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Ewa Dziwok
University of Economics in Katowice
e-mail: ewa.dziwok@ue.katowice.pl
ORCID: 0000-0003-3584-1398

THE ROLE OF A REFERENCE YIELD FITTING TECHNIQUE IN THE FUND TRANSFER PRICING MECHANISM

ZNACZENIE TECHNIKI MODELOWANIA W MECHANIZMIE CEN TRANSFEROWYCH

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Summary: The funds transfer pricing (FTP) structure has become the basis for the process of asset and liability management (ALM) in a modern bank. According to the supervisory documents, FTP is thus a regulatory constraint and an important tool in the ALM process. What is more, institutions should have an adequate internal transfer pricing mechanism based on the reference rate delivered from the market in the form of the yield curve. The fragility and sensitivity of the reference yield over time could have huge consequences for the liquidity risk management process. The aim of the article is to compare the methods of estimation FTP reference yield depending on the goodness-of-fit methodology (least square methods based on short rates (up to 1 year) and prices taken from the Polish market will be considered)). The data taken into account cover the period between 2005-2017 and the results obtained allow to point out the periods when disturbances on the market affected the goodness of a model's fit to real data and, consequently, have an effect on the fund transfer pricing mechanism.

Keywords: liquidity risk, risk measurement, fund transfer pricing.

Streszczenie: Struktura cen transferowych (FTP) stała się podstawą procesu zarządzania aktywami i pasywami (ALM) we współczesnym banku. Zgodnie z dokumentami nadzoru FTP jest zatem ograniczeniem regulacyjnym i ważnym narzędziem w procesie ALM. Co więcej, instytucje powinny mieć odpowiedni wewnętrzny mechanizm cen transferowych oparty na stopie referencyjnej dostarczanej z rynku w formie krzywej dochodowości. Wrażliwość referencyjnej stopy zwrotu w czasie może mieć ogromne konsekwencje dla procesu zarządzania ryzykiem płynności. Celem artykułu jest porównanie metod estymacji stopy referencyjnej służącej konstrukcji FTP w zależności od stopnia dopasowania (uwzględnione zostaną metody najmniejszych kwadratów oparte na stopach i cenach dla danych do 1 roku, pochodzących z rynku polskiego). Dane ujęte w analizie obejmują lata 2005-2017, a uzyskane wyniki wskazują na okresy, w których zakłócenia na rynku wpływały na jakość dopasowania modelu do danych rzeczywistych, a w konsekwencji – na mechanizm ustalania cen transferowych.

Słowa kluczowe: pomiar ryzyka, ryzyko płynności, ceny transferowe.

1. Introduction

The funds transfer pricing (FTP) mechanism has become the basis for the process of asset and liability management (ALM) in modern banking. The significance of pricing liquidity risk derives from the Basel Principles for Sound Liquidity Risk Management and Supervision [BCBS 2008]. In September 2009, the EU introduced the amendments to Annex V of the CRD and the Committee of European Banking Supervisors (CEBS) published Guidelines on Liquidity Cost Benefit Allocation [CEBS 2010]. According to these documents, FTP is thus a regulatory constraint and an important tool in the ALM process. Moreover, institutions should have an adequate internal transfer pricing mechanism based on the reference rate delivered from the market in the form of a yield curve.

Yield curve modeling is the process of building a continuous function from the market data, both securities and interest rate derivatives. The construction generally uses two types of models: parametric (evaluated by Nelson-Siegel and Svensson), and those based on B-splines (cubic splines). Both types provide a lot of possibilities for further analysis and forecasting.

The aim of the article is to compare the methods of estimation depending on the goodness-of-fit methodology (least square methods based on rates and prices will be analyzed). The data taken into account cover the period between 2005-2017 and the results obtained allow to point out the periods when disturbances on the market affected the goodness of a model's fit to real data, and consequently have an effect on the fund transfer pricing mechanism.

2. A yield curve construction

The idea of the fund transfer pricing mechanism is based on the reference rate which is often market determined in the form of fixing (i.e. WIBOR, bonds' fixing). The first step is the construction of a reference curve (or yield curve) through the interest rate term structure model. The next step is to take into account the institutions' own spread as well as the bid/ask spread, depending on the side of the transaction. While the above elements are done, the liquidity cost components are added.

There are plenty of methods, widely described by James, Weber [2000] that allow to construct the yield curve. To achieve a smoothing yield curve construction, two main groups of models should be taken into account: parametric ones introduced by Nelson and Siegel [1987] and those extended by Svensson [1994] as well as cubic splines developed by Fisher et al. [1995] and Waggoner [1997].

An idiosyncrasy of parametric models (which this article focuses on) involves their simplicity and the small number of parameters to be estimated. Additionally the functional form determines the three main features (smoothness, flexibility and stability) expected from the correctly estimated curve [Anderson, Sleath 2001].

Following the definition suggested by Nawalkha, Soto, Beliaeva [2005], the term structure of interest rates gives the relationship between the yield of the investment with the same credit quality but different term to maturity. Classical economic theories described by Fisher [1930], developed by Hicks [1946] and Cox et al. [1981] allow to define four main shapes of the term structure: positive, negative, flat and humped, that could be constructed under special circumstances.

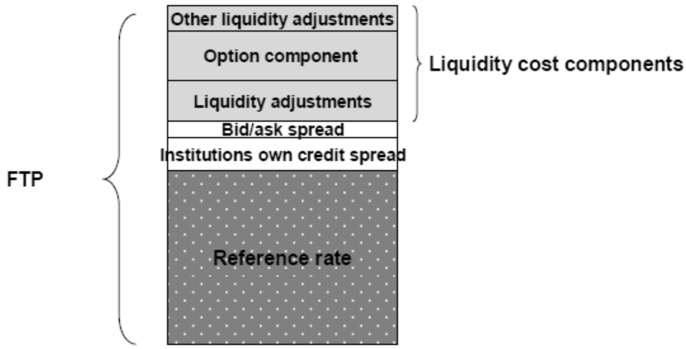

Fig. 1. FTP mechanism – a construction of the yield curve

Source: [CEBS 2010, p. 10].

Generally a term structure is typically built with a set of liquid and common assets; the problem arises in cases of a non-liquid market (as in Poland) with a small number of data. One solution is to analyze several types of models and then to choose the one which allows to achieve the best approximation.

Definition 1

A zero-coupon bond is an instrument with only two cashflows: first – at the beginning of the investment – called the price; the second one is cashflow which is paid at maturity.

Suppose the price of the zero-coupon bond is denoted as *P*, with cashflow c at maturity τ and yield to maturity $i(\tau)$ understood as the spot rate. If the continuously compound interest is taken into account, the price is the discounted value of future cashflow c:

$$P(\tau) = c \cdot e^{-i(\tau)\cdot\tau} , \tag{1}$$

where: $P(\tau)$ – price of the zero-coupon bond with maturity τ, c – cashflow at time τ, $i(\tau)$ – spot rate.

Following Audley et al. [2002], it is important that under continuous compounding, the spot rate is understood as the continuously compounded instantaneous rate of return. Graphically, the spot rate may be visualized as the yield corresponding to the point at which the spot yield curve intercepts the yield axis.

Definition 2

Function $\delta : \Re_+ \to (0\,;1\,]$ is called the discount function and is expressed as:

$$\delta(\tau) = e^{-i(\tau)\cdot\tau} \tag{2}$$

Lemma:
Every default-free coupon bond can be described as a portfolio of zero-coupon bonds (with the maturities appropriate to the payment dates).

Proof:
If P is a coupon bond with a set of future cashflows c_j, observed at time τ_j, $j = 1,2,...,k$ and let (for simplicity) spot rates $i_j(\tau_j) = i_j$, $j = 1,2,...,k$, then the price of a coupon bond could be expressed as the present value of cashflows:

$$P = c_1 \cdot e^{-i_1\tau_1} + c_2 \cdot e^{-i_2\tau_2} + ... + c_k \cdot e^{-i_k\tau_k}$$

According to formula 1, the coupon bond can be described as a linear combination of discount factors δ_j, $j = 1,2,...,k$:

$$P = c_1 \cdot \delta_1 + c_2 \cdot \delta_2 + ... + c_k \cdot \delta_k\,.$$

Definition 3

The instantaneous forward rate $f(\tau) \equiv f_{\tau,\tau+\Delta\tau}$, defined by de La Grandville [de La Grandville 2001], is understood as the marginal rate of return implied for an infinitesimally short period (length of investment) $\Delta\tau \to 0$.

$$i(\tau) = \frac{1}{\tau}\int_0^\tau f(m)\,dm \tag{3}$$

The existence of an inter-relation between discount factor $\delta(\tau)$, spot rate $i(\tau)$ and the forward one $f(\tau)$ (in continuous time) could be – after formulas (1) to (3) illustrated as below:

$$P(\tau) = \delta(\tau) = e^{-\,i(\tau)\cdot\tau} = e^{-\int_0^\tau f(m)dm}\,, \tag{4}$$

where: $P(\tau)$ – price of a bond, $\delta(\tau)$ – discount factor, $i(\tau)$ – spot rate, $f(\tau)$ – forward rate, τ – term to maturity.

The term structure construction begins by gathering the sample of the instrument to be used. In the Polish money market which is analyzed here, there is a lack of short-term data (apart from money market fixing quotations), which is why all the available quotations were taken into account with no quality check.

Suppose that there is a set of *k* instruments, with market values P_l, $l = 1,2,...,k$ and cashflows $c_{l,j}$ for bond *l* at time τ_j, $j = 1,2,...,k$. Let $C = \{c_{l,j}\}_{l=1,..,k, j=1,..,k}$ is a cashflow matrix, generally a sparse one with most entries zero and $P = \{P_l\}_{l=1,..,k}$ is the price vector. The knowledge of *C* and *P* determines the discount factors:

$$P \; = C \cdot \left[\delta(\tau_1) \quad \delta(\tau_2) \quad \dots \quad \delta(\tau_k)\right]^T. \tag{5}$$

To fit the curve it is necessary to choose an interpolation method, (a form of the theoretical function) which allows to obtain discount factors $\overline{\delta}(\tau)$ for all the maturities (between zero and infinity). McCulloch [1971, 1975] used a piecewise polynomial function, but the main problem was the instability of this model and the high possibility of unrealistic, negative forward rates (through formula 4).

The utilization of a parametric model (Nelson-Siegel) allows to calculate forward rates directly (and then with formula 4, obtain discount factors). It guarantees different shapes of theoretical term structure.

For further analysis, the Nelson-Siegel model with four parameters $\overline{\delta}(\tau) = \overline{\delta}(\tau | \beta_0, \beta_1, \beta_2, v)$ is taken into account:

$$f(\tau) = \beta_0 + (\beta_1 + \beta_2 \frac{\tau}{v}) \cdot e^{-\frac{\tau}{v}}, \tag{6}$$

where: $f(\tau)$ – instantaneous forward rate, $[\beta_0, \beta_1, \beta_2, v]$– vector of parameters describing the curve: β_0 – parameter which shows the limit in infinity, $\beta_0 > 0$, β_1 – parameter which shows the limit in infinity, $\beta_0 + \beta_1 \geq 0$, β_2 – parameter which shows the strength of curvature, υ_1 – parameter which shows the place of curvature, $\upsilon_1 > 0$.

According to formula (4), a whole set of discount factors (for all cashflows) could be calculated from forward rates. Then a vector of theoretical prices $\overline{P} = \{\overline{P_l}\}_{l=1,..,k}$ can be described as a product of cash flow matrix C and a vector of discount factors (in a functional form):

$$\overline{P} \; = C \cdot \left[\overline{\delta}(\tau_1) \quad \overline{\delta}(\tau_2) \quad \dots \quad \overline{\delta}(\tau_k)\right]^T. \tag{7}$$

A set of parameters $[\beta_0, \beta_1, \beta_2, v]$ is estimated by minimizing mean square errors between market and theoretical prices (taken from the fitted curve):

$$\frac{\sum_{l=1}^{k}(P_l-\overline{P_l})^2}{k} \to \min, \tag{8a}$$

and between market and theoretical rates:

$$\frac{\sum_{l=1}^{k}(i_l-\overline{i_l})^2}{k} \to \min, \tag{8b}$$

where: $P_l - \overline{P_l}$ – price error of l-th asset, $i_l - \overline{i_l}$ – yield error of l-th asset, k – number of bonds.

The goodness-of-fit comparison (for prices and yields respectively) is possible by the calculation of errors through time. A low mean value proves the flexibility of the model and shows its ability to fit the data quite accurately.

3. Data and results

For this research, Polish money market rates were taken into account. They are represented by WIBOR (Warsaw InterBank Offered Rate) – a panel of interbank lending rates calculated and published daily at around 11.00 a.m. Warsaw time by ACI Poland (until 30.06.2017) and since then by GPW Benchmark S.A.. Contrary to the LIBOR, the WIBOR rate is the average of the quotations provided by the chosen banks which have received the status of the so-called Primary Dealers.

The maturities of WIBOR rates have changed in recent years and nowadays they range from overnight to one year. As a representation of the interbank market, the WIBOR rates reflect default risk affected by the quoting bank's condition (an interbank loan is unsecured) and the liquidity of the market. Because the shortest, overnight rate illustrates the demand for liquidity and strongly depends on the obligatory reserve maintenance period, its volatility is very high. For the purposes of following research, daily rates from T/N to one year were taken (eight in total: T/N, 1 week, 2 weeks, 1month, 3, 6, and 9 months, one year) from the beginning of 2005 to the end of 2017.

One should remember that the vector of theoretical prices $\overline{P}$ could be expressed as a product of a cash flow matrix and a vector of discount factors (formula 7), the process of fitting the term structure starts from the construction of a cash flow matrix. For the money market it forms a square diagonal one (with eight columns and rows in this analysis) because each of these instruments has only one cash flow – the principal to be repaid at maturity.

Considering two different ways of MSE error calculation (8a, 8b) and following the Nelson-Siegel parametric model, two sets of instantaneous rates can be found. To achieve these results two macros were written in VBA code that helped to receive four theoretical prices for each of analyzed days. As a result, two vectors of MSE were calculated.

The plots of errors for the chosen methods allow to analyze the sensitivity of the model to disturbances in the market (Figures 2 and 3).

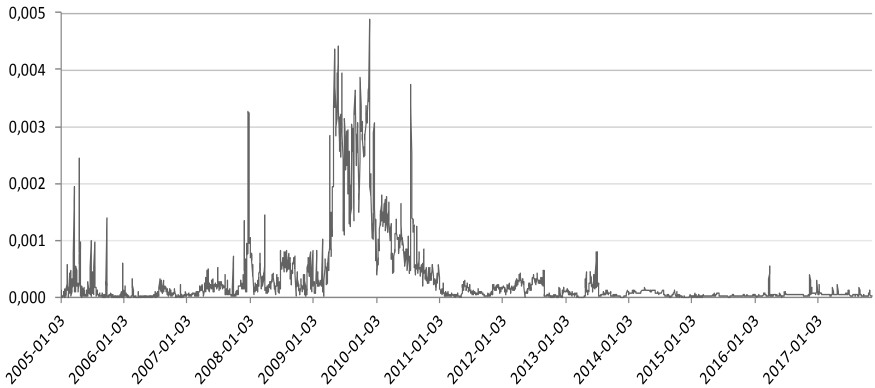

Fig. 2. MSE errors between market and theoretical rates

Source: own calculations.

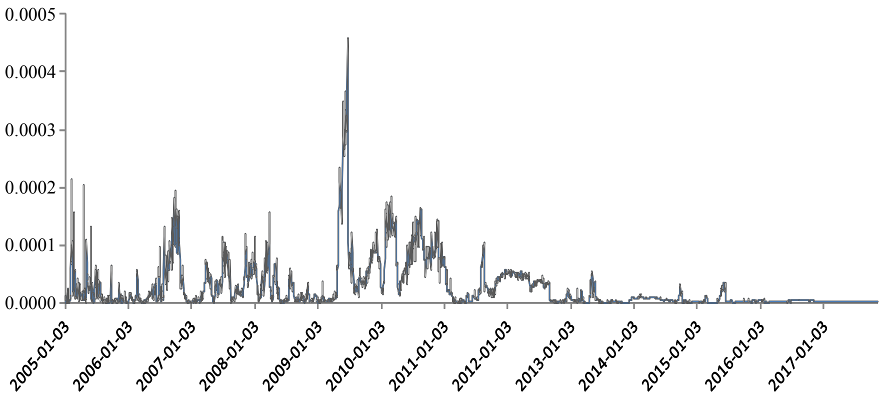

Fig. 3. MSE errors between market and theoretical prices

Source: own calculations.

From the beginning of the financial crisis, the volatility of assets' rates became very high which caused problems with data fitting. The highest value of errors was observed during financial crises and accelerated in 2009-2010. Since 2014 the observed volatility started to be lower – mainly as a result of lower market confidence and the introduction (in 2016) of the banking tax.

The selection problem presented here (how to find the best method of the reference yield construction by adopting a comparison of errors) shows that the best results were achieved by the implementation of the MSE price methodology (through a minimizing of the sum of squared errors of the market and theoretical prices).

4. Conclusion

In order for the FTP mechanism to be effective, it must demonstrate neutrality to changes in market rates. In addition, the FTP system must take into account time lag, because the designated curve serves as a reference yield for earlier price decisions. This is why the fragility and sensitivity of the reference yield in time could have huge consequences for the liquidity risk management process.

Two different fitting techniques were applied here (based on price errors and rate errors minimizing procedure) to compare the quality of the parametric model and its effectiveness in the FTP mechanism. According to the analysis the most flexible and accurate fitting method (represented through a low value for the error) is the procedure which utilizes the parametric Nelson-Siegel model with MSE based on prices. In addition, this model was much more resistant to market disturbances, especially in the beginning of 2008. This contrasts with the high level of errors during 2009-2010 when the interest rates were unusually volatile.

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