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*Bartosz Olesiński**, *Marek Rozkrut***, *Andrzej Torój****

HOW TIME-VARYING ELASTICITIES OF DEMAND TRANSLATE INTO THE EXCISE-RELATED LAFFER SURFACE

In an environment of growing real prices and changing consumption patterns in the tobacco market, the question arises whether the price elasticities of demand may be estimated as constant parameters over multi-annual samples. The authors develop a methodological framework for estimating time-varying demand elasticities in a state-space model, estimated via maximum likelihood based on the Kalman filter. This model is applied to evaluate various, alternative paths of tobacco excise tax rates. Importantly, both in estimation and in simulations, the authors account not only for changes in the level, but also in the structure of excise tax by exploring the market segmentation into a lower and a higher end of the market. This allows the authors to contribute to the existing literature about the optimum structuring of the tax between the specific and *ad valorem* rates and to analyse the Laffer surface (rather than a curve). The measurement results indicate some growth in the magnitude of price elasticity of demand since 2005, and the simulations show that the differences between the actual and the optimum taxation policy for tobacco products were marginal in the 2014-2018 period.

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1. INTRODUCTION

The indirect taxation of tobacco products constitutes a significant part of government revenues in many EU countries. In Poland, for example, in 2017 the revenue from the excise duty on tobacco products accounted for 5.4 per cent of state budget revenues, 8.3 per cent of total indirect tax revenues and 27.5 per cent of overall excise tax revenues. Consequently, seeking an optimum level of the excise rate should play an important role in tax policy. According to the concept of the Laffer curve, if the policy objective was to maximise revenues, the tax rate cannot be too low but it also should not be

* Warsaw School of Economics, EY Poland.
** EY Poland.
*** Institute of Econometrics, Warsaw School of Economics.

too high, as the rising unit cost of a product reduces potential demanded quantity and hence the value of tax revenues (Laffer, 2004). Therefore, while pursuing optimum revenues, policymakers should be aware of the tobacco demand function of households, which is often summarised in a single parameter – price elasticity of demand.

Another constraint imposed on the tax policy of the European Union Member States is compliance with the EU regulations. In particular, the excise tax on tobacco products has to have a hybrid structure in all EU countries (see Council Directive 2011/64/EU of 11 June 2011 on the structure and rates of excise duty applied to manufactured tobacco, OJ L 176/24). It consists of a specific part (i.e. due for each piece or pack in a specified amount) and *ad valorem* part (i.e. due as a fraction of the retail sales price). This results in a non-linear relationship between the price of a cigarette pack and the amount of excise tax to be paid. The share of the specific rate should remain between 7.5 per cent and 76.5 per cent. In addition, the excise on a single pack is restricted to be higher or equal to 60 per cent of the last year's weighted average retail price (WAP), as well as higher or equal to 90 euros per 1000 pieces. Within these regulatory constraints, policymakers may adjust the overall level of taxation with any of the following components: specific, *ad valorem* and minimum rate. However, under heterogeneous demand, the same shift in the overall taxation may be achieved through different combinations of the above policy instruments, which in turn may have a different impact on various market segments, not least due to changes in relative prices of tobacco products in these segments. As a result, it is not only the level of the tax, but also the way it is established (its structure) that determine the ultimate budgetary outcome of the policy.

The aim of this paper is to develop a methodological framework for finding an optimum policy. The authors take the perspective of fiscal policymakers, hereinafter defined as pursuing the revenue-maximizing policy. On top of the excise tax, the authors also take into account the VAT revenues and apply this methodology to the Polish market.

It should be mentioned that this is one of the two possible perspectives that could be adopted by policymakers. The other could additionally take into account the impact of tax policy on public health, and the financial implications of improvement or deterioration in public health (e.g. via the direct and indirect costs of smoking-related diseases). The resulting, alternative objective function should therefore encompass the public expenditures related to the direct costs of smoking-related diseases, as well as the foregone revenues due to the negative health shock in the labour market.

This issue requires extensive additional discussion, remaining beyond the scope of this paper.

There is a lot of literature focusing on the economics of smoking and the interplay of the consumer and government strategies in the tobacco market (see, e.g. Chaloupka and Warner, 2000; Cawley and Ruhm, 2011; IARC, 2011 for extensive overviews). One strand of this literature is focused on non-price factors, such as smoking restrictions (Gallet, 2007; Anger, Kvasnicka, Siedler, 2011), while another is related to tax policy with respect to tobacco products (with more recent contributions of Goel and Nelson, 2012; Irvine and Sims, 2014), and in particular – to finding an optimum level in the spirit of the Laffer curve (van Walbeek, 1996; Strateickus, Fadejeva, Kaze, 2011). This paper mainly contributes to the latter type of literature by allowing for heterogeneous demand for tobacco products in the revenue-maximising framework. Namely, the authors introduce the split into distinct segments and account for substitution effects, which is rarely studied in the context of the tobacco market (with notable exceptions provided by the product-level discrete choice literature, e.g. Ciliberto and Kuminoff, 2010; Min, 2011; Liu et al., 2015).

The authors also allow for time-variability in price elasticities by adopting an agnostic approach to estimation, i.e. without assuming *ex ante* any specific hypothesised dependency map or functional form that could describe such variability. However, the paper analyses *ex post* the possible drivers of changes in price elasticities of demand by carrying out additional SVEC analysis (see Section 3.2.1). While applying the Kalman filter maximum likelihood approach, the authors draw on the existing works focusing on estimation of macroeconomic quantities such as potential output and output gap (Kuttner, 1994; Ozbek and Ozlale, 2005; Basistha and Nelson, 2007; Konuki, 2010), natural unemployment rate (Apel and Jansson, 1999) and natural rate of interest (Laubach and Williams, 2003; Garnier and Wilhelmsen, 2009; Brzoza-Brzezina, 2004). While the Kalman filtering approach has already been used in the context of demand for tobacco products (e.g. by Mazzocchi, 2006, or Park, 2010) those interesting contributions are not followed by a public policy analysis, which is the main focus of this paper.

The rest of the paper is organised as follows. Section 2 explains how the demand model is constructed and how time-varying price elasticities of demand are measured. Section 3 presents the application of the model in a simulation analysis for Poland over the period 2015-2018, with various strategies regarding the excise tax rates and components. As a matter of sensitivity check, the authors consider different strategies on the manufacturer side (price setting mechanisms) and consumer side (constant and price-level-dependent elasticities of demand). Section 4 concludes.

2. ECONOMETRIC ANALYSIS: STATE-SPACE MODEL

This analysis starts with a discussion of the neoclassical theory of consumer choice underlying the empirical demand model for tobacco products. Next, the authors present their data sources, and further the structure of the econometric model with a technical description of the applied state space specification. This demand system consists of three segments (two for factory-made cigarettes, as discussed below, and one for fine-cut tobacco), which was directly related to data availability with respect to retail market volumes and prices in the Polish market at the moment of the analysis. The authors conclude this section by demonstrating how this model fits the data.

2.1. The underlying microeconomic theory

This paper neither aspires to rigorously test the assumptions of the neoclassical demand theory nor intends to investigate the features of the prevailing utility functions of the cigarette consumers. The authors use some classical results as a basis for our demand system empirical specification and assume that households face the following standard utility maximisation problem:

$$\max_{\mathbf{Q}} u\left(\mathbf{Q}\right) \; s.t. \; \mathbf{P}'\mathbf{Q} = Y, \tag{1}$$

in which $\mathbf{Q}$ is the $n \times 1$ vector of quantities of goods, $\mathbf{P}$ denotes the corresponding vector of retail (nominal) prices and Y is a scalar denoting the nominal income (see e.g. Barnett and Serletis, 2008). The solution to this problem is the system of Marshallian ordinary demand functions of the form:

$$\mathbf{Q} = \mathbf{Q}\left(\mathbf{P}, Y\right). \tag{2}$$

Marshallian demand satisfies the following properties (Barnett and Serletis, 2008): (i) positivity; (ii) adding-up (or summability) $\mathbf{P}'\mathbf{Q} = Y$, (iii) homogeneity of degree zero in $\left(\mathbf{P}, Y\right)$, i.e. $\mathbf{Q}\left(t\mathbf{P}, tY\right) = \mathbf{Q}\left(\mathbf{P}, Y\right)$ for any $t > 0$ (implying there is no money illusion in this model); finally, (iv) the matrix of substitution effects is symmetric and negative semidefinite. For each good i one can take the total differential of the corresponding demand function:

$$dQ_i = \sum_{j=1}^{n} \frac{\partial Q_i}{\partial P_j} dP_j + \frac{\partial Q_i}{\partial Y} dY. \tag{3}$$

As a next step, we divide both sides of (3) by Q_i to obtain:

$$\frac{dQ_i}{Q_i} = \sum_{j=1}^{n} \frac{\partial Q_i}{\partial P_j} \frac{dP_j}{Q_i} \frac{P_j}{P_j} + \frac{\partial Q_i}{\partial Y} \frac{dY}{Q_i} \frac{Y}{Y}, \tag{4}$$

in which we substitute $dlog(z) = dz/z$ and the price and income elasticities of demand: $\eta_{ij} = (\partial Q_i/\partial P_j)(P_j/Q_i)$ and $\eta_{iY} = (\partial Q_i/\partial Y)(Y/Q_i)$, respectively, to obtain:

$$dlog(Q_i) = \sum_{j=1}^{n} \eta_{ij} dlog(P_j) + \eta_{iY} dlog(Y). \tag{5}$$

In this case, we have three tobacco market segments and a composite external good, reflecting all the goods and services except for cigarettes and tobacco consumed by households. Using the homogeneity assumption, i.e. $\sum_{j=1}^{n} \eta_{ij} + \eta_{iY} = 0$, one can carry out the following operation related to the external good (for which $i = n$):

$$\begin{aligned} dlog(Q_i) &= \sum_{j=1}^{n} \eta_{ij} dlog(P_j) + \eta_{iY} dlog(Y) - \left[\sum_{j=1}^{n} \eta_{ij} + \eta_{iY}\right] dlog(P_n) \\ &= \sum_{j=1}^{n-1} \eta_{ij} dlog(P_j / P_n) + \eta_{iY} dlog(Y / P_n). \end{aligned} \tag{6}$$

Assuming, for simplicity, that the price of the external good is proportional to the CPI index, one can use the CPI-deflated prices and disposable income as regressors in this econometric analysis.

It will prove useful to transform equation (6) further in order to account for some properties of the government's problem to maximise tobacco-related revenues. To see this, consider a simplified problem in which there is only *ad valorem* tax $\tau \in (0;1]$, all the monetary values are already CPI-deflated and there is no external good (so the number of cigarette segments equals n). The government revenues from segment *i* can be calculated as follows:

$$R_i(\tau) = \tau P_i(\tau) Q_i[\mathbf{P}(\tau)]. \tag{7}$$

The problem faced by the government is formulated as follows:

$$\max_{\tau} \sum_{i=1}^{n} R_i(\tau) \tag{8}$$

and it can be shown that the first-order condition simplifies to (see Appendix for the derivation):

$$\tau = -\frac{1}{\sum_{i=1}^{n}\left[s_i\left(\sum_{j=1}^{n}\eta_{ij}\right)\right]}, \tag{9}$$

where s_i stands for the share of segment i in the overall tobacco expenditures in the duty-paid market. Note that $\sum_{i=1}^{n} s_i\left(\sum_{j=1}^{n}\eta_{ij}\right)$ is the expenditure-weighted average of sums of all the price elasticities of demand in each segment. There is a valid solution only if this weighted elasticity is lower than -1. If, in contrast, $\sum_{i=1}^{n} s_i\left(\sum_{j=1}^{n}\eta_{ij}\right) \in (-1;0)$, the optimum *ad valorem rate* is higher than 100 per cent and if $\sum_{i=1}^{n} s_i\left(\sum_{j=1}^{n}\eta_{ij}\right) > 0$, the optimum *ad valorem rate* is negative (such taxation levels are infeasible). At the same time, at this stage nothing is said about the signs of particular components of this weighted average, i.e. the own and cross-price elasticities of demand for particular segments. Note that it is a simplified result – the formulas taking into account both *the ad valorem* and the specific rate still need to be derived. Nevertheless, this exercise helps formulate some intuition with respect to the necessary conditions for the optimum government revenues level to exist.

In equation (9), the key parameter is $\sum_{j=1}^{n}\eta_{ij}$ and it can be estimated directly as one of the linear parameters of the demand system. To see this, transform equation (6) by adding and subtracting $\left(\sum_{j\neq i}\eta_{ij}\right) dlog\left(P_i/P_n\right)$ on the right-hand side to obtain:

$$\begin{aligned} dlog(Q_i) &= \sum_{j=1}^{n-1}\eta_{ij} dlog\left(\frac{P_j}{P_n}\right) + \left(\sum_{j\neq i}\eta_{ij}\right) dlog\left(\frac{P_i}{P_n}\right) - \left(\sum_{j\neq i}\eta_{ij}\right) dlog\left(\frac{P_i}{P_n}\right) + \eta_{iY} dlog\left(\frac{Y}{P_n}\right) \\ &= \left(\eta_{ii} + \sum_{j\neq i}\eta_{ij}\right) dlog\left(\frac{P_i}{P_n}\right) + \sum_{j\neq i}\eta_{ij}\left(dlog\left(\frac{P_j}{P_i}\right)\right) + \eta_{iY} dlog\left(\frac{Y}{P_n}\right) \\ &= \left(\sum_{j=1}^{n}\eta_{ij}\right) dlog\left(\frac{P_i}{P_n}\right) + \sum_{j\neq i}\eta_{ij}\left(dlog\left(\frac{P_j}{P_i}\right)\right) + \eta_{iY} dlog\left(\frac{Y}{P_n}\right). \end{aligned} \tag{10}$$

Another motivation for such a transformation in the case of the tobacco market is the fact that the same tax rates apply to all the segments, giving rise to the price collinearity problem. The authors deal with this by using price ratios in different segments as the regressors being responsible for capturing the substitution effects.

2.2. Data sources

The monthly time series cover the period from August 2005 to June 2014. The data includes retail sales volumes and weighted average retail prices of cigarettes in two market segments: low-price (LOW), and high-price (HIGH), that were defined by the authors based on the information obtained from BAT about price positioning of particular brands on the Polish market. The aggregation into final segments is done by the authors. Moreover, the authors include in the analysis the data on fine-cut tobacco (TOB).

The source of retail price data is Nielsen (Nielsen, Retail Index for Cigarettes and Tobacco categories, representing purchases made by the final customers in the retail channels monitored by Nielsen in the retail market for Total Poland for the 2005-2014 period). The volume data combines retail sales information from Nielsen (with analogous coverage of the volume sales data as in the case of the Nielsen price data) and wholesale market information provided by BAT ($Q_t^{i,N}$ and $Q_t^{i,W}$ respectively, for period t and segment i). The authors use the wholesale market data to capture the level because Nielsen data does not cover all the entities operating in the retail market in Poland over the 2005-2014 sample period. As a matter of fact, any pack of cigarettes (or portion of fine-cut tobacco) sold in the retail market must have been captured in the wholesale data in the respective month or one of the previous months. However, lags in supply to the retail market and seasonal stock building make the wholesale volumes yet another imperfect source of information. In order to get the best of the two data sources, the Nielsen data is used as a source of short-term variability, but it is additionally rescaled with a ratio between centred moving averages of wholesale and retail volumes so as to take account of the partial, and seasonal, coverage of the retail market in the volume data:

$$Q_t^i = Q_t^{i,N} \frac{\frac{1}{2}Q_{t-6}^{i,W} + \sum_{j=-5}^{5} Q_{t-j}^{i,W} + \frac{1}{2}Q_{t+6}^{i,W}}{\frac{1}{2}Q_{t-6}^{i,N} + \sum_{j=-5}^{5} Q_{t-j}^{i,N} + \frac{1}{2}Q_{t+6}^{i,N}}. \tag{11}$$

It is assumed that that the mid-term ratio is kept constant for the final 12 months of the sample. Note that under log-difference transformation applied to the data (see below), this correction is significant only to the extent that the coverage of the retail market by Nielsen varies over time. However, this approximation suggests that it does vary to such an extent that it might affect the results, and hence the correction is non-negligible.

The prices were deflated with HP-filtered CPI index. Moreover, to capture the income effects, driven e.g. by changes in the macroeconomic environment, the paper uses the data on real net disposable income available from the National Accounts (via Eurostat). These data were quadratically interpolated from quarterly to monthly frequency[1].

As the study considers a dynamic filtering problem with regards to non-stationary variables (real prices of tobacco products are generally increasing and the market volume is generally shrinking in the sample period), all the above-mentioned variables were log-transformed and year-on-year differences were computed, which approximately yield annual growth rates $dlog_{yoy}\left(Q_t\right)=log\left(Q_t\right)-log\left(Q_{t-12}\right)$. The authors used a year-on-year rather than month-on-month log dynamics in order to account for the seasonal patterns. Hereinafter, variables in levels (not in seasonal log-differences) will be indicated with capital letters in variable names.

2.3. The empirical specification and additional model assumptions

This empirical system is based on equation (10), which is essentially short-term in nature (both in the month-on-month and year-on-year variant). This is because a short-term specification provides a natural framework for estimating the short-term changes of time-varying parameters of demand.

The empirical system comprises the following set of demand equations, estimated as measurement equations in the state-space model:

$$\begin{aligned} dlog_{yoy}\left(Q_t^{LOW}\right) = \gamma_0 + \alpha_t^{LOW} dlog_{yoy}\left(P_t^{LOW}\right) + \\ +\beta_1 dlog_{yoy}\left(P_t^{HIGH} / P_t^{LOW}\right) + \\ +\beta_2 dlog_{yoy}\left(P_t^{TOB} / P_t^{LOW}\right) + \kappa_1 dlog_{yoy}\left(Y_t\right) + \varepsilon_t^{LOW}, \end{aligned} \tag{12}$$

[1] When interpolating the data using additional high-frequency time series (such as high-frequency wages data), additional noise was introduced into the model, which affected the stability of the estimation of the state-space model. Hence, one can consider this alternative approach as detrimental to the final results.

$$\begin{aligned} dlog_{yoy}\left(Q_t^{HIGH}\right) = \gamma_0 + \alpha_t^{HIGH} dlog_{yoy}\left(P_t^{HIGH}\right) + \\ +\beta_1 \frac{Q_{t-12}^{LOW}}{Q_{t-12}^{HIGH}} dlog_{yoy}\left(P_t^{LOW} / P_t^{HIGH}\right) + \\ +\beta_3 dlog_{yoy}\left(P_t^{TOB} / P_t^{HIGH}\right) + \kappa_2 dlog_{yoy}\left(Y_t\right) + \varepsilon_t^{HIGH}, \end{aligned} \tag{13}$$

$$\begin{aligned} dlog_{yoy}\left(Q_t^{TOB}\right) = \gamma_t + \beta_2 \frac{Q_{t-12}^{LOW}}{Q_{t-12}^{TOB}} dlog_{yoy}\left(P_t^{LOW} / P_t^{TOB}\right) + \\ +\beta_3 \frac{Q_{t-12}^{HIGH}}{Q_{t-12}^{TOB}} dlog_{yoy}\left(P_t^{HIGH} / P_t^{TOB}\right) + \\ +\kappa_3 dlog_{yoy}\left(Y_t\right) + \varepsilon_t^{TOB}. \end{aligned} \tag{14}$$

In any of the equations, the evolution of the respective volumes $dlog_{yoy}\left(Q_t\right)$ is explained with the evolution of real price in the same segments $dlog_{yoy}\left(P_t\right)$, the price ratios for the competitive segments and the segment in question, as well as the disposable income $dlog_{yoy}\left(Y_t\right)$. Note that by such a formulation we are making some additional model assumptions, going beyond the Marshallian specification of demand.

Firstly, it is only the own price elasticities of demand in both cigarette segments that we allow to vary over time (α_t^{LOW} and α_t^{HIGH}, which are the empirical variants of the crucial $\sum_{j=1}^{n} \eta_{ij}$ parameter discussed in the context of equations (9) and (10)). In contrast, this model assumes constant cross-price elasticities and income elasticities. This is for the sake of estimation efficiency and feasibility, as one cannot allow any parameter to vary freely and one should also bear in mind the limited sample size. The third time-varying parameter here is the intercept in fine-cut tobacco equation, γ_t. This is due to the fact that this market segment was expanding very dynamically in the sample period and driven not only by the demand-side, but also supply-side factors. Fine-cut tobacco was not commonly used and widely available in the Polish market before 2005 so the demand for this product was largely driven by non-price factors related to the product life cycle. The inclusion of the time-varying intercept allows to deal with the resulting potential omitted variable bias.

Furthermore, the authors impose restrictions on cross-price elasticities, enforcing that an outflow from e.g. LOW to HIGH due to an unfavourable (for LOW) change in relative prices should be – in terms of quantity – equal to inflow to HIGH from LOW due to the same change in relative prices (favourable for HIGH). Due to log-difference transformation, this is not ensured just by setting β_1 in equation (12) equal to β_1 in equation (13) (taking advantage of the property (iv) of the Marshallian demand functions, see Subsection 2.1), but also one of these coefficients has to be rescaled by the ratio of absolute volumes 12 months earlier (e.g. $Q_{t-12}^{LOW} / Q_{t-12}^{HIGH}$). The analogous approach is applied to β_2 and β_3 .

Moreover, one has to be aware that the problem of the shrinking tobacco market is not only related to the growing real retail prices of tobacco products, but also to other factors influencing consumption trends, such as health considerations, fashion, increasingly restrictive regulations related to smoking in public places, etc. These are not easily separable, for such trends have been going hand in hand with the growing retail prices over the 2005-2014 period. As a result, the only observable factor that allows such a separation is the faster or the slower pace of price increases at a monthly frequency. To alleviate the problem of inefficient estimation under multicollinearity, the authors assume equality between constants in equations (12) and (13), i.e. autonomous growth rates related to non-price factors.

Finally, note that equation (14) differs from (12)-(13) by not including any own price elasticity, but only relative prices. This is motivated both economically and econometrically. Fine-cut tobacco may be seen as a product inferior to cigarettes, yielding comparable (or slightly lower) utility, but at the cost of an additional labour input from the consumer. As a result, its price relationship against cigarettes seems to be far more important than the price level of this good itself, especially given the fact that this market used to be very limited in size before 2005. Econometrically, the variance of both P_t^{TOB} and P_t^{LOW} / P_t^{TOB} (or P_t^{HIGH} / P_t^{TOB}) is dominated by the same single event: a substantial level shift in retail prices of tobacco, effective in 2009 and resulting from excise tax adjustment. As a consequence, the above-mentioned variables exhibit correlation of near -1, causing heavy multicollinearity problems when all of them are included in the equation. Skipping P_t^{TOB} seems to alleviate the problem, while β_2 and β_3 are estimated more precisely thanks to cross-equation restrictions.

One important issue that arises while estimating the system described in equations (12)-(14) is the possible endogeneity of prices in particular segments. For instance, one could argue that both the price increases and the duty-paid cigarette sales decreases that took place over the 2005-2014 period in Poland were driven by certain overall negative sentiment towards smoking. However, the principal factor leading to the price increases in Poland were the tobacco excise hikes, which in turn were part of the effort of the Polish authorities to implement the Council Directive (2011/64/EU) that obliged the EU Member States to harmonize tobacco taxation, among others, in terms of minimum taxation level (90 euros per 1000 cigarettes). Therefore, the price increases in that period arose from factors unrelated to cigarette demand developments in Poland, i.e. they are exogenous.

The unobservable variables α_t^{LOW}, α_t^{HIGH} (price elasticities of demand for cigarettes in the LOW and HIGH segment, respectively) and $\gamma_{1,t}$ (supply-side intercept in the fine-cut tobacco market) are assumed to evolve according to the following state equations:

$$\alpha_t^{LOW} = \alpha_{t-1}^{LOW} + \Delta\alpha_t^{LOW}, \tag{15}$$

$$\Delta\alpha_t^{LOW} = \rho_1 \Delta\alpha_{t-1}^{LOW} + \left(1-\rho_1\right)c_1 + \eta_t^{LOW}, \tag{16}$$

$$\alpha_t^{HIGH} = \alpha_{t-1}^{HIGH} + \Delta\alpha_t^{HIGH}, \tag{17}$$

$$\Delta\alpha_t^{HIGH} = \rho_1 \Delta\alpha_{t-1}^{HIGH} + \left(1-\rho_1\right)c_2 + \eta_t^{HIGH}, \tag{18}$$

$$\gamma_t = \gamma_{t-1} + \Delta\gamma_t, \tag{19}$$

$$\Delta\gamma_t = \rho_2 \Delta\gamma_{t-1} + \left(1-\rho_2\right)c_3 + \eta_t^{TOB.}. \tag{20}$$

In line with these equations, the increments to the state variables evolve as autoregressive processes that also include a constant and an error term. This specification is analogous to the models for estimating the output gap or natural unemployment rate, referred to in the previous section. Moreover, in line with that literature, the authors apply restrictions in the form of variance ratios between residuals from different equations. In this case, the variance ratios are defined pairwise between residuals from (12) and (16), (13) and (18), as well as (14) and (20), and this ensures that the estimated states are smooth enough, i.e. do not fluctuate excessively from month to month. This could occur as a result of overfitting the measurement equations and, at the same time, violate the economic intuition of price elasticity of demand as presumably an inertial category. A proportion of 2% is assumed for each ratio between the residual variance from the state equation and its counterpart from the measurement equation, and hence, for each $i \in \{LOW, HIGH, TOB\}$:

$$Var\left(\eta^i\right) = 0.02Var\left(\varepsilon^i\right). \tag{21}$$

This assumption is subject to the sensitivity analysis in the following subsection.

Finally, the Kalman filter needs to be initialised. This is done using the constant-parameter estimates obtained with the aggregate market data, specifically by regressing the (log) volume of the aggregate cigarette market on the (log) price and a constant. The estimated value of the parameter for (log) price is –0.57 and is used to initialise the values of own-price elasticities of demand for the LOW and HIGH segments in the initial, 'zero' period (α_0^{LOW} and α_0^{HIGH}). The standard error of this estimate (0.0152) is squared to initialise the variances of these states. The variances of $\Delta\alpha_0^{LOW}$ and $\Delta\alpha_0^{HIGH}$ are set arbitrarily to 0.01. γ_t is initialised in an analogous way, based on the constant from the regression of fine-cut tobacco market volume on the price (both expressed as year-on-year dynamics). The initial values of $\Delta\alpha_0^{LOW}$, $\Delta\alpha_0^{HIGH}$ and $\Delta\gamma_0$ were all set to 0. The analysis is insensitive to the choice of initial points, over a reasonable range of parameters.

2.4. Measurement of demand elasticities and sensitivity analysis

Most of the constant parameters that were estimated in the model are significant and take the expected signs (see Table 1). The value of γ_0 suggests that both segments of the cigarette market shrink, on average, by 4 per cent in annual terms due to non-price factors.

All the parameters controlling the cross-segment elasticities take the expected positive signs (β_1, β_2, β_3). Two of them are significant at 0.01 level, i.e. the elasticities between 'neighbourhood' segments LOW-HIGH and TOB-LOW. The cross-price elasticity between the 'remote' segments, i.e. TOB and HIGH, is both the weakest in magnitude (0.52) and significant only at 0.10 level.

In line with expectations, the parameters of income elasticity were also estimated as positive, though in the tobacco segment the parameter is insignificant. Note that on the macroeconomic level, i.e. with time-series data aggregated over market segments, these estimates cannot be viewed as standard income elasticities, but as a measure mostly capturing business cycle effects. In this respect, the tobacco segment appears to be acyclical, LOW – procyclical to a limited extent (estimate of 0.392 significant only at 0.10) and HIGH – the most procyclical (estimate 0.531 significant at 0.01).

The respective estimates of the state equations are shown in Table 2.

Table 1

State-space model: the point estimates of constant parameters of the measurement equations

	Parameter and the corresponding segment	Estimate	p-value
Trend	γ_0 (LOW and HIGH)	–0.040***	0.005
cross-price elasticities	β_1 (LOW-HIGH)	1.101***	0.000
	β_2 (LOW-TOB)	0.855***	0.007
	β_3 (TOB-HIGH)	0.525*	0.082
income elasticities	κ_1 (LOW)	0.392*	0.068
	κ_2 (HIGH)	0.531***	0.010
	κ_3 (TOB)	0.690	0.384
logs of residual variances	$\ln\left(Var\left(\varepsilon^{LOW}\right)\right)$	–6.245	0.000
	$\ln\left(Var\left(\varepsilon^{HIGH}\right)\right)$	–6.393	0.000
	$\ln\left(Var\left(\varepsilon^{TOB}\right)\right)$	–5.361	0.000

Notes: The measurement equations have the following form:

$$dlog_{yoy}\left(Q_t^{LOW}\right) = \gamma_0 + \alpha_t^{LOW} dlog_{yoy}\left(P_t^{LOW}\right) + \beta_1 dlog_{yoy}\left(P_t^{HIGH} / P_t^{LOW}\right) + \\ + \beta_2 dlog_{yoy}\left(P_t^{TOB} / P_t^{LOW}\right) + \kappa_1 dlog_{yoy}\left(Y_t\right) + \varepsilon_t^{LOW},$$

$$dlog_{yoy}\left(Q_t^{HIGH}\right) = \gamma_0 + \alpha_t^{HIGH} dlog_{yoy}\left(P_t^{HIGH}\right) + \beta_1 \frac{Q_{t-12}^{LOW}}{Q_{t-12}^{HIGH}} dlog_{yoy}\left(P_t^{LOW} / P_t^{HIGH}\right) + \\ + \beta_3 dlog_{yoy}\left(P_t^{TOB} / P_t^{HIGH}\right) + \kappa_2 dlog_{yoy}\left(Y_t\right) + \varepsilon_t^{HIGH},$$

$$dlog_{yoy}\left(Q_t^{TOB}\right) = \gamma_t + \beta_2 \frac{Q_{t-12}^{LOW}}{Q_{t-12}^{TOB}} dlog_{yoy}\left(P_t^{LOW} / P_t^{TOB}\right) + \\ + \beta_3 \frac{Q_{t-12}^{HIGH}}{Q_{t-12}^{TOB}} dlog_{yoy}\left(P_t^{HIGH} / P_t^{TOB}\right) + \kappa_3 dlog_{yoy}\left(Y_t\right) + \varepsilon_t^{TOB}.$$

Source: authors' own elaboration.

Table 2

State-space model: the point estimates of constant parameters of the state equations

Parameter	Estimate	p-value
c_1	–0.006	0.758
c_2	–0.019	0.414
c_3	0.001	0.968
ρ_1	0.957***	0.000
ρ_2	0.914***	0.000

Notes: The state equations have the following form:

$$\alpha_t^{LOW} = \alpha_{t-1}^{LOW} + \Delta\alpha_t^{LOW},$$
$$\Delta\alpha_t^{LOW} = \rho_1 \Delta\alpha_{t-1}^{LOW} + (1-\rho_1)c_1 + \eta_t^{LOW},$$
$$\alpha_t^{HIGH} = \alpha_{t-1}^{HIGH} + \Delta\alpha_t^{HIGH},$$
$$\Delta\alpha_t^{HIGH} = \rho_1 \Delta\alpha_{t-1}^{HIGH} + (1-\rho_1)c_2 + \eta_t^{HIGH},$$
$$\gamma_t = \gamma_{t-1} + \Delta\gamma_t,$$
$$\Delta\gamma_t = \rho_2 \Delta\gamma_{t-1} + (1-\rho_2)c_3 + \eta_t^{TOB}.$$

Source: authors' own elaboration.

The estimated price elasticities of demand in LOW and HIGH segments (α_t^{LOW} and α_t^{HIGH}), computed as smoothed states from the state-space model, along with their 90 per cent confidence intervals, are presented in Figure 1. One can observe a gradual decline in both segments, i.e. in LOW since 2010 and in HIGH since 2012. The LOW own price elasticity seems to have stabilised in the proximity of –1, while the elasticity in the HIGH segment seems to be falling quite sharply until June 2014. At the end of the sample under analysis, both values do not differ significantly from –1 and from each other. However, historically, this was not the case. Up to approximately 2011, both confidence intervals were fully within the range from 0 to –1 and seemed to be relatively stable. Note that these results provide evidence against a model with a constant price elasticity of demand.

Figure 1 shows a temporary upward shift in price elasticities in both segments. This phenomenon does not seem to be any regular pattern, but a symptom of insufficient control for specific market circumstances. In this model, one does not (and often cannot) control for a number of substitutes, including – above all – electronic cigarettes (for which only low-frequency, rough estimates were available) and the shadow market (which is unobservable

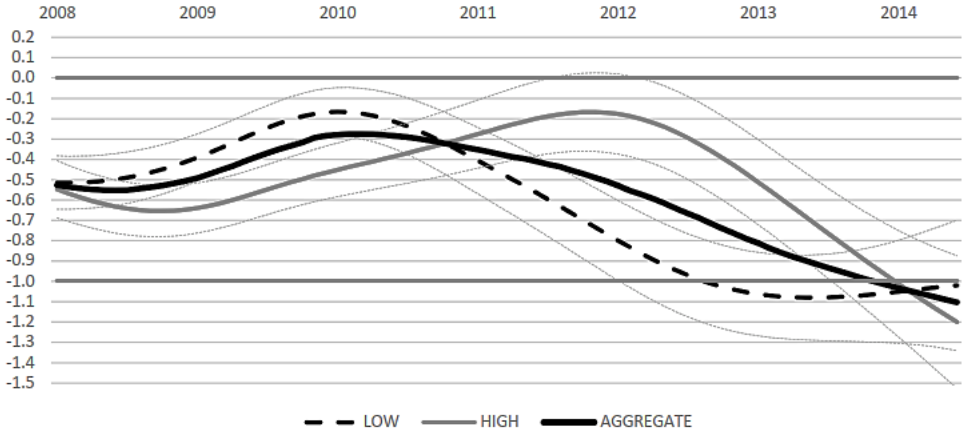

Fig. 1. Price elasticities of demand (smoothed states estimated using Kalman filter) in two cigarette market segments (LOW, HIGH, with 90 per cent confidence intervals) and in the overall factory-made cigarette market (AGGREGATE).

Source: authors' own elaboration.

by nature, but is estimated in other studies to have been growing sharply over the analysed sample period). The evolving life cycle of some tobacco-related products is the authors' main hypothesis explaining this hump-shapedness of both graphs, as some consumers may have become more responsive to price fluctuations when e-cigarettes or shadow market products were gaining popularity.

Remember that equation (9) requires that the weighted average of own price elasticities of demand in all the segments be lower than –1. It is important to note the 90 per cent confidence intervals for the estimates of price elasticities of demand in both cigarette market segments are wide enough to cover values around –1 at the end of the sample period, and hence include the values for which the optimum taxation level does not exist. Below, the authors focus on the point estimates of own price elasticities of demand, as they indicate the existence of the maximum of Laffer's surface as more likely than non-existence, especially under higher tax rates, the resulting higher prices and the likely higher elasticities (see Subsection 3.2.1).

In this estimation, the authors calibrated the variance ratios between the related measurement and state equations at 2 per cent. Sensitivity to this assumption is tested in Figure 2. As expected, the higher the variance ratio, the more volatility allowed in the state equation residuals, and the more

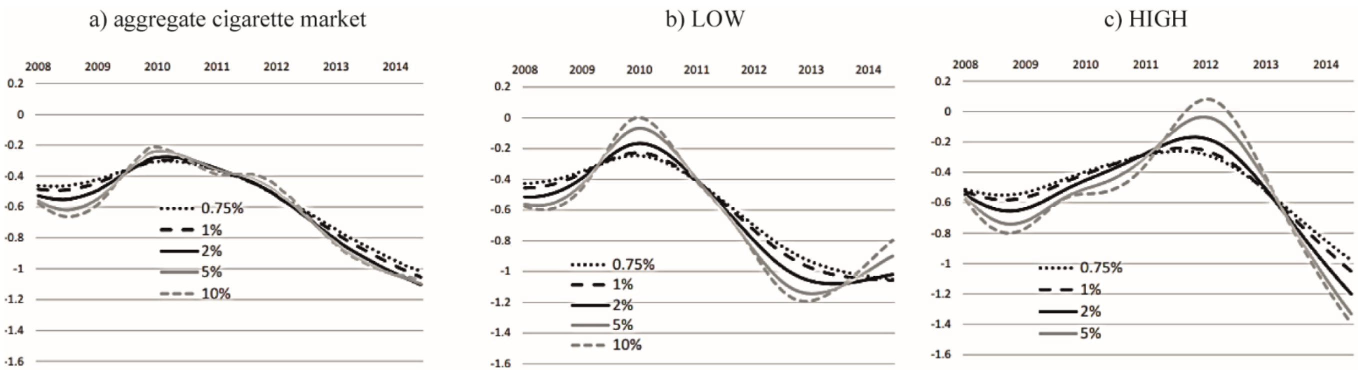

Fig. 2. Sensitivity analysis of price elasticities of demand in two cigarette market segments (LOW, HIGH) and in the overall factory-made cigarette market to changes in the value of the variance ratio used in the Kalman filter estimation.

Source: authors' own elaboration.

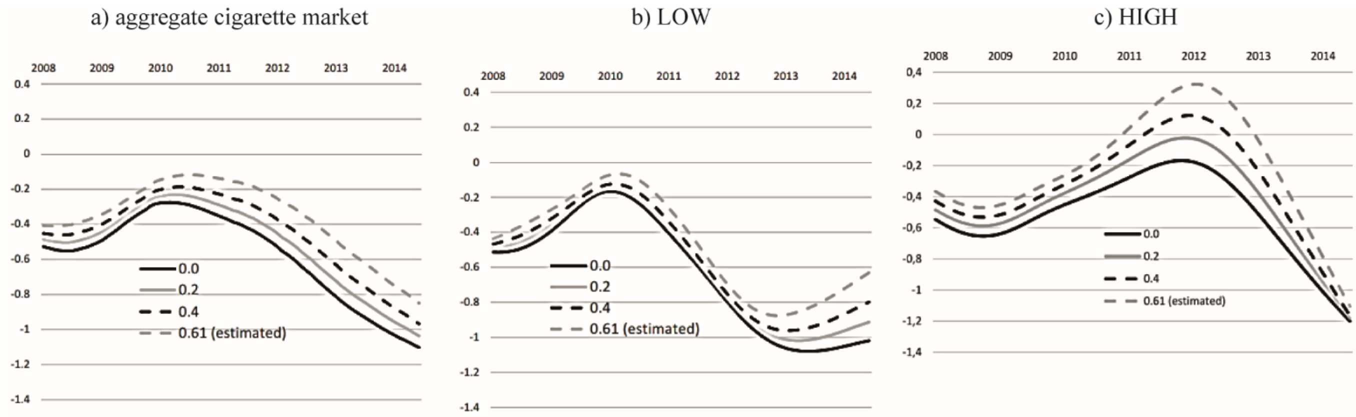

Fig. 3. Sensitivity analysis of price elasticities of demand in two cigarette market segments (LOW, HIGH) and in the overall factory-made cigarette market to changes in the value of correlation of measurement errors.

Source: authors' own elaboration.

volatile becomes the smoothed path of elasticity. This is true over the entire tested range (from 0.75 per cent to 10 per cent), however it does not change significantly the shape of the graphs. Differences are particularly marginal when the market is analysed at the aggregate level. This is the result of deviations from the baseline assumptions in both subsegments cancelling out when aggregated to the level of entire market (i.e. average own-price elasticity calculated by weighting α_t^{LOW} and α_t^{HIGH} by volumes in the corresponding segments).

Moreover, the authors test the assumption that residuals in the individual measurement equations remain uncorrelated against the alternative of a single correlation coefficient for any related pair of residuals. Raising this coefficient to 0.61 (i.e. the level at which it was estimated, though at the cost of a lower precision of all other estimates) neither changes the shape nor the trends observed for the elasticities over time. However, it leads to some upward shift in the time path for both segments.

Usually, policy analyses of the indirect tax rates are based on a crucial assumption of a given product price elasticity of demand being below or above –1. However, in this case the problem cannot be reduced to that issue. There is no homogeneous tobacco product, and – given the complex structure of excise taxation – goods in different segments are taxed differently. Consequently, substitution effects between segments – if induced – may generate fiscal effects that could significantly alter the budgetary outcome projected exclusively on the basis of own price elasticities. Therefore, in the analysis of different government policies, the paper will account for the cross-segment elasticities, estimated in this section. Furthermore, while having obtained the estimates of price elasticities of demand for the 2005–2014 period, one has to investigate how these elasticities may have further evolved in response to the considered changes in tax policy within a given time horizon. The authors deal with these issues in a comprehensive simulation analysis conducted in Section 3.

3. SIMULATION RESULTS

The econometric analysis focused on the in-sample consumer behaviour within particular segments of the duty-paid tobacco market. The purpose of the simulation analysis is to build on these results in order to quantify the impact of the excise tax policy on government revenues over the July 2014–December 2018 period. The simulation model is populated by the three types of tobacco market players:

- the government responsible for indirect tax policy, namely the structure and levels of excise rates as well as VAT rate for tobacco products;
- manufacturers (suppliers), who set the cigarette and tobacco prices (largely influenced by excise rates);
- consumers, who adjust their demand for tobacco products, influenced by absolute and relative prices of different tobacco products.

The remainder of the section is organised as follows. First, the authors describe the behaviour of manufacturers (suppliers) and consumers, which allows to draw the Laffer surfaces. Next, the authors define different tax policies and discuss their impact on government revenues in light of the results produced by the simulation model.

3.1. Pricing strategies of manufacturers (suppliers)

In the simulation model, manufacturers (suppliers) set a separate retail price P^i per pack for each tobacco market segment considered in the econometric analysis ($i = LOW$, $HIGH$, TOB for the low-price, high-price and fine-cut tobacco segments, respectively). In the case of both cigarettes and fine-cut tobacco, the authors use the concept of a standardised pack of 20 cigarettes and assume that a single cigarette from a standardised pack rolled from fine-cut tobacco contains 0.925 g of tobacco. Only a small fraction of the retail price of a single pack of tobacco products is left after taxation; this component is referred to as net consumer price $\overline{P}^i$ and defined as follows:

$$\overline{P}^i = P^i - T^i, \tag{22}$$

where T^i stands for the total indirect taxation. It can be assumed that manufacturers use a particular, desired value of net consumer price $\overline{P}^i$ (either in absolute or relative terms) as a basis for the targets for retail prices P^i. Companies re-calculate these targets on an annual basis in order to account for any changes in T^i and to prevent deviations of the net consumer prices from the desired levels. In order to achieve this, the strategies of manufacturers (suppliers) with respect to P^i must include the details about the structure of component T^i, which is described below.

3.1.1. Indirect taxation

The formula according to which the authors calculate T^i in this model is based on Polish tax law, which differentiates between factory-made cigarettes and fine-cut tobacco. For the former, total indirect tax can be expressed as follows:

$$T^{CIG} = \begin{cases} \left(\theta^{CIG} + \frac{\nu}{1+\nu}\right)P^{CIG} + \delta^{CIG} & \text{if} \quad P^{CIG} \geq WAP \\ \mu & \text{if} \quad P^{CIG} < WAP, \end{cases} \tag{23}$$

where $\theta^{CIG} \in [0;1]$, $\delta^{CIG} \in [0;+\infty)$ and $\mu \in [0;+\infty)$ denote the *ad valorem*, specific and minimum excise tax rates on cigarettes, respectively; $\nu \in [0;1]$ stands for the VAT tax rate applicable to tobacco products (assumed at the 23 per cent level throughout the analysis) and WAP denotes the official weighted average price of cigarettes. The last parameter constitutes an effective threshold for the retail price P^{CIG} below which the minimum excise rate applies. WAP for year t is calculated based on the cigarette prices in the duty-paid market in the first 10 months of year $t-1$. WAP is not only the threshold used for the calculation of the minimum excise rate – it is also used for the calculation of μ according to the following formula:

$$\mu = \left(\theta^{CIG} + \frac{\nu}{1+\nu}\right)WAP + \delta^{CIG}. \tag{24}$$

In such a formulation, even with no changes in the *ad valorem* and specific excise tax rates, the minimum excise rate μ is readjusted automatically each year, based on equation (24) that uses not only the contemporaneous specific and *ad valorem* rates, but also the WAP calculated using the prices from the preceding year.

In case of fine-cut tobacco, there is no minimum excise rate, so the total indirect tax is calculated as follows:

$$T^{TOB} = \left(\theta^{TOB} + \frac{\nu}{1+\nu}\right)P^{TOB} + \delta^{TOB}, \tag{25}$$

where $\theta^{TOB} \in [0;1]$ and $\delta^{TOB} \in [0;+\infty)$ stand for the *ad valorem* and specific excise tax rates on fine-cut tobacco, respectively.

3.1.2. Price targets

In the simulation, manufacturers (suppliers) use the parameters of indirect taxation described above in order to prevent undesired changes in $\overline{P}^i$. The retail price targets in this model are set in January each year and are based either directly on a particular value of $\overline{P}^i$ or on its targeted relation to the retail price, $\overline{P}^i / P^i = \Pi^i$. The authors assume that the prices are fully adjusted to the new targets in February – this delay is related to the fact that it takes some time for the packs subject to old excise values to be replaced with products charged with new excise values. The authors refer to the price strategies based on $\overline{P}^i$ and Π^i as to the absolute and the relative price mechanism, respectively.

3.1.3. The absolute price mechanism

Under the absolute price mechanism, manufacturers (suppliers) aim at maintaining a pre-defined desired value of $\overline{P}^i$. However, keeping $\overline{P}^i$ constant, irrespective of circumstances, would be unrealistic, since net consumer prices might be subject to exogenous changes for the following reasons:

- consumer inflation (CPI, smoothed with the Hodrick-Prescott filter);
- changes in the production costs and technology;
- other factors influencing the price strategies.

Under the absolute price mechanism, in January each year the net consumer price is evaluated as of December ($\overline{P}^i_{DEC}$) in the preceding year, using equation (22), and adjusted for an average annual inflation in the previous year ($\overline{CPI}$) and for an additional factor (x, assumed to be equal to 3 per cent for all the segments, which is calibrated basing on historical observations). The result is the target net consumer price for the new year, which is then used to calculate the new retail price that is implemented in February according to the following formula:

$$P^i = \max\left\{ \frac{\left(1+\overline{CPI}+x\right)\overline{P}^i_{DEC}+\delta^i}{1-\theta^i-\dfrac{\nu}{1+\nu}}, \frac{\left(1+\overline{CPI}+x\right)\overline{P}^i_{DEC}+\mu}{1-\dfrac{\nu}{1+\nu}} \right\}. \tag{26}$$

The specification of equation (26) using the max() function guarantees that $\overline{P}^i$, calculated using equation (22), remains constant over the February–

December period, fully accounting for the non-linearity induced by the minimum excise mechanism built in the Polish excise tax system (see equations (23)-(24)). Note that equation (26) works for fine-cut tobacco as well – with $\mu = 0$, it simplifies to:

$$P_i = \frac{\left(1 + \overline{CPI} + x\right)\overline{P}^i_{DEC} + \delta^i}{1 - \theta^i - \frac{\nu}{1+\nu}}.$$

3.1.4. The relative price mechanism

Under the relative price mechanism in the simulations, the manufacturers (suppliers) aim at maintaining a pre-defined share of the net consumer price in retail price Π^i. However, the real level of net consumer price is defended against the CPI inflation as well, even if there are no changes in tax policy. Under this mechanism, the share of the net consumer price in the retail price in December in the preceding year (Π^i_{DEC}) is calculated, which is then used to calculate a new retail price that is implemented in February according to the following formula:

$$P^i = \max\left\{\frac{\delta^i}{1 - \theta^i - \frac{\nu}{1+\nu} - \Pi^i_{DEC}}, \frac{\mu}{1 - \frac{\nu}{1+\nu} - \Pi^i_{DEC}}, \frac{\left(1 + \overline{CPI}\right)\overline{P}^i_{DEC} + \delta^i}{1 - \theta^i - \frac{\nu}{1+\nu}}, \frac{\left(1 + \overline{CPI}\right)\overline{P}^i_{DEC} + \mu}{1 - \frac{\nu}{1+\nu}}\right\}. \tag{27}$$

The specification of equation (27) using the $\max()$ function guarantees that Π^i does not decrease over the February-December period, once again accounting for the non-linearity of the Polish excise tax system. However, an additional complication arises, as we have two additional terms within the $\max()$ function that contain the indexation with $\overline{CPI}$. Without these additional terms, if there was inflation and no excise tax hikes, the real value of $\overline{P}^i$ would decrease – this is not allowed to happen in the simulation. Under such circumstances, upward movements in Π^i_{DEC} are necessary and this is what equation (27) ensures. Note once again that equation (27) works for fine-cut tobacco as well – with $\mu = 0$, it simplifies to:

$$P_i = \max\left\{ \frac{\delta^i}{1-\theta^i - \frac{\nu}{1+\nu} - \Pi^i_{DEC}}, \frac{\left(1+\overline{CPI}\right)\overline{P}^i_{DEC} + \delta^i}{1-\theta^i - \frac{\nu}{1+\nu}} \right\}.$$

3.2. The out-of-sample behaviour of consumers

The econometric analysis focuses on the in-sample behaviour of consumers and proves that price elasticities of demand for cigarettes in the LOW and HIGH segments change over time. However, for the purpose of the simulation analysis, one also needs a model of the out-of-sample behaviour of the price elasticities of demand in individual market segments. This issue is addressed below.

The hypotheses of non-linearity of demand function with respect to prices and of 'switching effects' inherent in the consumer behaviour imply that there is a relationship between the measured price elasticity (over the sample period) and the price level. The switching effects might be related to the reactions of groups of consumers that consider certain price levels excessive. When the tobacco price exceeds such a level, the reaction of consumers to price hikes might be much stronger than their reaction to analogous price increases in the past.

3.2.1. The out-of-sample model

A number of econometric issues have to be addressed before one moves to simulating the out-of-sample price elasticities of demand. First of all, given the specification of the state equations (15) and (17), one cannot assume that the estimate of α_t^{LOW} and α_t^{HIGH} will be stationary because the price levels might have an impact on the price elasticities of demand. Secondly, one expects that the relationships discussed above work one-way, i.e. it is the price that affects the elasticities, not the other way round, yet this exogeneity presumption can only be tested using a multivariate model. These issues can be addressed with the Vector Error Correction Model (VECM).

In the VECM framework one can consider two variables: time-varying price elasticity of demand, α_t^i, and CPI-deflated price level, P_t^i, in particular market segments ($i = LOW$, $HIGH$). The authors provide the general form of the VECM system of order p for these two variables below:

$$\begin{aligned}\begin{bmatrix}\Delta\alpha_t^i \\ \Delta P_t^i\end{bmatrix} = \begin{bmatrix}\omega_1 \\ \omega_2\end{bmatrix}\begin{bmatrix}\phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22}\end{bmatrix}\begin{bmatrix}\alpha_{t-1}^i \\ P_{t-1}^i\end{bmatrix} + \mathbf{\Gamma}_0 + \mathbf{\Gamma}_1\begin{bmatrix}\Delta\alpha_{t-1}^i \\ \Delta P_{t-1}^i\end{bmatrix} + \ldots + \\ + \mathbf{\Gamma}_p\begin{bmatrix}\Delta\alpha_{t-p}^i \\ \Delta P_{t-p}^i\end{bmatrix} + \begin{bmatrix}\xi_{1t} \\ \xi_{2t}\end{bmatrix},\end{aligned} \tag{28}$$

in which $\mathbf{\Gamma}_0$ is a two-element vector, and $\mathbf{\Gamma}_1,\ldots,\mathbf{\Gamma}_p$ are 2×2 matrices of coefficients. The analysis of this model begins with some initial tests, setting the maximum lag order p equal to 4 in all the cases (see Table 3). Such a parsimonious specification is justified when using a relatively short sample, ranging from January 2008 to June 2014. Such a short sample is necessary because of very imprecise estimates of the time-varying price elasticities for the 2005–2008 period.

Table 3

Test results for the VECM models for both cigarette market segments

Test	Null hypothesis	p-value for the LOW segment	p-value for the HIGH segment
Trace test	No cointegrating relationships	0.052	0.070
	1 independent cointegrating relationship	0.879	0.536
Max-eigenvalue test	No cointegrating relationships	0.033	0.053
	1 independent cointegrating relationship	0.879	0.536
LR test for exogeneity restrictions	$\omega_2 = 0, \phi_{11} = 1$	0.936	0.554
LM test for serial correlation	No serial correlation at lag order 5	0.145	0.072

Notes: The VECM model with two cointegrating relationships:

$$\begin{bmatrix}\Delta\alpha_t^i \\ \Delta P_t^i\end{bmatrix} = \begin{bmatrix}\omega_1 \\ \omega_2\end{bmatrix}\begin{bmatrix}\phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22}\end{bmatrix}\begin{bmatrix}\alpha_{t-1}^i \\ P_{t-1}^i\end{bmatrix} + \mathbf{\Gamma}_0 + \mathbf{\Gamma}_1\begin{bmatrix}\Delta\alpha_{t-1}^i \\ \Delta P_{t-1}^i\end{bmatrix} + \ldots + \mathbf{\Gamma}_p\begin{bmatrix}\Delta\alpha_{t-p}^i \\ \Delta P_{t-p}^i\end{bmatrix} + \begin{bmatrix}\xi_{1t} \\ \xi_{2t}\end{bmatrix}.$$

Source: authors' own elaboration.

Under 0.1 significance level, both the trace test and the max-eigenvalue test reject the null hypotheses of no cointegrating relationships in the VECM system, and they do not reject the null of 1 independent cointegration relationship. At the same time, the LR test for exogeneity restrictions does not reject the null hypothesis stating that $\omega_2 = 0$ and $\phi_{11} = 1$, i.e. this is P_t^i that shapes α_t^i and not the other way round. These results are the same for both the LOW and the HIGH cigarette market segment. At the same time, the test results suggest that serial correlation is a problem in one of our VECM models: the LM test does reject the null of no serial correlation in the case of the HIGH elasticity, but the test does not reject the null of no serial correlation in the case of the LOW elasticity. Unfortunately, serial correlation cannot be eradicated due to the small sample size (78 observations).

In spite of these problems, the test results allow one to confirm the following cointegrating relationship between the time-varying price elasticity of demand and CPI-deflated price level in both cigarette market segments (yet one should treat the parameters for the HIGH segment with caution, due to the above-mentioned serial correlation problem):

$$\alpha_t^i = \tilde{\phi}_0 + \tilde{\phi}_1 P_t^i + \varepsilon_t, \tag{29}$$

where ε_t stands for the deviation from the long-term equilibrium implied by the coefficients $\tilde{\phi}_0$ (an intercept) and $\tilde{\phi}_1 = -\phi_{12}$ (see equation (28)). Note that

Table 4

The key estimation results for the key parameters of the VECM systems for both cigarette market segments (t-statistics are given in parentheses if available).

Estimated parameter	The LOW segment	The HIGH segment
$\tilde{\phi}_0$	0.974	1.044
$\tilde{\phi}_1$	-0.223 [-12.534]	-0.186 [-4.578]
ω_1	-0.000292 [-3.84707]	0.000143 [-3.63038]

Notes: The single cointegrating relationship normalized with respect to α_t^i : $\alpha_t^i = \tilde{\phi}_0 + \tilde{\phi}_1 P_t^i + \varepsilon_t$.
The corresponding VECM model:

$$\begin{bmatrix} \Delta\alpha_t^i \\ \Delta P_t^i \end{bmatrix} = [\omega_1][1 \quad -\tilde{\phi}_1]\begin{bmatrix} \alpha_{t-1}^i \\ P_{t-1}^i \end{bmatrix} + \mathbf{\Gamma}_0 + \mathbf{\Gamma}_1\begin{bmatrix} \Delta\alpha_{t-1}^i \\ \Delta P_{t-1}^i \end{bmatrix} + \ldots + \mathbf{\Gamma}_p\begin{bmatrix} \Delta\alpha_{t-p}^i \\ \Delta P_{t-p}^i \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix}.$$

Source: authors' own elaboration.

equation (29) presents a cointegrating relationship that was normalised with respect to the appropriate price elasticity of demand, whereas equation (28) presents the non-normalised VECM model.

The coefficient estimates for equation (29) are outlined in Table 4. The estimates of the ω_1 parameter are negative and statistically significant for both segments, which means that an error correction process towards the long-term equilibria takes place in each of the VECM systems. One can therefore use these results to simulate the out-of-sample developments of price elasticities of demand for the 2014–2018 period.

3.2.2. A limiting scenario

Apart from scenarios based on time-varying price elasticities of demand for tobacco products, the authors additionally consider a scenario with constant price elasticities within LOW and HIGH segments. These elasticities are calculated as the average estimated elasticity over the last 12 months, for which cigarette market data is available (July 2013–June 2014), and amount to: –1.05 for the lower price segment (LOW) and –0.99 for the higher price segment (HIGH). However, taking into account the results of the estimation of time-varying price elasticities of demand, it seems more reasonable to allow for the changes in price elasticities of demand rather than keep them constant in the simulations. Nevertheless, one might claim that changes in the out-of-sample price elasticities of demand might be weaker than over the 2005–2014 period, e.g. due to the fact that cigarette consumers most sensitive to price movements may have already left the duty-paid market and stopped smoking or moved to cheaper substitute products or to the illegal market. If the latter was true, it would mean that the elasticities might not continue to decline to the level significantly below the –1 threshold. Therefore, it is useful to include in these simulations a variant with a constant price elasticity of demand as a limiting scenario.

In all the cases, the authors assume that in the out-of-sample intercept in the fine-cut tobacco equation will be equal to the average value of the corresponding time-varying parameter over the final 12 months available in this sample, i.e. to –0.16.

3.3. Laffer surfaces for 2015

The point estimates of price elasticities of demand at the end of our sample (i.e. in mid-2014), coupled with the simulated price-setting strategies of the manufacturers (suppliers), imply the existence of a classical, concave function that relates government revenues to tax rates – in the spirit of the Laffer's curve, which allows to visualise the Laffer's surface. The word 'surface' is used to emphasise the contrast between the authors' results, spanning over the two-dimensional domain of the *ad valorem* and specific tax rate combinations, and the previous literature focusing mostly on a single, illustrative tax rate, but without the analysis of the optimum tax structure. Remember that different combinations of the specific and *ad valorem* rates, even though yielding the same average level of the overall taxation, may trigger different price (and demand) movements in distinct market segments, which in turn may result in a different budgetary impact.

For illustrative purposes, the authors show the Laffer surfaces for the single year 2015 rather than for the entire 2015–2018 period. Bearing in mind two simulated price setting strategies of manufacturers (the absolute and relative price mechanism, see equations (26) and (27)) and two sets of assumptions in the case of consumers (constant and price-dependent elasticities of demand described in equation (29)), four distinct Laffer surfaces for the year 2015 are considered. In each case, the paper considers a vertical view of the Laffer's surface, with the current and the optimum structure of taxation (reflecting the maximum of the Laffer's surface for that year), as well as a corresponding three-dimensional illustration (see Figures 4 to 7). The authors do not report revenues for such combinations of excise rates, where:

- the overall revenues are lower than PLN 12 bn;
- the overall level of taxation is below the minimum required by the EU law;
- the proportions of the given rates in entire taxation violate the EU law (the share of the specific rate within the total taxation of the cigarettes should remain between 7.5 per cent and 76.5 per cent and the minimum tax rate should be no lower than 90 euros per 1000 cigarettes, or 60 per cent of the WAP, but until January 2018 Poland could use lowered limits of 64 euros or 57 per cent of the WAP).

For all the four combinations of manufacturers' and consumers' behaviour, the actual 2014–2018 mix of the level and structure of excise rates seemed to be very close to the maximum of the Laffer surface. Any (minor) shifts towards the optimum would have been related to changing the structure of the excise tax rather than its level (i.e. moving to North-West or South-East in two-dimensional graphs). With price elasticities of demand depending on the price level, a slight re-structuring towards a lower specific and a higher *ad valorem* rates were suggested by the modelling result.

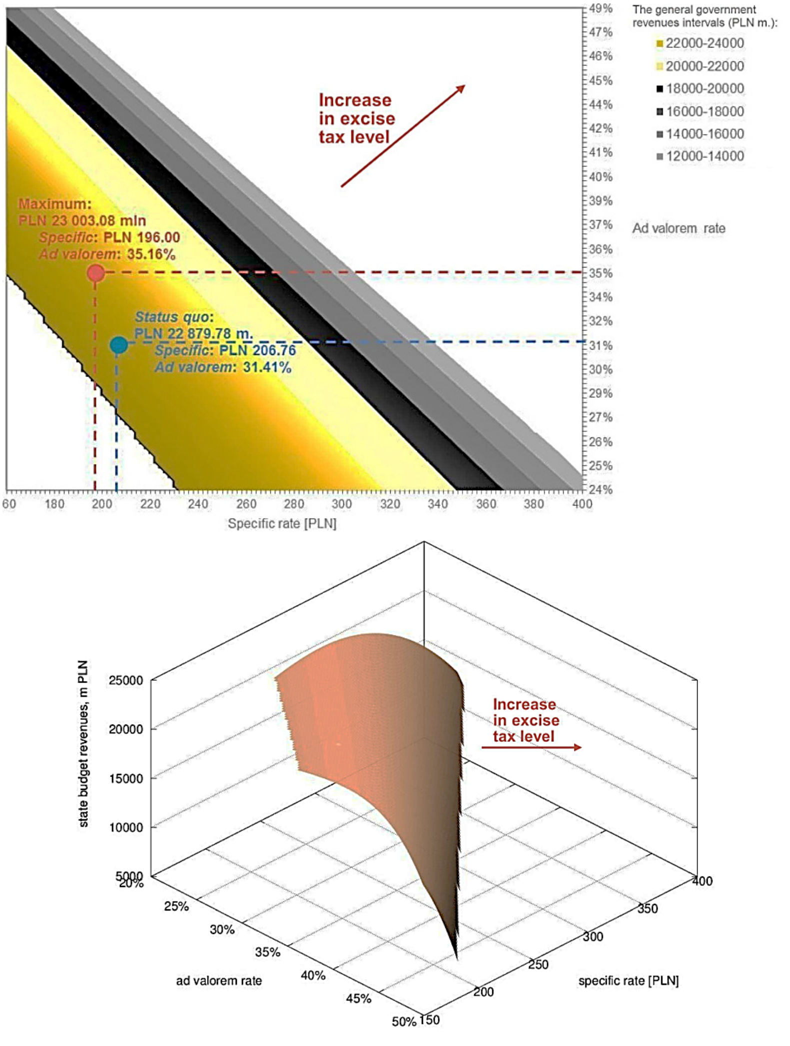

Fig. 4. The relationship between tobacco excise rates (ad valorem and specific rate) and the general government revenues from excise and VAT on tobacco products (the Laffer's surface) – resulting from the combination of price dependent price elasticities of demand in both cigarette market segments and the absolute price mechanism.

Notes: in the upper panel, the authors compare the level of general government revenues from excise and VAT on tobacco products under the "no more hikes" or status quo policy (specific and *ad valorem* excise rates at their 2014 levels, blue dots) and under the optimum policy (which yields the maximum possible government revenues from excise and VAT on tobacco products, red dots).

Source: authors' own elaboration.

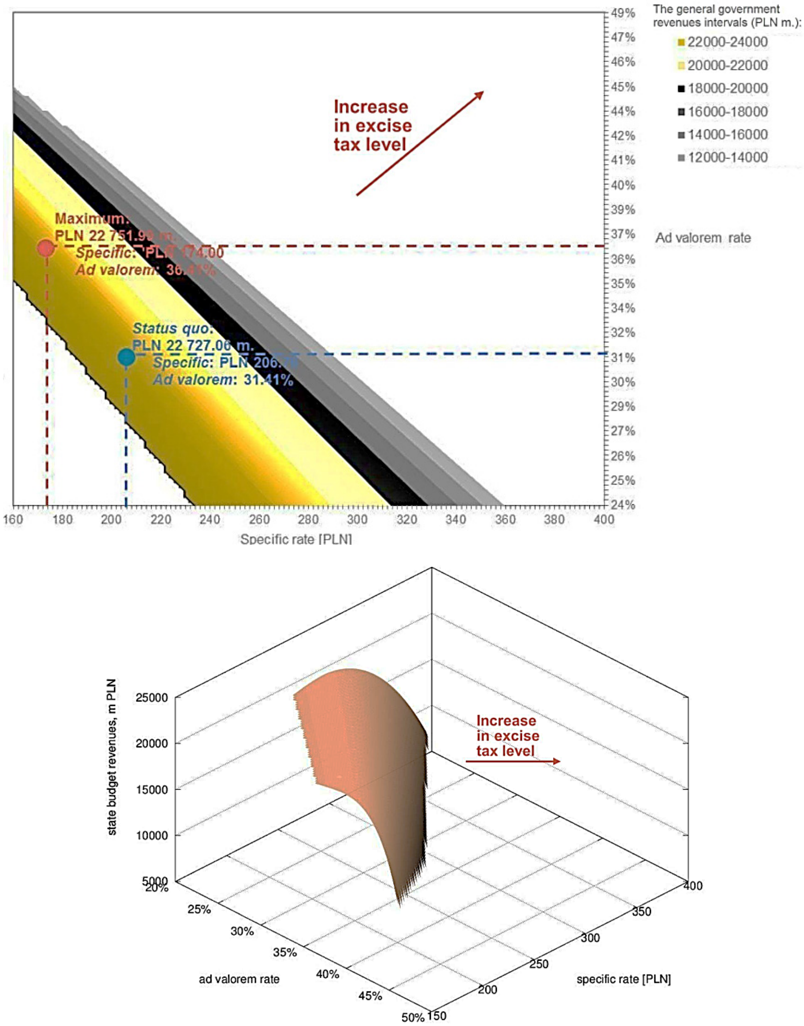

Fig. 5. The relationship between tobacco excise rates (ad valorem and specific rate) and the general government revenues from excise and VAT on tobacco products (the Laffer's surface) – resulting from the combination of price dependent price elasticities of demand in both cigarette market segments and the relative price mechanism.

Notes: in the upper panel, the authors compare the level of general government revenues from excise and VAT on tobacco products under the "no more hikes" or status quo policy (specific and *ad valorem* excise rates at their 2014 levels, blue dots) and under the optimum policy (which yields the maximum possible government revenues from excise and VAT on tobacco products, red dots).

Source: authors' own elaboration.

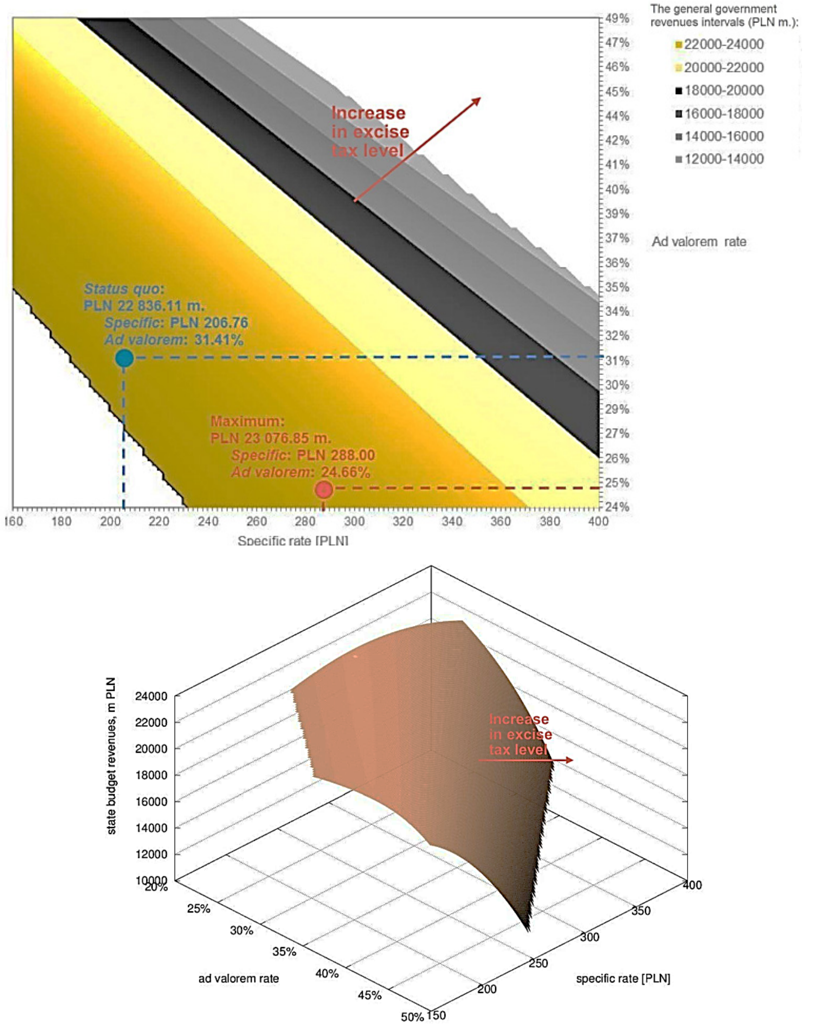

Fig. 6. The relationship between tobacco excise rates (*ad valorem* and specific rate) and the general government revenues from excise and VAT on tobacco products (the Laffer's surface) – resulting from the combination of constant price elasticities of demand in both cigarette market segments and the absolute price mechanism.

Notes: in the upper panel, the authors compare the level of general government revenues from excise and VAT on tobacco products under the "no more hikes" or status quo policy (specific and *ad valorem* excise rates at their 2014 levels, blue dots) and under the optimum policy (which yields the maximum possible government revenues from excise and VAT on tobacco products, red dots).

Source: authors' own elaboration.

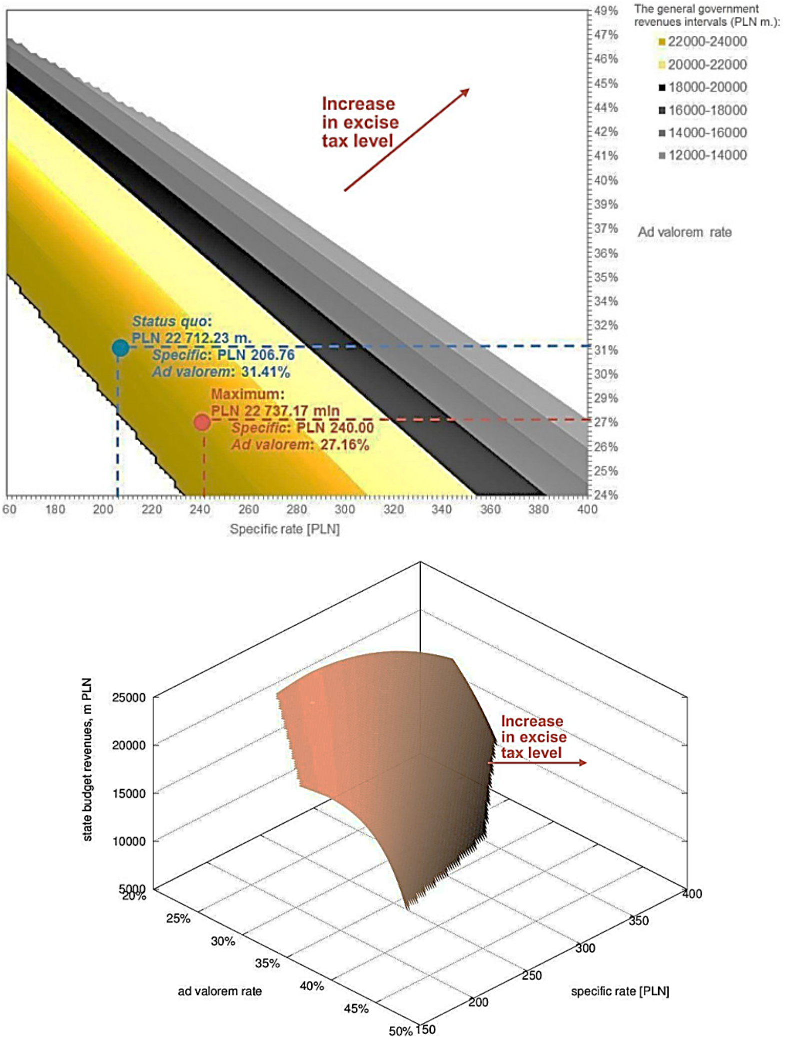

Fig. 7. The relationship between tobacco excise rates (*ad valorem* and specific rate) and the general government revenues from excise and VAT on tobacco products (the Laffer's surface) – resulting from the combination of constant price elasticities of demand in both cigarette market segments and the relative price mechanism.

Notes: in the upper panel, the authors compare the level of general government revenues from excise and VAT on tobacco products under the "no more hikes" or status quo policy (specific and *ad valorem* excise rates at their 2014 levels, blue dots) and under the optimum policy (which yields the maximum possible government revenues from excise and VAT on tobacco products, red dots).

Source: authors' own elaboration.

However, the opposite would have been the case if one had 'frozen' the price elasticities of demand at the average level over the last 12 months (in the estimation sample period).

3.4. Considered strategies of the government in the 2015-2018 period

In this simulation model, the government shapes its tax policy with respect to tobacco products through the parameters θ^i (the *ad valorem* rate) and δ^i (the specific rate, see equations (23)-(25)). Let us remember that the government does not shape the minimum rate μ directly – it is calculated using other excise tax rates. Assume that $\delta^{TOB} \big/ \delta^{CIG} = const$ and $\theta^{CIG} = \theta^{TOB}$, i.e. the excise rates for fine-cut tobacco follow the changes in the corresponding rates on cigarettes. Therefore, this section focuses on the excise rates set for the cigarette market.

For the period 2015–2018, the paper analyses the following policies of the government:

- Policy 1: 'no more hikes' or *status quo*;
- Policy 2: revenue maximisation on a year-by-year basis;
- Policy 3: multi-annual, upfront revenue maximisation for the entire 2015-2018 period.

Under the 'no more hikes' policy, it can be assumed that there are no changes to the excise law as regards the tobacco market – the specific (δ^{CIG}) and *ad valorem* (θ^{CIG}) rates remain constant at the PLN 206.76 and 31.41 per cent levels, respectively, throughout the 2015–2018 period. However, even with no changes in the excise bill, the minimum excise rate, μ, is readjusted automatically each year, based on the formula that uses the contemporaneous specific and *ad valorem* rates, but also the WAP calculated for the previous year (see equation (24)). This upward movement of the minimum excise rate would come to a halt, provided that no exogenous price increases occurred for an extended period (this is because only some segments are affected by the minimum excise rate, which means that each minimum excise rate hike is passed through to WAP only partially).

Under the revenue maximisation on a year-by-year basis policy, the government aims to find the optimum excise rates each time before the new fiscal year (1-year horizon optimisation). Under this policy, the 2015 government revenues are calculated using the range of possible 2015 rates, among which the optimal mix is chosen. The effects of this policy, not least the change in the market size, are taken into account while carrying out optimisation for the year 2016. The same logic applies for 2017 and 2018. The government uses the grid-search approach basing on different Laffer surfaces – one for each year.

The final considered policy consists in the multi-annual, upfront revenue maximisation for the entire 2015–2018 period. Under this policy, the government carries out maximisation of the present value of the cumulative revenues over the 2015–2018 period with respect to eight variables: the specific and *ad valorem rates* for each year. The discount rate is equal to the ratio of the Polish sovereign debt servicing costs to the overall value of the Polish sovereign debt in 2013. This discount rate is equal to 4.82 per cent and is kept constant for the purposes of the simulation analysis. The result is the optimum path of excise rates for the period 2015–2018. This policy differs from the previous one as the government does not use single Laffer surfaces for each year, but adopts a multi-annual optimisation procedure (without limitations of the grid search approach).

Having specified the behaviour patterns of all the tobacco market players, one can now move to the simulation results.

3.5. Simulation results: government revenues in the 2015–2018 period

Taking into account the consumers' and manufacturers' behaviour, we arrive at distinct results in four different market settings. In addition, the government might choose between three different strategies, which gives a total of 12 scenarios. The results for the 2015–2018 period are reported in Tables 5 to 8.

The simulation results for the 2015-2018 horizon in the case of price-dependent elasticities of demand (Tables 5 to 6) confirm the findings from the analysis of the 2015 Laffer surfaces that the actual excise rate was close to the maximum. At the same time, the statistical uncertainty around the obtained results, as well as the results of the sensitivity analysis (to the assumptions made during the econometric analysis) do not allow for making strong conclusions whether the actual position was to the 'left' or to the 'right' of the maximum. However, the analysis implies that there was some, although limited, room for improvement on the revenue side through an increase in the *ad valorem* rate and a reduction in the specific rate. This is because a lower specific component and a higher *ad valorem* component reduce the price of cigarettes in the LOW segment relative to cigarettes in the HIGH segment. Remember that the LOW segment has exhibited over the sample period higher (in modulus) price elasticities and a stronger

Table 5

The simulation results for the 2015–2018 period, price-dependent elasticities, the absolute price mechanism

		2015	2016	2017	2018	2015-2018
“No more hikes” (Policy 1)	Specific rate (PLN)	206.76	206.76	206.76	206.76	
	Ad valorem rate	31.41%	31.41%	31.41%	31.41%	
	Revenues (excise, VAT, PLN m.)	22879.78	22599.59	22372.64	22150.51	22514.86
Short-term revenue maximization (Policy 2)	Specific rate (PLN)	196.00	198.00	192.00	194.00	
	Ad valorem rate	35.16%	36.41%	38.16%	38.66%	
	Revenues (excise, VAT, PLN m.)	23003.08	22755.82	22546.32	22333.43	22672.73
Mid-term revenue maximization (Policy 3)	Specific rate (PLN)	205.18	186.07	188.55	204.07	
	Ad valorem rate	33.02%	36.89%	37.64%	36.79%	
	Revenues (excise, VAT, PLN m.)	22983.01	22768.41	22599.85	22408.93	22701.18

Notes: The 2015–2018 column consists of the average revenues weighted using the 4.82 per cent discount rate.

Source: authors’ own elaboration.

Table 6

The simulation results for the 2015–2018 period, price-dependent elasticities, the relative price mechanism

		2015	2016	2017	2018	2015-2018
“No more hikes” (Policy 1)	Specific rate (PLN)	206.76	206.76	206.76	206.76	
	Ad valorem rate	31.41%	31.41%	31.41%	31.41%	
	Revenues (excise, VAT, PLN m.)	22727.06	22537.28	22387.04	22236.07	22481.43
Short-term revenue maximization (Policy 2)	Specific rate (PLN)	174.00	182.00	200.00	200.00	
	Ad valorem rate	36.41%	35.91%	34.16%	34.91%	
	Revenues (excise, VAT, PLN m.)	22751.99	22601.10	22519.10	22457.54	22588.14
Mid-term revenue maximization (Policy 3)	Specific rate (PLN)	163.19	136.32	186.86	210.54	
	Ad valorem rate	37.90%	41.95%	35.59%	33.24%	
	Revenues (excise, VAT, PLN m.)	22751.59	22598.58	22535.93	22479.26	22596.54

Notes: The 2015–2018 column consists of the average revenues weighted using the 4.82 per cent discount rate.

Source: authors’ own elaboration.

Table 7

The simulation results for the 2015–2018 period, constant elasticities, the absolute price mechanism

		2015	2016	2017	2018	2015-2018
“No more hikes” (Policy 1)	Specific rate (PLN)	206.76	206.76	206.76	206.76	
	Ad valorem rate	31.41%	31.41%	31.41%	31.41%	
	Revenues (excise, VAT, PLN m.)	22836.11	22599.17	22334.55	22113.50	22474.93
Short-term revenue maximization (Policy 2)	Specific rate (PLN)	288.00	340.00	390.00	400.00	
	Ad valorem rate	24.66%	23.66%	23.66%	27.91%	
	Revenues (excise, VAT, PLN m.)	23076.85	22954.35	22878.70	22782.26	22928.69
Mid-term revenue maximization (Policy 3)	Specific rate (PLN)	286.52	399.40	545.41	620.48	
	Ad valorem rate	23.65%	13.79%	2.98%	3.49%	
	Revenues (excise, VAT, PLN m.)	23061.51	22968.77	22938.28	22880.48	22965.64

Notes: The 2015-2018 column consists of the average revenues weighted using the 4.82 per cent discount rate.

Source: authors’ own elaboration.

Table 8

The simulation results for the 2015–2018 period, constant elasticities, the relative price mechanism

		2015	2016	2017	2018	2015-2018
“No more hikes” (Policy 1)	Specific rate (PLN)	206.76	206.76	206.76	206.76	
	Ad valorem rate	31.41%	31.41%	31.41%	31.41%	
	Revenues (excise, VAT, PLN m.)	22712.23	22536.51	22405.98	22282.56	22492.69
Short-term revenue maximization (Policy 2)	Specific rate (PLN)	240.00	274.00	284.00	294.00	
	Ad valorem rate	27.16%	23.66%	23.66%	23.66%	
	Revenues (excise, VAT, PLN m.)	22737.17	22584.54	22494.23	22427.04	22566.78
Mid-term revenue maximization (Policy 3)	Specific rate (PLN)	201.68	208.16	290.62	345.75	
	Ad valorem rate	32.50%	32.02%	21.43%	15.77%	
	Revenues (excise, VAT, PLN m.)	22734.85	22584.56	22517.91	22464.90	22580.74

Notes: The 2015-2018 column consists of the average revenues weighted using the 4.82 per cent discount rate.

Source: authors’ own elaboration.

causality between price levels and price elasticities. As a result, high specific rates affect the LOW segment more strongly than the HIGH segment in terms of the market volume, leading to a stronger reduction in government revenues. However, in quantitative terms, any effects achievable through an increase in the *ad valorem* rate and a reduction in the specific rate would have been marginal. Taking that into account, as well as the statistical uncertainty around the obtained estimates, the risk of any significant policy adjustment in the simulation horizon does not seem to be justified.

The results are different if we assume a constant level of price elasticities (at the level estimated over the last 12 months in the sample period, i.e. July 2013–June 2014, Tables 7 to 8). In this scenario the government may seem to have had some room for changing the excise structure towards a higher specific rate. Under this assumption, price elasticities of demand are lower (in modulus) than under time-varying elasticities over the simulation period. As a result, revenue losses from a shrinking market volume (especially in the LOW segment – due to an increase in the specific rate) are more than compensated for by additional revenues from the higher end of the market, where the unit taxation is higher than in the LOW segment. Again, potential gains from the above change would have been marginal, while the statistical uncertainty around the obtained estimates only reinforces the call for a cautious approach as regards any excise tax adjustments. Moreover, one should remember that an assumption of constant elasticities seems to be less realistic than time-varying elasticities, as demonstrated by the authors' econometric analysis.

Finally, the simulation results show that under both time-varying and constant elasticities, adopting a four-year policy horizon is superior to a policy based on a year-by-year optimisation of tax rates. The rationale is as follows: a mid-term objective function of the government (Policy 3) results in more gradual tax hikes, which helps to maintain a relatively higher duty-paid market volume over the path to 2018. In contrast, with a short-term objective function (Policy 2), policymakers attempt to achieve the highest possible one-year ahead level of budget revenues, which results in stronger hikes and thus a more rapidly shrinking duty-paid market (consumers shifting to substitute products or to the illegal market, some reducing or stopping smoking). Such an approach reduces the tax base for the next year, which, however, is not taken into account in a one-year-ahead approach. In a new situation the government tries to make up for a duty-paid market shrinkage through yet stronger tax hikes, which results in yet faster duty-paid market contraction relative to the developments observed under

Policy 3. The process continues over the whole simulation period. At the beginning of the simulation horizon, the year-by-year optimisation policy results in higher government revenues than a policy based on the multi-annual approach. However, over the whole period considered, the latter policy proves to be superior to the former. Thus it is yet more evidence showing that short-termism brings short-term benefits but causes mid-term damage.

3.6. *Ex post* analysis

The results described above are based on an econometric sample that ends in June 2014 and concerns a simulation period of July 2014–December 2018. Depending on the time perspective, the modelling results can be interpreted as a forecast into the future, but also – given the information set available after December 2018 – as a "what-if" analysis for the discussed simulation period.

In the real world, the 'no more hikes' strategy was followed by the Polish authorities over the 2015-2018 period, i.e. the specific rate remained at the level of PLN 206.76, the *ad valorem* rate remained at the level of 31.41 per cent, and the minimum rate was updated automatically. According to the data of the Polish Ministry of Finance, the tobacco excise revenues increased from PLN 17.9 bn in 2014 to PLN 18.8 bn in 2017, while according to the conducted simulations, under the 'no more hikes' policy, the government revenues should have dropped from PLN 17.9 bn in 2014 to about PLN 17.2 bn in 2017. This deviation of the modelling results from the actual developments of tobacco excise revenues stems from the structural break in excise tax policy initiated in 2015. In particular, over most of the 2005–2014 period, a series of excise hikes took place accompanied by a corresponding retail price growth, while from 2015 onwards, an unprecedented period of stable excise rates started. If the estimation additionally covered the period of 2015–2018, it might have taken account of the related behavioural development among consumers. From that perspective, the presented simulation results under the 'no more hikes' policy should be considered less accurate (both *ex ante* and *ex post)* than the results for Policies 2 and 3 that included further excise hikes. In addition, the demand for cigarettes in Poland is driven by non-price factors. Such factors (e.g. significant immigration from Ukraine) could have led to stronger cigarette demand and thus to higher government revenues from those simulated in this model.

CONCLUSIONS

The authors developed a methodological framework for analysing the excise tax policy on tobacco products. It allows for time-varying own price elasticities of demand, which are measured in a state-space model as unobservable state variables. The model incorporates the split of the tobacco market into distinct segments, which allowed to draw conclusions not only about the optimum level, but also about the optimum structure of taxation, i.e. how the tax burden should be distributed between *ad valorem* and specific rates. Different combinations of the specific and *ad valorem* rates, even though yielding the same average level of the overall taxation, may trigger different price (and demand) movements in distinct market segments, which in turn may result in a different budgetary impact. Consequently, the authors drew the Laffer surface (rather than a curve) over this two-dimensional tax domain. The authors illustrated their approach by fitting the model to the Polish data (2005–2014) and using it to simulate different excise tax policies over the period 2015–2018. The econometric analysis confirmed considerable time-variability of the price elasticity of demand for tobacco products growing (in modulus) in line with the increasing real prices of tobacco products. Moreover, price elasticities of demand (and their trends) turned out to be historically different for individual market segments of tobacco products. This is an important contribution of the authors' study in providing recent estimates of price elasticities of demand in the Polish tobacco market, for which only limited evidence is available.

The point estimates of price elasticities of demand towards the end of the statistical sample (i.e. in mid-2014), coupled with the simulated price-setting strategies of the manufacturers (suppliers), imply the existence of a classical, concave function that relates government revenues to tax rates – in the spirit of the Laffer curve. Consequently, there exists a revenue-maximising taxation level, above which the higher excise tax rates lead to a decline in tax revenues due to a rapidly falling market volume (lower demand). Note, however, that the statistical uncertainty around the point estimates is relatively high and the confidence interval also covers the area where the likely maximum does not exist.

Since the paper considered not only the level, but also the mix of both the level and structure of excise rates, the optimisation problem is two-dimensional and its solution is illustrated by the Laffer surface, not the curve. The results show that the actual mix of cigarette tax rates in 2015–

2018 was located close to the maximum on the Laffer surface. However, taking into account the statistical uncertainty around these estimates, it is currently difficult to determine whether the actual position was ‘to the left’ or ‘to the right’ of the maximum. Moreover, the simulation results suggest that the differences between the actual and the optimum taxation policy for tobacco products were marginal. One thus concludes that the excise tax adjustments should not be significant and be implemented with caution. Finally, the research shows that adopting a mid-term perspective is superior to a policy based on the one-year-ahead maximisation of government revenues, for the latter leads to a stronger contraction of the legal tobacco market, which carries forward and reduces the tax base in the following years. It is yet more evidence indicating that short-termism brings short-term benefits but causes mid-term damage.

As many countries in the world use a mixture of specific and *ad valorem* taxation of tobacco products, this framework could be also used in other countries in order to monitor the tax policy performance on a continuous basis. The most natural candidates are other EU member states to which a similar tobacco excise tax framework applies, enforced at EU level.

As a natural direction of further research the authors can indicate an econometric estimation that covers the extended period of stable excise rates in Poland. A sample that includes both the periods of relative price stability and the periods of considerable price hikes could be used to produce a much more versatile econometric model of demand. Furthermore, cointegration-based models of demand could be explored to cast some light on the issue of the long-term tobacco control policy in Poland. Importantly, in such a setting, the economic rationale for changes in time-varying price elasticities of demand would be different than discussed in this paper. Lastly, the future study could be extended to the area of direct and indirect costs of smoking-related diseases. This would lead to a modified objective function of the general government that would take into account the following components of the general government balance, not covered in this paper: (1) expenditure of the general government sector on the treatment of diseases related to smoking (direct cost of smoking-related diseases from the general government perspective) and (2) decrease in general government revenues related to the negative health shock in the labour supply market and the resulting drop in selected revenues, mainly PIT and social security contributions (fiscal consequences of indirect costs of the smoking-related diseases).

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APPENDIX

Consider a simplified problem of the government maximising tobacco-related revenues in which there is only *ad valorem* tax $\tau \in \left(0;1\right]$. The government revenues from segment i are calculated as follows:

$$R_i\left(\tau\right) = \tau P_i\left(\tau\right) Q_i\left(\mathbf{P}\left(\tau\right)\right), \tag{30}$$

in which P_i is retail price in segment i, Q_i is Marshallian demand for segment i and $\mathbf{P}$ is an $n \times 1$ vector of retail prices in all the segments. Under a single tax, the retail price in segment i is defined as follows:

$$P_i\left(\tau\right) = \frac{\overline{P}^i}{1-\tau}, \tag{31}$$

with $\overline{P}^i$ being the net consumer price. For the purposes of this proof it is assumed that it remains constant as in case of tax changes, manufacturers defend it by adjusting retail price. Taking derivatives of equations (30) and (31) with respect to τ which gives:

$$\frac{\partial R_i}{\partial \tau} = P_i\left(\tau\right) Q_i\left(\mathbf{P}\left(\tau\right)\right) + \tau Q_i\left(\mathbf{P}\left(\tau\right)\right) \frac{\partial P_i}{\partial \tau} + \tau P_i\left(\tau\right) \sum_{j=1}^{n} \frac{\partial Q_i}{\partial P_j} \frac{\partial P_j}{\partial \tau} \tag{32}$$

and

$$\frac{\partial P_i}{\partial \tau} = \frac{\overline{P}^i}{(1-\tau)^2} = \frac{P_i\left(\tau\right)}{1-\tau}. \tag{33}$$

The problem faced by the government is defined as follows:

Note that $E=\sum_{i=1}^{n} P_i Q_i$ is the total expenditures of consumers on cigarettes and use equation (33) to obtain:

$$\tau\sum_{i=1}^{n} Q_i \frac{P_i}{1-\tau}+\tau\sum_{i=1}^{n} P_i \sum_{j=1}^{n} \frac{\partial Q_i}{\partial P_j}\frac{P_j}{1-\tau}=-E. \tag{37}$$

Divide both sides of equation (37) by E and multiply the second term of the left-hand side of equation (under summation operator for i) by Q_i/Q_i (the denominator can enter under summation operator for j):

$$\tau\sum_{i=1}^{n} \frac{P_i Q_i}{E}\frac{1}{1-\tau}+\tau\sum_{i=1}^{n} \frac{P_i}{E}\frac{Q_i}{Q_i}\sum_{j=1}^{n} \frac{\partial Q_i}{\partial P_j}\frac{P_j}{1-\tau}=-1. \tag{38}$$

Note that $s_i=P_i Q_i/E$ is the share of segment i in the overall cigarettes expenditure whereas $\eta_{ij}=\left(\partial Q_i/\partial P_j\right)\left(P_j/Q_i\right)$ is the price elasticity of demand for segment i with respect to price of segment j in order to reach the following equation:

$$\tau\sum_{i=1}^{n} \frac{s_i}{1-\tau}+\tau\sum_{i=1}^{n} s_i \sum_{j=1}^{n} \frac{\eta_{ij}}{1-\tau}=-1. \tag{39}$$

This equation is solved with respect to τ to reach equation (9):

$$\tau=-\frac{1}{\sum_{i=1}^{n}\left[s_i\left(\sum_{j=1}^{n}\eta_{ij}\right)\right]}. \tag{40}$$