THE OPTIMIZATION ON THE VIATICAL INSURANCE MARKET*

ŚLĄSKI PRZEGLĄD STATYSTYCZNY
Nr 18(24)

ISSN 1644-6739
e-ISSN 2449-9765

Stanisław Heilpern, Joanna Dębicka
Wroclaw University of Economics and Business, Poland
e-mails: stanislaw.heilpern@ue.wroc.pl; joanna.debicka@ue.wroc.pl

Quote as: Heilpern, S., and Dębicka, J. (2020). The optimization on the viatical insurance market. *Śląski Przegląd Statystyczny, 18*(24).

DOI: 10.15611/sps.2020.18.15
JEL Classification: D81, G22, G120, C41

We assume that an *x*-year old man has whole-life insurance. In the insurance period, he becomes terminally ill and wants to resell the rights to the death benefit. The insured considers the possibility of reselling only part α of the rights to death benefit c. If he resells his right to the policy, he receives the viatical settlement payment (VSP) from the investor. The policyholder is obliged to pay annual premiums p at the beginning of each year of the insurance period. He would like to choose α for the maximum benefit from his point of view. The paper considered the above optimization problem which is due to the insured's behaviour under risky conditions and the value of VSP.

To model viatical contracts, the authors used the multiple state model introduced by Dębicka and Heilpern in [Dębicka, Heilpern 2017], which consists of the space of each state $S = \{1, 2, ..., 5\}$ for the following five states:

1 – the insured is terminally ill and his expected lifetime is less than 4 years.
2 – the insured is terminally ill and his expected lifetime is less than 3 years.
3 – the insured is terminally ill and his expected lifetime is less than 2 years.
4 – the insured is terminally ill and his expected lifetime is less than 1 year.
5 – the insured died being terminally ill with a fatal disease.

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* 25th Scientific Statistical Seminar „Marburg-Wrocław”. Gollhofen, 23-26 September 2019. Extended abstract.

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Let X be the random variable describing the total cash flows after resale policy. The distribution of random variable X takes the following form:

$$P(X = x_k) = q_k,$$

where $k = 1, 2, 3, 4$, $q_1 = 1 - p_{12}$, $q_2 = p_{12}(1 - p_{23})$, $q_3 = p_{12}p_{23}(1 - p_{34})$, $q_4 = p_{12}p_{23}p_{34}$, and p_{ij} are the transition probability from state i to state j. The values x_k of the cash flow equal $x_k = \alpha VSP + (1 - \alpha)b_k$, where $b_k = v^k c - p\sum_{i=0}^{k-1} v^i$.

The expected value of this cash flow is

$$EX = \sum_{k=1}^{4} x_k q_k = \alpha VSP + (1-\alpha)\sum_{k=1}^{4} b_k q_k.$$

If $VSP < \sum_{k=1}^{4} b_k q_k$, then we obtain a maximum of EX when $\alpha = 0$. For $VSP > \sum_{k=1}^{4} b_k q_k$ we have maximum when $\alpha = 1$, and for $VSP = \sum_{k=1}^{4} b_k q_k$ all α's are just as ‘good’.

Now let us take into consideration different kinds of behaviour of the insured under risky conditions. When the insured follows *the utility theory* [von Neumann, Morgenstern 1944], then he maximizes the expected utility of cash flow:

$$E(U(X)) = \sum_{k=1}^{4} U(x_k) q_k,$$

where $U(x)$ is a utility function. We use the discounted utility model (Hey 2004) and obtain

$$U(x_k) = u(\alpha VSP - (1-\alpha)p) + u((\alpha-1)p)\sum_{i=1}^{k-1} v^i + u((1-\alpha)c)v^k,$$

where u is continuous, concave (risk aversion), increasing function, such that $u(0) = 0$. When the utility function has the power form:

$$u_1(x) = (x + p)^\beta - p^\beta,$$

we obtain the maximum of $E(U(X))$ for

$$\alpha_0 = \frac{(cA_c)^{1/(\beta-1)}(c+p)}{((VSP+p)^\beta + p^\beta A_p)^{1/(\beta-1)} + c^{\beta/(\beta-1)} A_c^{1/(\beta-1)}},$$

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where $A_p = \sum_{k=2}^{4} \left(\sum_{i=1}^{k-1} v^i \right) q_k$ and $A_c = \sum_{k=1}^{4} v^k q_k$.

The rank dependent expected utility (RDEU) *theory* [Quiggin 1982] is based on the Choquet integral, the utility function $u(x)$ and distorted probability function $w(q)$, where w: [0, 1] → [0, 1] is increasing and $w(0) = 0$, $w(1) = 1$. The RDEU is

$$E_w(u(X)) = \int_0^{\infty} w(S(x))dx - \int_{-\infty}^{0} (1 - w(S(x)))dx = \sum_{k=1}^{4} u(x_k) w_k ,$$

where $S(x)$ is the survival function of random variable $Y = u(X)$, $w_1 = w(q_1)$, $w_2 = w(q_1 + q_2) - w(q_1)$, $w_3 = w(1 - q_4) - w(q_1 + q_2)$ and $w_4 = 1 - w(1 - q_4)$. We obtain a similar situation as in the usual utility theory; only the probabilities are distorted.

The insured, who follows *the cumulative prospect theory* [Tversky, Kahneman 1992] maximizes the generalized expected utility

$$E_{w_+, w_-}(u(X)) = \int_0^{\infty} w_+(S(x))dx - \int_{-\infty}^{0} w_-(F(x))dx,$$

where $F(x)$ is a cumulative distribution function of Y. Yet in this case the values of the cash flow $x_k > 0$, then we use the distorted probabilities $w_{+.k}$ only. The authors in [Kahneman, Tversky 1979] proposed the utility function

$$u_2(x) = \begin{cases} x^{\beta} & x \geq 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases},$$

where $\beta = 0.88$ and $\lambda = 2.25$ and the distorted function

$$w(q) = \frac{q^{\gamma}}{(q^{\gamma} + (1-q)^{\gamma})^{1/\gamma}},$$

where $\gamma_+ = 0.61$ and $\gamma_- = 0.69$. If the insured follows the cumulative prospect theory, the expected utility of the cash flow takes the following form

$$E_{w+}(U(X)) = \begin{cases} (\alpha VSP - (1-\alpha)p)^{\beta} + (1-\alpha)^{\beta} B & \alpha \geq \alpha_V \\ (1-\alpha)^{\beta} B - \lambda((1-\alpha)p - \alpha VSP)^{\beta} & \alpha < \alpha_V \end{cases},$$

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where $\alpha_V = \dfrac{p}{VSP + p}$, $B = A_c c^{\beta} - \lambda A_p p^{\beta}$, $A_p = \sum_{k=2}^{4}\left(\sum_{i=1}^{k-1} v^i\right) w_{+,k}$ and $A_c = \sum_{k=1}^{4} v^k w_{+,k}$. The maximum value of the expected utility of the cash flow can be at points $\alpha = 0$ or $\dfrac{p(VSP + p)^{1/(\beta-1)} + B^{1/(\beta-1)}}{(VSP + p)^{\beta/(\beta-1)} + B^{1/(\beta-1)}}$ when $\alpha \geq \alpha_V$ and at $\alpha = 0$ or $\dfrac{B^{1/(\beta-1)} - p(VSP + p)^{1/(\beta-1)}}{B^{1/(\beta-1)} - \lambda^{1/(\beta-1)}(VSP + p)^{\beta/(\beta-1)}}$ when $\alpha < \alpha_V$.

Example. A 60-year man fell ill, he established a policy at $x = 20$ years. Let $c = 100$, $p = 0{,}013c$ and $v = 0.98$. Then $59.200 < VSP < 96.899$.

A. Thus $E(X) = \alpha VSP + 95.687(1 - \alpha)$. Therefore, for $VSP < 95.914$, $\alpha = 0$ guarantees the maximal value of $E(X)$ and for $VSP > 95.687$ we have $\alpha = 1$; but for $VSP = 95.687$, every $0 \leq \alpha \leq 1$ is just as ‘good’.

B. For power utility function $u_1(x)$ and $\beta = 0.6$ we obtain

$$E(U(X)) = ((VSP + 1.4817)^{0.6} + 0.3201)\alpha^{0.6} + 0.9754(101.4817 - 100\alpha)^{0.6} - 2.8211$$

and we have a maximum of $E(U(X))$ for

$$\alpha_0 = \frac{0.001080}{((VSP + 1.4817)^{0.6} + 0.3201)^{-2.5} + 0.001064}.$$

The greater values of VSP imply a greater value of the optimal parameter α.

C. We use the utility function $u_1(x)$ with $\beta = 0.6$ and the distortion probability function $w(q)$ with $\gamma = 0.61$ and obtain the RDEU of cash flow

$$E_w(U(X)) = ((VSP + 1.4817)^{0.6} + 0.8476)\alpha^{0.6} + 0.9673(101.4817 - 100\alpha)^{0.6} - 3.3383.$$

The maximum of $E_w(U(X))$ is reached at point

$$\alpha_0 = \frac{0.001071}{((VSP + 1.4817)^{0.6} + 0.8476)^{-2.5} + 0.001087}$$

We obtain similar results as in b), but we obtain the greater values of the optimal α_0 and smaller values of the expected utility of the cash flow.

D. For utility function $u_2(x)$ with $\beta = 0.88$, $\lambda = 2.25$ and the distorted probability function $w(q)$ with $\gamma_+ = 0.61$ the expected utility of the cash flow is

$$E(U(X)) = \begin{cases} (\alpha VSP - 1.4817(1-\alpha))^{0.88} + 53.5312(1-\alpha)^{0.88} & \alpha \geq \alpha_V \\ 53.5312(1-\alpha)^{0.88} - 2.25(1.4817(1-\alpha) - \alpha VSP)^{0.88} & \alpha < \alpha_V \end{cases},$$

where $\alpha_V = \dfrac{1.4817}{VSP + 1.4817}$. When $x = 20$ for acceptable values of VSP, we obtain similar results to the classic utility case, but for $x = 55$, we have $14.254 < VSP < 93.482$ and we observe a different situation. For $VSP \leq 20.556$ the optimal value $\alpha_0 = 0$. The graph of the values of optimal α with respect to VSP is not continuous in point $VSP = 19.475$. The greater values of λ, reflecting the insured's approach to losses, imply a greater value of the optimal parameter α.

Proof of the presented results and more examples can be found in [Dębicka, Heilpern 2020].

Acknowledgements

The support of the grant scheme NON-STANDARD MULTILIFE INSURANCE PRODUCTS WITH DEPENDENCE BETWEEN INSURED 2013/09/B/HS4/00490 is gratefully acknowledged.

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Keywords: viatical settlement, multistate model, Bernoulli principle, Rank Dependent Expected Utility Theory, Cumulative Prospect Theory.