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RAILWAY CREW SCHEDULING IN PASSENGER TRANSPORTATION – OPTIMIZATION APPROACH

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Abstract: In this paper, a railway crew scheduling problem is presented. The basic requirements related to the drivers' working time, which should be considered in the presented topic, are characterized. The literature review of existing models for crew planning was also performed as well. In the main part of the article, the decision model, which will allow to plan a driver for a train, is defined. The article also contains an assessment of the possibility to use the presented model in the scheduling the work of rail crews in practice.

Keywords: driver scheduling, train transportation, exact model optimization.

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1. Introduction

Making decisions on the planning and organization of transport processes in railway companies is a key issue in terms of their efficiency.

As part of passenger transport planning in rail transport, four basic stages should be identified:

1. Train Planning of train timetabling.
2. Planning of rolling stock circulation.
3. Crew scheduling (pairing).
4. Crew rostering.

The construction of train timetables consists in determining, for a given train set, the frequency of running and departure times from the starting station for each train, arrival time and departure to each station on the railway network in such a way as to meet safety restrictions and to achieve other goals, such as minimizing travel time and maximizing profit. In passenger transportation cyclical timetables are mainly used, for which the departure times of trains from the starting station take place at regular intervals.

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In the second phase, based on the timetable, passenger trains circulation is created. The essence of the issue is that a supporting vehicle should be allocated to each train in such a way that both the costs of the tasks which are to be performed, and the number of rolling stock needed to service, are minimal.

The next stage is crew scheduling. This stage is complicated, as planners have to take into account the normative acts and legal regulations of carriers, regarding working time and other restrictions such as the number of employees in the depot, in order to maximize the time of effective work performance, and/or to minimize the number of duties.

The last stage related to railway traffic planning is crew rostering. This problem involves assigning crews to all trains scheduled to run so as to minimize the costs of the tasks. The provisions of the Labour Code and a number of other assumptions that limit the possibility of running trains by individual employees should be taken into account, e.g. knowledge of the route by the driver.

Planning the transport process is carried out in stages in the order presented above, in accordance with a hierarchical approach where a higher-level plan (train timetables) provides a framework for lower-level plans, i.e. the planning of circuits, planning of duties and assigning staff to scheduled duties.

This article focuses on the problem of scheduling drivers. When analyzing the current needs of railway companies, it can be seen that there are many additional conditions which are required to be considered when assigning duties. In the model presented below, additional conditions specific to the railway industry were included such as additional drivers and additional licenses on selected routes and the possibility of returning to the starting nodes which is taken into working hours and is crucial for increasing effectiveness.

2. Literature review

The first attempt to solve the problem of duties allocation in the aviation industry was presented by [Minoux 1984]. In this approach the reduction of the size of the model is made by solving the shortest path problem in the network, in which the arcs represent the connection (flights) and is referred to as the flight network approach. The solution took into account the constraints of connecting flights but did not consider the connection conditions for the whole flight path.

Another proposal is the paper by [Lavoie et al. 1988], in which the problem was modeled as a network where nodes are duties and arcs are connections. This approach is referred to as the duty network approach. A similar approach for Air France was proposed by [Desaulniers et al. 1997].

The effectiveness of both methods of solving the problem of the assignment of duties (flight network or duty network) is difficult to compare, because the first approach needs more time to solve the problem of pricing while the duty network variant requires more memory.

A solution to the problem related to the amount of memory necessary to obtain a solution in the duty network variant was attempted by [Hjorring, Hansen 1999], using a two-type network. Depending on the needs, the part of network was redefined from flight to the duty variant.

In subsequent years, we can point to the successful use of optimization methods in the assignment of duties. For example, in the works of [Caprara et al. 1997] and [Caprara et al. 1999] the three-phase algorithm was presented. The most important second phase uses the set partitioning approach with Lagrange relaxation.

Another very important publication in this area is the work of [Kroon, Fischetti 2001]. In solving the problem of assigning duties, they used constraint programming, which together with the increase in computing power significantly improved the obtained solutions.

An interesting method of enumerating all duties along with the heuristics for filtering them was presented by [Wren et al. 2003]. The result of this work was the creation of the TRACS II system for planning services in the UK.

Alternative mathematical formulations for the tactical level crew capacity planning problem in railways were presented by [Suyabatmaz, Sahin 2015]. The authors determined the minimum required crew size in line with constraints for the feasibility and connectivity of the schedules. In addition the author provided evidence that arc-based formulation is a viable approach.

3. Problem formulation

The proposed model deals with drivers' scheduling, thereby focusing on the situation in the Lower Silesian Railways Company. The company is the regional operator of passenger trains in Lower Silesia, Poland. The plan of passenger electric train circuits obtained from the railway operator included 7 circuits, which included 100 segments in total. Each segment contained the following data: circulation number, train[1] number, departure station, arrival station, departure time and arrival time of the train to the station.

The problem of driver scheduling consists of finding a cost-minimum set of feasible driver duties (i.e. sequences of scheduled train movements) that covers all trains. Note that the resulting duties are not yet assigned to individual drivers.

The schedule of a single driver for a day is called a (driver) duty and consists of the several tasks the driver has to perform. An example of a task might be driving a train from the point of departure to the point of destination at the required time.

A duty is feasible if it meets all contractual and legal requirements, which regulate, e.g. maximum duty length, length of reading the orders of mobility, etc. We assume that the total working time of duty is from 8 to 12 hours and the time to get acquainted with the motor ordinances is 20 minutes. The time for other activities, i.e. cleaning, refueling and reviewing the plan are shown in the schedule of the trains circuits.

Each duty can start at one of three stations i.e. Wrocław (1), Legnica (2) or Węgliniec (3). At the end of the duty, the driver has to return to this station. To move from one station to the next, a driver can operate a train, but also use a deadhead, i.e. travel as passenger on a train or take a taxi (it is assumed that the travel time by car is the same as the average travel time of the train).

We also assume that the time interval between segments cannot be longer than two hours and the next day has the same plan (cyclicality).

4. Mathematical model

In the formulation written with formula (1) to (10), five groups of decision variables were used. The first group contains the decision variables x, y and z which are responsible for ensuring consistency of the routes obtained in the solution. The variable $x_{r1,r2,s}$ is a binary variable and is equal to the value of 1 if and only if segment r_2 is served directly after segment r_1 by duty s. Variables $y_{r,s}$ and $z_{r,s}$ are equal to 1 if the segment r is served as the first in a duty s (variable y) or is served as the last in a duty s (variable z). The next group of variables consists of variables $q_{r,s}$ and w_s, which are important for the times of the duties. The variable $q_{r,s}$ is equal to the duration of the duty in minutes from the beginning of the duty until segment r is realized. On the

[1] A train is not a physical object, but a timetabled train service with a unique train number.

other hand variable w_s is equal to the largest value of $q_{s,r}$, so it represents the duration of duty s.

The objective function expressed by formula (1) means that the solution will have the shortest length of duties – the sum of durations of all the duties expressed in the decision variable w_s.

$$\sum_{s\in S} w_s \to \min . \tag{1}$$

Constraints (2) and (3) are used to ensure that each segment will be served, i.e. it is served after another segment or is served first (2), and is followed by another segment or is handled last (3).

$$\sum_{s\in S} y_{r,s} + \sum_{s\in S, r1\in R} x_{r1,r,s} \geq 1 \text{ for every } r \in R , \tag{2}$$

$$\sum_{s\in S} z_{r,s} + \sum_{s\in S, r1\in R} x_{r,r1,s} \geq 1 \text{ for every } r \in R . \tag{3}$$

Constraint (4) initializes the value of the $q_{r,s}$ variable. If the duty starts from segment r (that is variable $y_{r,s}$ is equal to 1) then the value of variable $q_{r,s}$ after handling segment r must be equal to the length of the duration of segment r.

$$q_{r,s} \geq y_{r,s} \cdot \left(c_r^k - c_r^p\right) \text{ for every } r \in R, s \in S , \tag{4}$$

where:

c_r^k − end time of segment r,

c_r^p − end time of segment r.

Constraint (5) ensures the continuity of one duty in the time. If the value of the variable $x_{r1,r2,s}$ is 1 then this constraint is active and means that $q_{r2,s}$ is equal to the value of $q_{r1,s}$ increased by the duration of segment r_1 and the interval between segment r_1 and r_2. In addition, this restriction takes into account the situation when the segment begins one day before it ends.

$$q_{r,s} + y_{r,s} \cdot \left(c_{r'}^k - c_{r'}^p\right) + \begin{cases} c_{r'}^p - c_{r'}^k & \text{if } c_{r'}^p \geq c_{r'}^k \\ 24 - \left(c_{r'}^k - c_{r'}^p\right) & \text{if } c_{r'}^p \geq c_{r'}^k \end{cases} - 24 \cdot (1 - x_{r,r',s}) \leq q_{r',s} \tag{5}$$

for every $r \in R,\ r' \in R$.

Constraint (6) is used to combine the value of variable w_s with decision variables $q_{r,s}$. The value of variable w_s is equal to the largest of all values of $q_{r,s}$ for a given duty.

$$q_{r,s} + \sum_{r' \in R} y_{r',s} e^{p}_{g_s, r'} + \sum_{r' \in R} z_{r',s} e^{k}_{g_s, r'} \le w_s \text{ for every } s \in S, r \in R, \tag{6}$$

where:

g_s − depot of duty s,

$e^{p}_{g_s r}$ − travel time between g_s and start station of segment r,

$e^{k}_{g_s r}$ − travel time between g_s and end station of segment r.

Two further constraints, (7) and (8), ensure that the duration of the duty will not be less than 8 hours and longer than 12 decreased by 20 minutes, when the employee has to read the duty plan.

$$q_s \le 11.67 \cdot \sum_{r \in R} y_{r,s} \text{ for every } s \in S, \tag{7}$$

$$q_s \ge 7.67 \cdot \sum_{r \in R} y_{r,s} \text{ for every } s \in S. \tag{8}$$

The last group of constraints is related to the consistency of the segments. Constraints (9) and (10) ensure that the duty used to serve segments must have a beginning ($y_{r,s} = 1$) and an end ($z_{r,s}$ equal to 1).

$$\sum_{r \in R, r' \in R} x_{r',r,s} \le M \sum_{r' \in R} y_{r',s} \text{ for every } s \in S, \tag{9}$$

where:

M − very large number.

$$\sum_{r \in R, r' \in R} x_{r',r,s} \le M \sum_{r' \in R} z_{r',s} \text{ for every } s \in S. \tag{10}$$

Then constraints (11) and (12) ensure that the duty cannot start and end more than one segment.

$$\sum_{r \in R} y_{r,s} \le 1 \text{ for every } s \in S, \tag{11}$$

$$\sum_{r \in R} z_{r,s} \le 1 \text{ for every } s \in S. \tag{12}$$

Similarly, restrictions (13) and (14) ensure that the duty will not serve ($x_{r1,r2,s} = 1$) more than one segment.

$$\sum_{r'\in R} x_{r',r,s} \le 1 \text{ for every } s \in S, r \in R, \tag{13}$$

$$\sum_{r'\in R} x_{r,r',s} \le 1 \text{ for every } s \in S, r' \in R. \tag{14}$$

The last limitation (15) means that the sum of segments started by the duties is equal to the sum of the completed segments

$$\sum_{r'\in R} x_{r',r,s} + y_{r,s} = \sum_{r'\in R} x_{r,r',s} + z_{r,s} \text{ for every } s \in S, r \in R. \tag{15}$$

5. Computational results

In total, 5 instances were checked, each of which consisted of 3 circuits. In the first instance, cycles 1, 2 and 3 containing 48 segments were considered, in the second one cycles 2, 3 and 4 containing 40 segments were analyzed, in the third one there were 39 segments making up the 4, 5 and 6 cycles. In the fourth one circuits 5, 6 and 7 containing 42 segments were considered, while in the fifth one, the analyses of circuits 1, 2 and 5 containing 45 segments were analyzed.

The test instances were solved using the AIMMS optimization package. The results obtained are presented in Table 1. These results were compared with the schedules developed by the planner (Table 2) in order to determine the efficiency of drivers' working time. Table 3 presents a list of selected parameters for 5 instances.

The analysis of the data presented in the Table 3 demonstrates that the planner developed 3 drivers' working time schedules for 7 duties, and 2 schedules for 8 duties, while the model that was used to solve the 5 test instances, indicated that it is possible to develop working time schedules of 7 duties for each instance.

It is worth noting, that the duration of duties, according to the model, is shorter than the times in the plans of the planner. The average duration of duties according to the model is 64 h and 23 min, while according to the planner schedule, it equals 72 h and 17 min. The results indicate, on average, an approximately 11% improvement in the use of drivers' working time.

Table 1. The results of computational instances obtained as a result of the application of the model, solved using the AIMMS optimization package

Number of test instance	Number of duty	Number of the first segment in duty	Number of the last segment in duty	Number of the depot	Length of duty	Travel time from the depot to the starting station of the segment	Travel time of the terminal station to the depot
Test instance 1	1	1	9	1	8.65		
	2	10	48	1	11.59		0.92
	3	31	31	3	8.00		
	4	32	24	3	8.00		
	5	19	40	3	8.27		0.32
	6	25	30	3	8,00		
	7	41	18	3	11.43	0.32	
Test instance 2	1	25	30	1	10.01		0.92
	2	35	40	1	9.47		0.92
	3	31	34	1	8.19	0.92	
	4	14	6	3	8.00		
	5	7	12	3	8.00		
	6	13	13	3	8.00		
	7	1	24	2	11.72	0.89	0.89
Test instance 3	1	32	10	1	9.44		0.92
	2	1	4	1	8.19	0.92	
	3	11	31	1	9.50	0.92	
	4	18	39	3	9.03		0.50
	5	5	24	3	11.11	1.88	
	6	25	25	3	8.00		
	7	26	17	3	8.00		
Test instance 4	1	1	21	1	9.17	0.92	
	2	36	42	1	11.70		
	3	7	14	3	11.14	0.50	
	4	24	29	3	8.00		0.50
	5	30	6	3	8.47	0.50	0.50
	6	16	23	3	10.17		
	7	15	15	3	8.00		
Test instance 5	1	1	9	1	8.65		
	2	31	31	3	8.00		
	3	32	40	1	10.90	0.92	
	4	19	11	3	10.62		
	5	12	18	3	8.73		
	6	27	30	3	8.00	1.79	
	7	41	45	2	9.79	0.89	0.89

Source: own elaboration based on calculations.

Table 2. The results of computational instances obtained by the planner

Number of test instance	Number of duty	Number of the first segment in duty	Number of the last segment in duty	Number of the depot	Length of duty	Travel time from the depot to the starting station of the segment	Travel time of the terminal station to the depot
Test instance 1	1	32	11	3	11.65		
	2	19	41	3	10.62		0.85
	3	1	28	1	11.90		
	4	42	46	2	8.00		0.88
	5	12	31	3	9.73		
	6	29	30	1	8.00		1.88
	7	47	48	1	8.00		0.92
Test instance 2	1	1	8	3	10.48		1.88
	2	14	23	3	10.95		0.89
	3	31	9	1	11.60	0.92	0.92
	4	35	30	1	10.47		0.92
	5	24	29	2	11.48		2.37
	6	10	12	1	8.00	0.92	1.88
	7	13	13	3	8.00		
Test instance 3	1	1	31	1	10.17	0.92	
	2	11	19	1	10.91	0.92	
	3	26	27	3	8.00		
	4	32	39	1	10.10		2.43
	5	20	24	1	9.90		1.88
	6	5	10	1	9.47		0.92
	7	25	25	3	8.00		
	8	28	29	1	8.00		
Test instance 4	1	1	9	1	10.91	0.92	
	2	16	35	3	9.73		1.88
	3	10	14	1	9.90		1.88
	4	18	41	1	11.60		
	5	22	29	1	10.10		2.43
	6	30	32	3	8.00	0.5	
	7	15	15	3	8.00		
	8	42	42	1	8.00		
Test instance 5	1	1	28	1	11.90		
	2	10	17	1	11.10		1.88
	3	19	26	3	10.48		1.88
	4	32	40	1	10.91	0.92	
	5	41	31	1	11.38		1.88
	6	29	30	1	8.00		
	7	18	18	3	8.00		

Source: railway carrier data.

Table 3. Comparative summary of selected parameters for two variants of five instances

Number of test instance	1		2		3		4		5	
Plan variant	Model	Planner	Model	Planner	Model	Planner	Model	Planner	Model	Planner
Number of duties	7	7	7	7	7	8	7	8	7	7
Duration of duties	63.94	67.90	63.39	70.98	63.27	74.55	66.65	76.24	64.69	71.77
Minimum duty time	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
Maximum duty time	11.59	11.90	11.72	11.60	11.11	10.91	11.70	11.60	10.90	11.90
Number of duties beginning/ending in a given depot	4	3	3	1	2	3	3	3	4	3
The number of journeys from the depot to the starting station of the segment	1	0	2	2	3	2	3	2	3	1
The number of journeys from the terminal station to the depot	2	4	3	6	2	3	2	3	1	3
Travel time between the depot and the beginning/ terminal station of the segment	1.56	4.53	4.54	10.7	5.14	7.07	2.92	7.61	4.49	6.56
Solving time [min]	3.32	19	2.42	21	1.70	16	4.07	17	2.47	14

Source: own elaboration based on calculations.

As can be seen from Table 3, the number of journeys between the depot and the start or terminal station of the segment is smaller by 4 journeys in the schedules prepared using the optimization model, compared to that by the planner. The model used to solve the five instances, indicated that the travel times between the depot and the start or terminal station of the segment were shorter on average by 3 hours and 34 minutes from the times set in the planner schedules. This parameter is important from the point of view of the transport company because it is not connected with effective work. The data also demonstrates that the number of duties starting and ending in a given depot, without having to travel to or from the depot, is higher by 3 than in the case of duties indicated by the transport planner.

In addition, the optimizer needs less time to designate the solution than the planner. The optimization model presented solutions on average after 2 minutes and 48 seconds, while the planner needed 17 minutes and 24 seconds to set the schedules.

6. Conclusions

The article presents the problem of railway crew scheduling. The assumptions that should be included in the analyzed issue are indicated. Next, the model's proposal was presented, the use of which would make it possible to schedule plans of duties.

In the empirical part of the work, the introduced model was used to solve five instances. In order to compare and evaluate the results from the model, they were compiled with real data (i.e. the results obtained by the planner). For each variant of the instance solved using the model, shorter durations of train drivers' duties were achieved. The results indicated, on average, an approximately 11% improvement in the use of drivers' working time, which would reduce personnel costs in the company. In addition, the optimizer needed less time to plan the solution than the planner – on average 14 minutes and 37 seconds.

Unfortunately, at the current stage of the work it should be noted that limiting the number of circulations to 3, i.e. maximum 48 segments, quite significantly reduces the possibilities of using the proposed model. It should also be noted that this model has been used as an exact model, while its implementation in practice means the necessity of the frequent updating of its parameters and solving at least several of them during the day. Taking into account the above limitations, further research will aim to develop a heuristic model that will allow to determine a solution (schedule of services) for more than 3 circuits (48 segments).

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