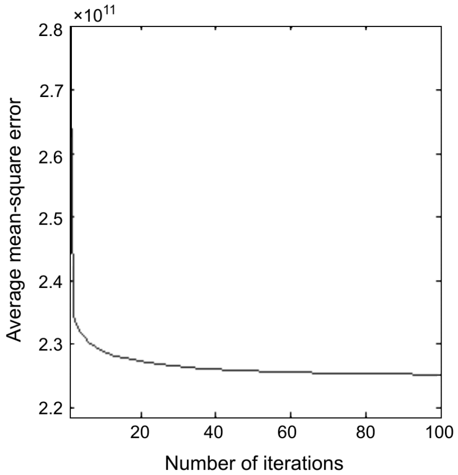

Fig. 3. Average mean-square error versus the number of iterations of the POCS algorithm in the simulation experiment.

Similarly, the average mean-square error in the input domain (x_1, y_1) can be expressed as

$$e_{2,n} = \frac{1}{M}\iint \left| \left| P_2\left[h'_{1,n}(x_1, y_1)\right]\right|^2 - \left|h'_{1,n}(x_1, y_1)\right|^2 \right|^2 \mathrm{d}x\mathrm{d}y \tag{9}$$

Two conditions for nondivergence of the error en should be satisfied. First, the cascaded Fresnel diffraction process should be an energy-conserving operator, which has been achieved in the present case by taking for $H_2(x_2, y_2)$ as a phase-only function. The second condition requires the two projected functions in the n-th iteration, $P_1[c_n(x_3, y_3)]$ and $P_2[h_{1,n}(x_1, y_1)]$ are the functions closest (in terms of the mean-square metric) to the functions $c_n(x_3, y_3)$ and $h_{1,n}(x_1, y_1)$, respectively, which is satisfied according to the Eqs. (6) and (7). So the error $e_{1,n}$, $e_{2,n}$ can be rewritten as

$$e_{1,n} = \frac{1}{M}\iint \left| \left| P_2\left[h'_{1,n}(x_1, y_1)\right]\right|^2 - \left|h_{1,n}(x_1, y_1)\right|^2 \right|^2 \mathrm{d}x\mathrm{d}y \tag{10}$$

$$e_{2,n} = \frac{1}{M}\iint \left| \left|c_{n+1}(x_3, y_3)\right|^2 - \left|c'_n(x_1, y_1)\right|^2 \right|^2 \mathrm{d}x\mathrm{d}y \tag{11}$$

In accordance with the second condition, we obtain

$$e_{1,n+1} \le e_{2,n} \le e_{1,n} \tag{12}$$

That is, the error can only decrease (or stay the same) at each iteration. The Figure 3 in the following simulation shows the reducing trend of the mean-square error.

3. Application of the CFDH to watermarking and its simulation experiment

In the application of the CFDH to watermarking, the input function $h_1(x_1, y_1)$ is replaced by the complex function $g_1(x_1, y_1)$ in the POCS-based iterative algorithm, which is the result of the random-phase only function $h_1(x_1, y_1)$ multiplied by the host image $A_1(x_1, y_1)$. The phase function $h_1(x_1, y_1)$ of $g_1(x_1, y_1)$ is adjusted at every iteration in the main POCS algorithm while the amplitude $A_1(x_1, y_1)$ of $g_1(x_1, y_1)$ is not changed. The filter function $H_2(x_2, y_2)$ is used for the reference key function. The setup shown in Fig. 1 is used to reveal the watermark and the watermark image can be reconstructed in the imaging plane P_3.

A simulation experiment has been done. Since the architecture is lensless, the beam propagating through the system may be somewhat divergent, the field sizes of the significant planes must be mismatched. Hence z_1 and z_2 should be chosen carefully to reduce the size mismatch regarding the propagation distance [4, 5]. Here we choose $z_1 = z_2 = 60$ mm, the wavelength λ of the input wave is 600 nm. The Fresnel diffraction calculation is based on the fractional Fourier transform algorithm. The two principal patterns used in the present study are shown in Fig. 4. The text "Demos" surrounded by a rectangle frame in Fig. 4**a** is the watermark pattern to be hidden, which is a binary image consisting of 85×85 pixels and the value of each pixel is 0 or 255. The host image is a dog's picture shown in Fig. 4**b**, a bit-map image consisting of

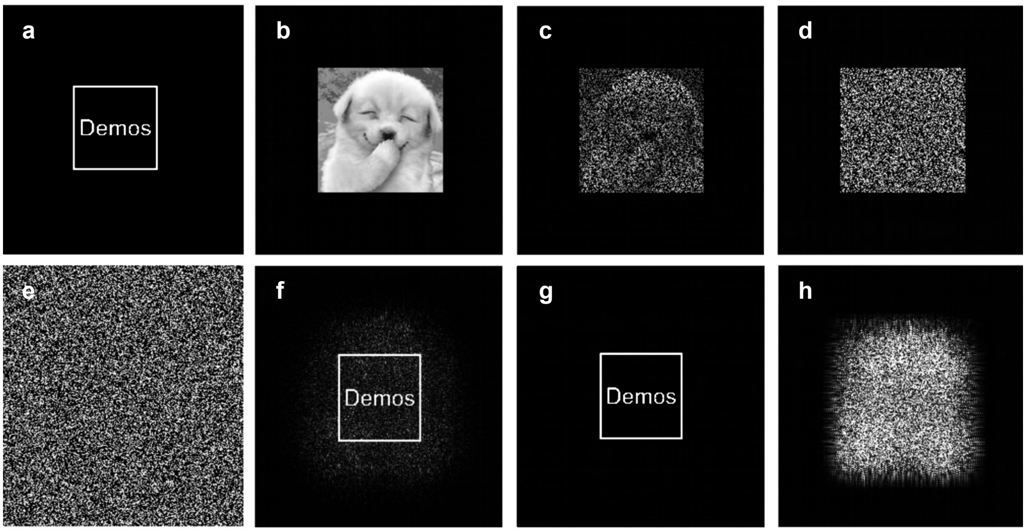

Fig. 4. Result of the computer simulation experiment: the watermark to be hidden (**a**), the host image (**b**), the host image multiplied by the phase function $h_1(x_1, y_1)$ (**c**), the phase function $h_1(x_1, y_1)$ of $g_1(x_1, y_1)$ (**d**), the random-phase only function $H_2(x_2, y_2)$ in the filter plane (**e**), the reconstructed image in the imaging plane with the right reference key $H_2(x_2, y_2)$ (**f**), the binarization of **f** (**g**), the reconstructed image in the imaging plane with the wrong phase function $H_2(x_2, y_2)$ of the input function $g_1(x_1, y_1)$ (**h**).

128×128 pixels and the value of each pixel is between 0 and 1, which is multiplied by the phase function and displayed in the central area of 128×128 pixels in the input plane, designated as the window *W*, the area out of which is padded with 0. The POCS-based iterative algorithm is tested with the watermark pattern “Demos”. After sufficient number of iterations of the POCS algorithm, a continuous-tone complex-valued matrix $g_1(x_1, y_1)$ is obtained, which is shown in Fig. 4**c** whose amplitude is the host image, and the watermark is encoded into its phase shown in Fig. 4**d**. The random-phase only function $H_2(x_2, y_2)$ used as the deciphering reference key is shown in Fig. 4**e**. The three planes in the Fig. 1 have 256×256 pixels. The watermark image can be reconstructed in the imaging plane of the setup in Fig. 1 with the continuous-tone complex-valued matrix $g_1(x_1, y_1)$ displayed in the input plane and the deciphering reference key in the filter plane. The reconstructed image is shown in Fig. 4**f**, while after being binarized, it is shown in Fig. 4**g**. The reconstructed image would be a noise one shown in Fig. 4**h** without the right reference key $H_2(x_2, y_2)$. The simulation experiment shows that the quality of the reconstructed image is good, especially after binarization, the error between the reconstructed image and the desired one is close to zero if the pattern to be hidden is a binary picture. So the security of the system proposed in this article can be guaranteed.

4. Conclusions

In this letter a cascaded Fresnel hologram system has been proposed and its application to watermarking has been demonstrated by a simulation procedure. In this technique, an arbitrary image can be encoded into the phase of a continuous-tone complex-valued image, reconstructed in the imaging plane of the cascaded Fresnel hologram setup from Fig. 1 with the right reference key $H_2(x_2, y_2)$ in the filter plane. Compared with that in Reference [11–15], this method is an architecture without lens, which reduce the hard-ware requirement and is easier to be implemented.

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Received October 28, 2005