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A DYNAMIC APPROACH TO RELATIVE TAXONOMY IN THE ASSESSMENT OF CHANGES IN THE SOCIAL COHESION OF POLISH PROVINCES IN 2010-2018

The classic approach to relative taxonomy was proposed by S. Wydymus (2013), and its positional version developed by J. Lira (2015). Although they are applied to panel data, both methods of relative taxonomy are static. This article presents a dynamic version of relative taxonomy. In contrast to the static approach, the dynamic approach not only shows relations between objects in individual periods but also changes in the phenomenon of interest that occurred between the objects throughout the reference period. The static version, described by Wydymus and Lira, does not account for the presence of missing (NA) values in a data array, in contrast to the proposed dynamic modification, which does. The proposed dynamic approach to relative taxonomy was used to assess the social cohesion of Polish provinces (voivodeships) in the period 2010-2018. The analysis indicates that social cohesion across the provinces systematically increased, while differences between them decreased. The rate at which social cohesion improved was faster in those provinces in which it was lower at the start of the reference period, i.e. Kujawsko-Pomorskie, Lubuskie, Warmińsko-Mazurskie and Zachodniopomorskie.

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1. INTRODUCTION – RATIONALE FOR THE STUDY

Many structures of aggregate measures have been developed in the source literature. Depending on the degree of compensation, three types of methods can be identified (El Gibari, Gómez, Ruiz 2019, p. 3): compensatory, partially

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compensatory, non-compensatory. Compensatory methods assume that a poor performance in an individual indicator can be compensated for by high performance in the other individual indicator and may not be reflected in the aggregated performance of a composite indicator (Banihabib et al. 2017). The task of partially-compensatory methods is to reduce the influence of the compensation phenomenon when constructing an aggregate measure. The core of the non-compensatory methods is to develop a ranking algorithm which is more consistent than the linear aggregation rule, where no compensation is allowed among the indicators, and thus all the weights reflect the relative importance of each indicator instead of a trade-off ratio (El Gibari, Gómez, Ruiz 2019, p. 15). In the literature on multi-criteria decision making (MCDM), non-compensatory methods are used (see Munda 2008; Munda, Nardo 2009). The article focused on the aggregate measures of compensatory and a partially compensatory nature.

The development of the methods of linear ordering of a set of objects dates back to the late 1960s, and most of these methods are the product of Polish taxonomic theory. The first paper in English addressing the concept of the pattern of development and the measure of development was given by Professor Zdzisław Hellwig at the UNESCO conference in Warsaw in 1967 (Hellwig 1967). The article describing the concept was published in English in a monograph edited by Z. Gostkowski (Hellwig 1972). The first article in Polish on the same topic appeared in the journal "Przegląd Statystyczny" [Statistical Review] in 1968 (Hellwig 1968). Both articles contained definitions of basic concepts, such as stimulants and destimulants, the pattern of development and the measure of development defined as a distance from the pattern of development.

It would be no exaggeration to argue that Hellwig's idea gave rise to a large number of new methods of linear ordering. These proposals (cf. Borys, Strahl and Walesiak 1990; Pociecha and Zając 1990) were designed to:

- propose different ways of normalising variable values,
- introduce nominants in the dataset,
- show different ways of determining the pattern of development (the benchmark),
- provide the possibility of selecting different formulas of the aggregate measure (the so-called measure of development),
- suggest the possibility of using fuzzy sets in the construction of the aggregate measure.

Since the original proposal concerning the measure of development was not normalized in a closed interval $[0; 1]$, Hellwig (1981) proposed an aggregate measure based on the pattern and anti-pattern of development, which satisfies this condition. Moreover, the proposal introduces the notion of a set axis (a line connecting the pattern with the anti-pattern object) and isoquants of development (curves of equal development) drawn at certain distances from the pattern object. Objects located between isoquants represent a similar level of development. The same level can be achieved by objects located at different points along the same isoquant of development (due to a different configuration of variable values). In the article by Siedlecka and Siedlecki (1990), the authors presented a formula for an orthogonal projection of objects to the set axis through isoquants. Hwang and Yoon (1981) proposed a somewhat different form of the TOPSIS aggregate measure, which includes both types of the pattern object.

In recent years, the following new concepts and applications of linear ordering have been put forward:

- those based on fuzzy numbers (cf. e.g. Łuczak, Wysocki 2006; Wysocki 2010; Jefmański and Dudek 2016),
- those based on symbolic interval-valued data (Młodak 2014),
- those that account for spatial relationships (Antczak 2013; Pietrzak 2014),
- those that combine multidimensional scaling with linear ordering for classic data (Walesiak 2016b) and for symbolic interval-valued data (Walesiak and Dehnel 2018; Dehnel and Walesiak 2019),
- those that employ principal component analysis (PCA) for linear ordering of objects based on the value of the first principal component (Bąk 2018; Perkal 1967),
- those based on aggregate measures with a penalty function (Mazziotta and Pareto 2016; 2018),
- those based on aggregate measures which account for the neighbourhood of an object (Łysoń, Szymkowiak and Wawrowski 2016),
- those based on an iterative approach to ranking sets of objects (Sokołowski and Markowska 2017),
- those that employ flexible linear ordering (Sokołowski and Markowska 2019).

This article focused on a method of relative taxonomy, which makes it possible to assess the level of a phenomenon of interest in a given object relative to other objects analysed in the study. In the classic approach, the method was proposed by Wydymus (2013), while its positional version was developed by Lira (2015). In his monograph, Lira (2019, pp. 26-33) presented solutions concerning modifications of aggregate measures in relative taxonomy that represent both classic and positional approaches. Both versions of relative taxonomy (classic and positional) are based on the static approach although they are applied to panel data. In line with the assumptions of this approach, the level of the phenomenon of interest in a given object in period t is relativized to the level of the phenomenon in other objects measured only in period t. This article describes a dynamic version of relative taxonomy, to which this limitation does not apply because the level of the phenomenon in a given object is assessed relative to the levels of the phenomenon in all objects throughout the entire reference period.

Moreover, the static version proposed by Wydymus and Lira does not allow for the presence of NA values in a data matrix. In contrast, the proposed dynamic modification does account for missing data. The article presents the possibilities and limitations of using this method of relative taxonomy in the linear ordering of a set of objects. In the empirical part of the study, the proposed dynamic version of relative taxonomy was used to assess the social cohesion of Poland's provinces in the period 2010-2018.

2. THE CONCEPT OF SOCIAL COHESION

The task of defining and measuring social cohesion is far from easy. The difficulty is the result of a multitude of different approaches to its conceptualisation. The definitions proposed in the literature differ in terms of the areas of life they focus on, periods they refer to, political ideas they represent and methods they employ to foster cohesion (Council of Europe 2005, p. 23).

The complexity of the concept of social cohesion is manifested by the variety of approaches that can be found in the literature (Jenson 1998, 2010; Bernard 1999; Beauvais and Jenson 2002; Chan et al. 2006; Hulse and Stone 2007, Dickes et al. 2010; Novy et al. 2012; Klein 2013; Ariely 2014; Langer et al. 2017; Fonseca et al. 2019). All of them refer, to a varying degree, to six dimensions: social relations, identification, orientation towards the common good, shared values, quality of life, and (in)equality, though it is often indicated that the last three are antecedents or consequences of social cohesion rather than its core dimensions (Schiefer and Noll 2017). Although various approaches involve different areas of social cohesion, refer to different political views and are informed by different ideologies or concerns of policy makers, the majority of them overlap, covering similar dimensions (Council of Europe 2005, s. 23). All the existing approaches that propose definitions of social cohesion can be divided into two discourses: the academic and the policy one (Chan et al. 2006).

The academic discourse is closely connected with social sciences, such as sociology and psychology. Studies referring to the academic discourse focus on the processes of social integration and stability and social exclusion, while ignoring the dilemmas associated with the definition of social cohesion (Berger 1998, Gough and Olofsson 1999, Friedkin 2004).

The policy discourse refers to policies undertaken by governments and various national and international institutions (the European Union, the Council of Europe, the World Bank and the OECD). In this case, social cohesion is viewed as a prerequisite of economic well-being. This perspective highlights the numerous economic and social problems resulting from unequal income distribution, employment, housing issues, limited access to health care and education, and participation in politics and public life. The policy discourse can therefore be described as problem-driven (Chan et al. 2006, Dickes et al. 2010, Bottoni 2018). The key problems described in the literature include unemployment, poverty and social exclusion.

When analysing conceptualisations of social cohesion, it is possible to track the direction of changes which reflect the increasing role of the socio-cultural and political indicators and the omission of the economic sphere.

Both discourses on social cohesion are multidimensional, but in each case the key concept is different, which is the main source of the existing discrepancies in measuring, and consequently, in assessing the level of social cohesion.

A great deal of attention has been paid in the literature to social cohesion. Most of the proposed methods of analysis involve various composite indicators that account for all types of social challenges (Friedkin 2004, Chan et al. 2006, Dickes et al. 2010). Thus, for example, Duhaime et al. (2004) attempted to measure the level of social cohesion in the Canadian Arctic using six sets of indices for perceptual and behavioural variables of social support, subjective well-being and quality of life. Based on the definition of social cohesion by Bernard (1999) and Chan et al. (2006), Acket et al. (2011) developed the VALCOS index of social cohesion for European countries. Langer et al. (2017) put forward two social cohesion indices: a national average SCI and a Social Cohesion Index Variance-Adjusted (SCIVA) to measure the national-level of social cohesion in 19 African countries in 2005, 2008 and 2012. In the Polish literature, Balcerzak (2015) analysed social cohesion in EU countries using a synthetic measure of development proposed by Hellwig (1972). In Dehnel et al. (2019), the level of social cohesion was assessed using a hybrid approach combining multidimensional scaling and linear ordering, applied separately to classic metric data and symbolic interval-valued data. There are a number of other publications that proposed composite indicators of social cohesion (e.g. Schaeffer 2013; Holgado et al. 2015). The level of social cohesion can also be assessed using non-compensatory composite indices (MPI – Mazziotta-Pareto Index, AMPI – the adjusted Mazziotta–Pareto index) proposed by Mazziotta and Pareto (2016; 2018).

A review of studies on social cohesion revealed that the emphasis is placed on spatial analysis of social cohesion, which takes into account not only the national but also the regional level. Moreover, the level of social cohesion in a given society can only be properly assessed in comparison to other territorial units and over time (Healy et al. 2016; *My Region, My Europe, Our Future* 2017; Sanchez et al. 2018). In the case of studies relating to countries and regions of the European Union, the EU regional Social Progress Index (EU-SPI)[1] has been used since 2016. For the purpose of this index, social progress is defined as “a society’s capacity to meet the

[1] https://ec.europa.eu/regional_policy/en/information/maps/social_progress

basic human needs of its citizens, to establish the basis for people and communities to improve and sustain their quality of life and to create the conditions for people to reach their full potential" (*My Region, My Europe, Our Future* 2017, p. 91). The EU-SPI is consistent with the overall framework of the global Social Progress Index, and is based on fifty indicators, primarily from Eurostat. It covers three dimensions of social progress (Annoni and Dijkstra 2016):

- basic human needs (nutrition and basic medical care, water and sanitation, shelter – housing, personal safety),
- foundations of well-being (access to basic knowledge, access to information and communication, health and wellness, environmental quality),
- opportunity (personal rights, personal freedom and choice, tolerance and inclusion, access to advanced education).

Given the nature of the statistical data (secondary data) used in the analysis, the empirical research described in the article was based on the approach adopted in studies conducted by EU countries using the EU-SPI.

3. A REVIEW OF LITERATURE ON RELATIVE TAXONOMY

The relative taxonomy method, despite being relatively new, has been used in a number of empirical applications. Wydymus (2013) employed his own version of relative taxonomy in a comparative analysis of disproportions between the level of economic development and the level wages in Poland and in other EU countries in the period 2004-2011. Lira et al. (2014) used Wydymus's method of relative taxonomy to study the process of eliminating the differences in the development of the technical infrastructure in rural areas across provinces in 2004-2012. Muszyńska and Müller-Frączek (2016) used the same method to assess the completion of tasks outlined in the flagship *initiative* for a "*resource-efficient Europe" by EU countries in the area of greenhouse gas emissions, renewable energy sources and energy efficiency*. Szopik-Depczyńska et al. (2017) applied the Wydymus method of relative taxonomy to analyse disproportions regarding the sustainable development of EU countries. Other applications of Wydymus's classic approach to relative taxonomy include the following studies by: Müller-Frączek and Muszyńska (2016), Wydymus (2017), Cheba (2017), Bąk and Cheba (2018), Głowicka-Wołoszyn et al. (2018), Cheba and Bąk (2019), Ziolo et al. (2019), Cheba (2019; 2020), Cheba and Szopik-Depczyńska (2018).

Lira (2015) proposed an aggregate measure based on Weber's spatial median. The classic approach of Wydymus (involving the arithmetic mean) and the positional one (based on the median) were used to examine the position of a given district relative to others in terms of the accessibility of the rural population to infrastructure services in Wielkopolskie province in 2013. Dolata and Lira (2017) used a method of relative taxonomy based on the positional approach to investigate the level of housing stocks and technical and sanitary equipment in dwellings in rural areas in Wielkopolskie province across its districts in the period 2004-2015.

4. CHARACTERISTICS OF THE RELATIVE TAXONOMY METHOD – THE STATIC APPROACH

The procedure in the static version of relative taxonomy (see Wydymus 2013) consists of the following steps:

1. Create T (period number, e.g. year) data matrices containing m variables describing n objects:

$$\left[x_{ijt}\right]_{n\times m} = \begin{bmatrix} x_{11t} & x_{12t} & \cdots & x_{1mt} \\ x_{21t} & x_{22t} & \cdots & x_{2mt} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1t} & x_{n2t} & \cdots & x_{nmt} \end{bmatrix}, \text{ for } t = 1, \ldots, T, \tag{1}$$

where: $i = 1, \ldots, n$ – object number (e.g. country),
$j = 1, \ldots, m$ – variable number,
$t = 1, \ldots, T$ – period number (e.g. year).

2. Identify stimulants (where higher values are more preferred), destimulants (where lower values are more preferred) and nominants (where the nominal value is the best value and lies within the variable range) in the set of variables. The definitions of stimulants and destimulants are presented in (Hellwig 1981, p. 48), while nominants are defined in (Borys 1984, p. 118). These definitions can also be found in (Walesiak 2016b). Destimulants and nominants are converted into stimulants according to the methods presented in Section 5.

3.The value of the j-th variable in period t for each object b is relativized to each object c according to the following formula[2]:

$$y_{(b/c)jt} = \begin{cases} x_{bjt} / x_{cjt} & x_{cjt} \neq 0; \\ 0 & x_{cjt} = 0; \end{cases} \tag{2}$$

where: $b = 1, \ldots, n$; $c = 1, \ldots, n$.

[2] This is an extended version of Wydymus's relativization (2013), proposed by Lira (2019), which includes objects for which the observations on variables equal zero.

In the study by Lira (2019, pp. 11-12), relativization formulas for stimulant, destimulant and nominant were proposed. In this situation, steps 2 and 3 take place in parallel. This solution is equivalent to steps 2 and 3 for this procedure.

The relativized values of variable j in period t are presented in the form of a matrix:

$$\mathbf{Y}_{jt} = \begin{bmatrix} 1 & y_{(2/1)jt} & \cdots & y_{(n/1)jt} \\ y_{(1/2)jt} & 1 & \cdots & y_{(n/2)jt} \\ \vdots & \vdots & \ddots & \vdots \\ y_{(1/n)jt} & y_{(2/n)jt} & \cdots & 1 \end{bmatrix}. \tag{3}$$

4. Calculate the matrix product:

$$\mathbf{Y}_{jt}^{*} = \mathbf{A} \cdot \mathbf{Y}_{jt}, \tag{4}$$

where $\mathbf{A}$ is a matrix of size $n \times n$, whose main diagonal contains zero values, while the remaining values are equal to $1/(n-1)$.

For each variable numbered $j = 1, \ldots, m$, elements of the main diagonal of matrix $\mathbf{Y}_{jt}^{*}$ are presented as column vectors in matrix $\mathbf{W}_t$ for subsequent periods *t*:

$$\mathbf{W}_t = \begin{bmatrix} w_{11t} & w_{12t} & \cdots & w_{1mt} \\ w_{21t} & w_{22t} & \cdots & w_{2mt} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1t} & w_{n2t} & \cdots & w_{nmt} \end{bmatrix}, \text{ for } t = 1, \ldots, T. \tag{5}$$

Matrix $\mathbf{W}_t$ is equivalent to a normalised matrix in multivariate statistical analysis for each period *t*.

Values w_{ijt} of matrix $\mathbf{W}_t$ represent the average similarity (based on the arithmetic mean) of a given relativized observation with respect to other relativized observations of the *j*-th variable in period *t*:

$$w_{ijt} = \frac{1}{n-1} \sum_{l=1, l \neq i}^{n} y_{(i/l)jt}. \tag{6}$$

Lira (2015) proposed a positional approach, in which average similarity w_{ijt} is calculated using the median (calculated from columns of matrix $\mathbf{Y}_{jt}$ excluding elements of the main diagonal):

$$w_{ijt} = \underset{l, l \neq i}{\operatorname{med}} \left(y_{(i/l)jt} \right), \text{ for } l = 1, \ldots, n. \tag{7}$$

The authors of this article are of the opinion that average similarity w_{ijt}, both in the classic approach (arithmetic mean) and in the positional approach (median), should be calculated by including elements of the main diagonal. Hence, in the classic approach $\mathbf{A} = \left[\frac{1}{n}, \dots, \frac{1}{n}\right]$, while in the positional approach the median is calculated from all values in each column of matrix $\mathbf{Y}_{jt}$. The benefits of this approach are illustrated with the example presented in Section 6.

5. Calculate aggregate measures SM_{it} for the i-th object in period t to order the objects:

a. Wydymus's classic approach:

$$SM_{it} = \sum_{j=1}^{m} \frac{1}{w_{ijt}} / m, \tag{8}$$

where: w_{ijt} is calculated according to formula (6).

b. Lira's approach:

$$SM_{it} = \text{med}\left(\frac{1}{w_{i1t}}, \cdots, \frac{1}{w_{imt}}\right), \tag{9}$$

where: w_{ijt} is calculated according to formula (7).

Lira (2019, p. 26) proposed that in the situation where $w_{ijt} = 0$, w_{ijt} should take the value close to 0, definitely lower than the minimum value determined for non-zero relativized values occurring for each variable in the analysed periods.

The values of aggregate measures SM_{it} can be greater or smaller than 1. The smaller the value of measure SM_{it}, the better the position of object i relative to other objects in period t.

Formulas (8) and (9) can be extended by the inclusion of different weights for variables expressing their significance in the description of the aggregate phenomenon:

$$SM_{it} = \sum_{j=1}^{m} \frac{1}{\alpha_j \cdot w_{ijt}} / m, \tag{10}$$

$$SM_{it} = \text{med}\left(\frac{1}{\alpha_1 \cdot w_{i1t}}, \cdots, \frac{1}{\alpha_m \cdot w_{imt}}\right), \tag{11}$$

where: α_j – weight for j-th variable satisfying the conditions $\alpha_j \in [0; m]$ and $\sum_{j=1}^{m} \alpha_j = m$.

There are three ways of determining variable weights in the literature (cf. Walesiak 2016a, p. 51). The weights can be determined either *a priori*, based on expert judgement, or using algorithms based on information included in primary (raw) data, or by combining both these methods. More information on the subject of variable weighting can be found in Bąk (1999), pp. 44-47; Borys (1984), p. 318-325; Abrahamowicz and Zając (1986); Grabiński (1984), pp. 25-30; Milligan (1989), Nowak (1990), pp. 33-35. The problem of determining variable weights has not yet been satisfactorily solved. Williams goes as far as to claim that variable weighting is a way of manipulating variable values (cf. Aldenderfer and Blashfield 1984, p. 21). For this reason, authors of empirical studies often assume that the variables are equally important from the perspective of the research problem. This view is maintained by, among others, Sneath and Sokal (1973).

Compared to many methods of linear ordering, the method of relative taxonomy (both its classic and positional version) has certain limitations:

- it can only be used for variables measured on the ratio scale[3] (possible values of such are included in the set of positive real numbers). Hence, it cannot be applied to interval variables. This is not a serious limitation, as ratio variables are by far the most common category of variables in the analysis of economic phenomena;
- the variability of relativized values, which are calculated using formula (2), has not been defined precisely: "quantities (…) are dimensionless and are close to 1" (cf. Wydymus 2013, p. 633). The relativization formula when the numerator is not greater than the denominator produces values included in the interval $(0;1]$, otherwise, values are included in the interval $(1;\infty)$;
- aggregate measures SM_{it} do not have an upper limit. This does not disqualify them;
- neither Wydymus's classic approach nor Lira's positional version allows for the presence of NA values in data matrices.

5. CHARACTERISTICS OF THE RELATIVE TAXONOMY METHOD – THE DYNAMIC APPROACH

In the static version (cf. Wydymus 2013, Lira 2015), relativization according to formula (2) is performed separately for each period $t = 1, \ldots, T$. In the dynamic version, values of the *j*-th variable are relativized based on all matrices from T periods. The dynamic procedure of relative taxonomy consists of the following steps:

1. Create a single data matrix containing m variables describing n objects in T periods:

[3] Measurement scales can be divided into metric and non-metric (Stevens 1946). Metric scales can be further subdivided into ratio and interval scales.

$$\left[x_{ijt}\right]_{n\cdot T\times m}=\begin{bmatrix} x_{111} & x_{121} & \cdots & x_{1m1} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n11} & x_{n21} & \cdots & x_{nm1} \\ \cdots & \cdots & \cdots & \cdots \\ x_{11T} & x_{12T} & \cdots & x_{1mT} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1T} & x_{n2T} & \cdots & x_{nmT} \end{bmatrix}, \tag{12}$$

where: $i=1,\ldots,n$ – object number (e.g. country),
$j=1,\ldots,m$ – variable number,
$t=1,\ldots,T$ – period number (e.g. year).

2. Identify the stimulants, destimulants and nominants in the set of variables. Destimulants D are converted into stimulants using the ratio transformation (cf. e.g. Walesiak 2002, p. 18):

$$x_{ijt}=\left(x_{ijt}^{D}\right)^{-1}. \tag{13}$$

Nominants N are converted into stimulants using the ratio transformation (cf. e.g. Walesiak 2002, p. 18):

$$x_{ijt}=\frac{\min\left\{nom_j;\, x_{ijt}^{N}\right\}}{\max\left\{nom_j;\, x_{ijt}^{N}\right\}}, \tag{14}$$

where: nom_j – nominal level of the *j*-th variable.

3. Relativize values of the *j*-th variable according to the following formula:

$$\left[\frac{x_{1j1}}{x_{ijt}},\ldots,\frac{x_{njT}}{x_{ijt}}\right],\ \text{for } i=1,\ldots n \ \text{ and } \ t=1,\ \ldots,\ T \tag{15}$$

which yields a $n\cdot T\times n\cdot T$ matrix of relativized values of the *j*-th variable. As a result of relativization, variable values are dimensionless.

4. Calculate the average similarity (using the arithmetic mean or median) of a given relativized observation with respect to other relativized observations of the *j*-th variable:

a. in the classic approach (arithmetic mean):

$$\left[z_{ijt}\right]=\begin{bmatrix} \frac{1}{n\cdot T}\sum_{t=1}^{T}\sum_{i=1}^{n}\frac{x_{111}}{x_{i1t}} & \cdots & \frac{1}{n\cdot T}\sum_{t=1}^{T}\sum_{i=1}^{n}\frac{x_{1m1}}{x_{imt}} \\ \vdots & \vdots & \vdots \\ \frac{1}{n\cdot T}\sum_{t=1}^{T}\sum_{i=1}^{n}\frac{x_{n1T}}{x_{i1t}} & \cdots & \frac{1}{n\cdot T}\sum_{t=1}^{T}\sum_{i=1}^{n}\frac{x_{nmT}}{x_{imt}} \end{bmatrix}, \tag{16}$$

b. in the positional approach (median):

$$\left[z_{ijt}\right]=\begin{bmatrix} \operatorname{med}\left(\frac{x_{111}}{x_{111}},\ldots,\frac{x_{111}}{x_{n1T}}\right) & \cdots & \operatorname{med}\left(\frac{x_{1m1}}{x_{1m1}},\ldots,\frac{x_{1m1}}{x_{nmT}}\right) \\ \vdots & \vdots & \vdots \\ \operatorname{med}\left(\frac{x_{n1T}}{x_{111}},\ldots,\frac{x_{n1T}}{x_{n1T}}\right) & \cdots & \operatorname{med}\left(\frac{x_{nmT}}{x_{1m1}},\ldots,\frac{x_{nmT}}{x_{nmT}}\right) \end{bmatrix}. \tag{17}$$

5. Calculate values of aggregate measure SM_{it} using one of these formulas:
a. the classic approach:

$$SM_{it} = \sum_{j=1}^{m}\frac{1}{z_{ijt}} / m, \tag{18}$$

b. the positional approach:

$$SM_{it} = \operatorname{med}\left(\frac{1}{z_{i1t}},\cdots,\frac{1}{z_{imt}}\right). \tag{19}$$

Values of SM_{it} given by (18) and (19) can be greater or smaller than 1. The smaller the value of measure SM_{it}, the better the position of object i relative to other objects in a time interval from $t=1$ to $t=T$. Unlike the static approach, the dynamic approach not only shows relations between the objects in specific periods, but also changes regarding the phenomenon of interest that took place between the objects over the entire reference period.

Formulas (18) and (19) can be extended by the inclusion of different weights for variables expressing their significance in the description of the aggregate phenomenon:

$$SM_{it} = \sum_{j=1}^{m}\frac{1}{\alpha_j \cdot z_{ijt}} / m, \tag{20}$$

$$SM_{it} = \text{med}\left(\frac{1}{\alpha_1 \cdot z_{i1t}}, \cdots, \frac{1}{\alpha_m \cdot z_{imt}} \right). \tag{21}$$

Neither version of the static approach proposed by Wydymus and Lira, allows for the presence of NA values in matrices, in contrast to the dynamic modification described in this article, which does. Data of this type are not taken into account when calculating the values of aggregate measure SM_{it} given by (18) and (19) as well as (20) and (21).

6. A COMPARISON BETWEEN THE STATIC AND DYNAMIC APPROACH TO RELATIVE TAXONOMY

The drawbacks of the static version of relative taxonomy are illustrated with a simple example. For simplicity, only one variable (stimulant) is used in three periods $t = 1, 2, 3$ (the j index is omitted in the notation). Values of the variable in period $t = 2$ are 50% lower than in period $t = 1$, while values in period $t = 3$ are 50% higher than in period $t = 1$.

t	$x_{t=1}$	$x_{t=2}$	$x_{t=3}$
1	20	10	30
2	10	5	15
3	40	20	60
4	80	40	120

After static relativization according to formulas (2) and (3) the following matrices $\mathbf{Y}_{t=1}$, $\mathbf{Y}_{t=2}$ and $\mathbf{Y}_{t=3}$ are obtained for each period:

$$\mathbf{Y}_{t=1} = \begin{bmatrix} 1 & 0.5 & 2 & 4 \\ 2 & 1 & 4 & 8 \\ 0.5 & 0.25 & 1 & 2 \\ 0.25 & 0.125 & 0,5 & 1 \end{bmatrix}, \quad \mathbf{Y}_{t=2} = \begin{bmatrix} 1 & 0.5 & 2 & 4 \\ 2 & 1 & 4 & 8 \\ 0.5 & 0.25 & 1 & 2 \\ 0.25 & 0.125 & 0.5 & 1 \end{bmatrix},$$

$$\mathbf{Y}_{t=3} = \begin{bmatrix} 1 & 0.5 & 2 & 4 \\ 2 & 1 & 4 & 8 \\ 0.5 & 0.25 & 1 & 2 \\ 0.25 & 0.125 & 0.5 & 1 \end{bmatrix}.$$

The average similarity of a given relativized observation is calculated (using the arithmetic mean) with respect to other relativized observations of the *j*-th variable in period t (elements of the main diagonal of matrix $\mathbf{Y}_t^*$).

Matrices $\mathbf{Y}_{t=1}^*$, $\mathbf{Y}_{t=2}^*$ and $\mathbf{Y}_{t=3}^*$ are calculated using formula (4):

$$\mathbf{Y}_{t=1}^* = \mathbf{A}\cdot\mathbf{Y}_{t=1} = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1/2 & 2 & 4 \\ 2 & 1 & 4 & 8 \\ 1/2 & 1/4 & 1 & 2 \\ 1/4 & 1/8 & 1/2 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 11/12 & & & \\ & 7/24 & & \\ & & 13/6 & \\ & & & 14/3 \end{bmatrix},$$

$$\mathbf{Y}_{t=1}^* = \mathbf{Y}_{t=2}^* = \mathbf{Y}_{t=3}^* .$$

Values of the static taxonomic measure SM_{it} given by (8) are shown in Table 1.

Table 1

Values of the static taxonomic measure SM_{it} given by (8) in different periods

Object *i*	**Values of SM_{it} given by (8)**		
	***t* = 1**	***t* = 2**	***t* = 3**
1	1.0909	1.0909	1.0909
2	3.4285	3.4285	3.4285
3	0.4615	0.4615	0.4615
4	0.2142	0.2142	0.2142

Source: own calculations made using R.

After analysing the values of the static aggregate measure SM_{it} given by (8), the following conclusions can be formulated:

a. Relations between objects in the three periods have not changed.

b. SM_{it} values show that the level of the phenomenon in different periods did not change although values of the variable in period $t=2$ are 50% lower than those in period $t=1$, while values of the variable in period $t=3$ are 50% higher than in period $t=1$.

It is better to multiply matrix $\mathbf{Y}_t$ by vector $\mathbf{A}=\left[\frac{1}{n},\ldots,\frac{1}{n}\right]$:

$$\mathbf{Y}^*_{t=1}=\mathbf{A}\cdot\mathbf{Y}_{t=1}=\begin{bmatrix}\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}\end{bmatrix}\cdot\begin{bmatrix}1 & 0.5 & 2 & 4\\ 2 & 1 & 4 & 8\\ 0.5 & 0.25 & 1 & 2\\ 0.25 & 0.125 & 0.5 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}0.9375 & 0.46875 & 1.875 & 3.75\end{bmatrix},$$

$$\mathbf{Y}^*_{t=1}=\mathbf{Y}^*_{t=2}=\mathbf{Y}^*_{t=3}.$$

Values of the static aggregate measure SM_{it} given by (8) for the modified matrix $\mathbf{A}$ are shown in Table 2.

Table 2

Values of the static aggregate measure SM_{it} (8) after modifying matrix $\mathbf{A}$

Object *i*	**Values of SM_{it} given by (8)**		
	***t* = 1**	***t* = 2**	***t* = 3**
1	1.066667	1.066667	1.066667
2	2.133333	2.133333	2.133333
3	0.533333	0.533333	0.533333
4	0.266667	0.266667	0.266667

Source: own calculations made using R.

By using matrix $\mathbf{A}=\left[\frac{1}{n},\ldots,\frac{1}{n}\right]$ in the calculation, it is possible to retain the proportions between the observations for the variables in different periods.

The same example is used to illustrate the application of the dynamic approach.

First, observations for the variable from three periods $t=1, t=2, t=3$ are combined to create one vector:

$$[x_{it}]_{n\cdot T\times 1}=\begin{bmatrix}20 & 10 & 40 & 80 & 10 & 5 & 20 & 40 & 30 & 15 & 60 & 120\end{bmatrix}'$$

and then the relativized values are calculated according to formula (15):

$$\begin{bmatrix}
1 & 0.5 & 2 & 4 & 0.5 & 0.25 & 1 & 2 & 1.5 & 0.75 & 3 & 6 \\
2 & 1 & 4 & 8 & 1 & 0.5 & 2 & 4 & 3 & 1.5 & 6 & 12 \\
0.5 & 0.25 & 1 & 2 & 0.25 & 0.125 & 0.5 & 1 & 0.75 & 0.375 & 1.5 & 3 \\
0.25 & 0.125 & 0.5 & 1 & 0.125 & 1/16 & 0.25 & 0.5 & 0.375 & 3/16 & 0.75 & 1.5 \\
2 & 1 & 4 & 8 & 1 & 0.5 & 2 & 4 & 3 & 1.5 & 6 & 12 \\
4 & 2 & 8 & 16 & 2 & 1 & 4 & 8 & 6 & 3 & 12 & 24 \\
1 & 0.5 & 2 & 4 & 0.5 & 0.25 & 1 & 2 & 1.5 & 0.75 & 3 & 6 \\
0.5 & 0.25 & 1 & 2 & 0.25 & 0.125 & 0.5 & 1 & 0.75 & 0.375 & 1.5 & 3 \\
2/3 & 1/3 & 4/3 & 8/3 & 1/3 & 1/6 & 2/3 & 4/3 & 1 & 0.5 & 2 & 4 \\
4/3 & 2/3 & 8/3 & 16/3 & 2/3 & 1/3 & 4/3 & 8/3 & 2 & 1 & 4 & 8 \\
1/3 & 1/6 & 2/3 & 4/3 & 1/6 & 1/12 & 1/3 & 2/3 & 0.5 & 0.25 & 1 & 2 \\
1/6 & 1/12 & 1/3 & 2/3 & 1/12 & 1/24 & 1/6 & 1/3 & 0.25 & 0.125 & 0.5 & 1
\end{bmatrix}.$$

Formula (16) was used to determine the average similarity (using the arithmetic mean) of a given relativized observation with respect to other relativized observations of the j-th variable:

$$[z_{it}]_{n \cdot T \times 1} = [1.1458 \quad 0.5729 \quad 2.2916 \quad 4.5833 \quad 0.5729 \quad 0.2864 \ldots \\ 1.1458 \quad 2.2916 \quad 1.7187 \quad 0.8593 \quad 3.4375 \quad 6.875]'.$$

Values of the aggregate measure SM_{it} given by (18) in the dynamic approach are shown in Table 3.

Table 3

Values of the aggregate measure SM_{it} given by (18) in the dynamic approach

Object *i*	**Values of SM_{it} given (18)**		
	***t* = 1**	***t* = 2**	***t* = 3**
1	0.872727	1.745455	0.581818
2	1.745455	3.490909	1.163636
3	0.436364	0.872727	0.290909
4	0.218182	0.436364	0.145455

Source: own calculations made using R.

If the values of the aggregate measure SM_{it} for each period (Table 3) are divided by the arithmetic mean, the obtained results of the static approach are shown in Table 4.

Table 4

Values of the aggregate measure in the static approach after normalising values of aggregate measure in the dynamic approach

Object *i*	**Normalised values of SM_{it}**		
	***t* = 1**	***t* = 2**	***t* = 3**
1	1.0667	1.0667	1.0667
2	2.1333	2.1333	2.1333
3	0.5333	0.5333	0.5333
4	0.2667	0.2667	0.2667

Source: own calculations made using R.

After analysing the values of the dynamic aggregate measure SM_{it} given by (18), the following conclusions can be formulated:

a. As seen in Table 3, the values of the aggregate measure represent changes in the level of the variable that occurred in the three periods, i.e. a decline by 50% in period $t=2$ compared to period $t=1$, and an increase by 50% in period $t=3$ compared to period $t=1$.

b. Relations between the objects in the reference periods have not changed (as can be seen in Table 4 – the static approach), but are at a different level (Table 3 – the dynamic approach).

7. AN APPLICATION OF THE DYNAMIC APPROACH TO RELATIVE TAXONOMY IN ASSESSING CHANGES IN SOCIAL COHESION OF POLISH PROVINCES IN THE PERIOD 2010-2018

The assessment of changes in social cohesion of Poland's provinces in the period 2010-2018 is based on the concept of the EU-SPI (Annoni and Dijkstra 2016), where three dimensions of social progress are distinguished: basic human needs, foundations of well-being and opportunities. The article used a substantive selection to establish the list of variables describing social cohesion. Therefore, when selecting the variables, statistical methods were not taken into account. Hence, the assessment of social cohesion in the provinces of Poland was carried out using 23 metric variables[4].

[4] A similar dataset was used in the study described in Dehnel et al. (2019);Walesiak and Dehnel (2020).

1. Basic human needs (eight variables):

x1 – mean gross monthly wage in PLN

x2 – total unemployment rate in %

x3 – mean useful floor area of a dwelling per inhabitant in m^2

x4 – average number of persons per room

x5 – length of the sewerage network in relation to the length of the water supply network in %

x6 – number of doctors and dentists per 10,000 population

x7 – crimes ascertained (criminal offences, against life and health, against property) per 10,000 population

x8 – road accidents per 100,000 population

2.Basis of well-being (nine variables):

x9 – users of water treatment services (% of total population)

x10 – percentage of all dwellings equipped with central heating

x11 – children enrolled in day-care centres per 1,000 children up to the age of 3

x12 – children enrolled in nursery schools per 1,000 children aged 3-5

x13 – students taking obligatory classes of English in primary and intermediate schools (% of all students)

x14 – people participating in cultural (artistic, entertainment, interdisciplinary, sports) events per 1,000 population

x15 – area of public greenspace (parks, residential greenspace) per 10,000 population (in ha)

x16 – length of municipal and district improved hard surface roads per 10,000 population (in km)

x17 – death rate among persons below the age of 60

3.Opportunities (six variables):

x18 – dependency ratio (ratio of the dependent and elderly population per 100 working age population)

x19 – percentage of women in the labour force

x20 – percentage of young adults (up to the age of 25) among registered unemployed

x21 – percentage of long-term unemployed (over 12 months) in the population of registered unemployed

x22 – places in stationary social welfare facilities per 10,000 population

x23 – beneficiaries of social assistance at the place of residence (below the means test threshold) per 1,000 population.

Variables x1, x3, x5, x6, x9-x16, and x22 represent stimulants (where higher values are more preferred), variables x2, x4, x7, x8, x17, x18, x20, x21 and x23 take the form of destimulants (where lower values are more preferred), and x19 is a nominant (with the nominal value of 50%). Statistical data on social cohesion including 23 variables about 16 provinces of Poland in the period 2010-2018 came from the Local Data Bank (BDL) maintained by Statistics Poland. There are missing data (NA) for variable x14 for the period 2010-2012. Destimulants were converted into stimulants using the ratio transformation (13), while the nominant x19 was converted into a stimulant using the ratio transformation (14).

The assessment of social cohesion was conducted using the dynamic approach to relative taxonomy (the positional version), described in Section 5.

Table 5 presents values of the aggregate measure SM_{it} indicating changes in the level of social cohesion in the provinces between 2010 and 2018. The positional version of the aggregate measure SM_{it} was employed, which involves the use of the median given by (19). The smaller the value of SM_{it}, the better the position of object i relative to the other objects over the entire reference period from 2010 to 2018. Hence, in contrast to the static approach, the dynamic approach not only shows the relations between the objects in specific periods, but also the changes regarding the phenomenon of interest that took place between objects over the entire reference period.

Table 5

Values of the aggregate measure given by (19) indicating changes in the level of social cohesion in the provinces of Poland in the period 2010-2018

No.	Province	2010	2011	2012	2013	2014	2015	2016	2017	2018
1	Dolnośląskie	1.0471	1.0188	0.9934	0.9650	0.9480	0.9346	0.9216	0.9051	0.8889
2	Kujawsko-Pomorskie	1.1644	1.1561	1.1239	1.0769	1.0411	0.9954	0.9932	0.9814	0.9204
3	Lubelskie	1.1098	1.0887	1.0880	1.0675	1.0308	1.0446	0.9989	0.9413	0.9281
4	Lubuskie	1.1547	1.0982	1.0512	1.0149	0.9868	0.9905	0.9804	0.9348	0.9136
5	Łódzkie	1.0763	1.0442	1.0511	1.0385	1.0193	0.9916	0.9446	0.9313	0.9184
6	Małopolskie	1.0557	1.0345	1.0218	1.0244	1.0108	0.9726	0.9433	0.9269	0.9207
7	Mazowieckie	1.0389	1.0297	1.0197	1.0070	0.9804	0.9306	0.9190	0.9093	0.9028
8	Opolskie	1.0084	0.9975	0.9829	0.9776	0.9521	0.9350	0.9216	0.8834	0.8768
9	Podkarpackie	1.1327	1.1251	1.0990	1.1154	1.0731	1.0380	1.0022	0.9747	0.9661
10	Podlaskie	1.0808	1.0531	1.0518	1.0474	1.0065	1.0000	0.9727	0.9236	0.9028
11	Pomorskie	1.0853	1.0654	1.0349	1.0278	1.0030	0.9637	0.9356	0.8782	0.8842
12	Śląskie	1.0333	1.0101	1.0025	0.9903	0.9861	0.9722	0.9583	0.9136	0.8798
13	Świętokrzyskie	1.1786	1.1390	1.1118	1.0833	1.0631	1.0226	0.9993	0.9556	0.9796
14	Warmińsko-Mazurskie	1.2145	1.2189	1.1597	1.1216	1.0633	1.0399	1.0328	1.0256	0.9833
15	Wielkopolskie	1.0991	1.0535	1.0415	1.0278	1.0000	0.9804	0.9514	0.9379	0.9059
16	Zachodniopomorskie	1.0840	1.0707	1.0272	1.0147	1.0000	0.9933	0.9707	0.8931	0.8672
Mean		1.0977	1.0752	1.0538	1.0375	1.0103	0.9878	0.9654	0.9322	0.9149
Standard deviation		0.0561	0.0579	0.0484	0.0441	0.0358	0.0353	0.0329	0.0372	0.0342
Median		1.0847	1.0594	1.0463	1.0278	1.0047	0.9911	0.9645	0.9291	0.9098
Absolute median deviation		0.0618	0.0508	0.0506	0.0431	0.0318	0.0343	0.0427	0.0325	0.0290
Kendall's tau correlation (year t + 1 to year t)		–	0.900	0.800	0.845	0.874	0.678	0.812	0.644	0.711

Source: own calculations made using R.

Throughout the reference period 2010-2018, the aggregate measure SM_{it} is close to 1, with specific values ranging from 0.8672 (in 2018 for Zachodniopomorskie) to 1.2145 (in 2010 for Warmińsko-Mazurskie). One can distinguish two subperiods separated by the year 2014 for which the mean value of the aggregate measure for all provinces is $\overline{SM}_{i2014} = 1.0103$, with a slight percent variation $CV_{SM} = 3.5\%$. Before 2014, values of the aggregate measure are greater than 1 (with the exception for some years of three provinces: Dolnośląskie, Opolskie and Śląskie), which means that in this period most provinces were characterised by a lower level of social cohesion than in the later years of the reference period. Starting from 2015, the majority of aggregate measure values were lower than 1, indicating an improvement in the level of social cohesion across the provinces. The highest level of social cohesion in 2010 was observed in Opolskie and Śląskie, and in 2018 these two provinces were overtaken by Zachodniopomorskie. The lowest assessments of social cohesion, both in 2010 and in 2018, were recorded for Warmińsko-Mazurskie and Świętokrzyskie.

Changes in the level of social cohesion for the provinces of Poland in the period 2010--2018 are illustrated by means of line charts (Figure 1) and box plots (Figure 2).

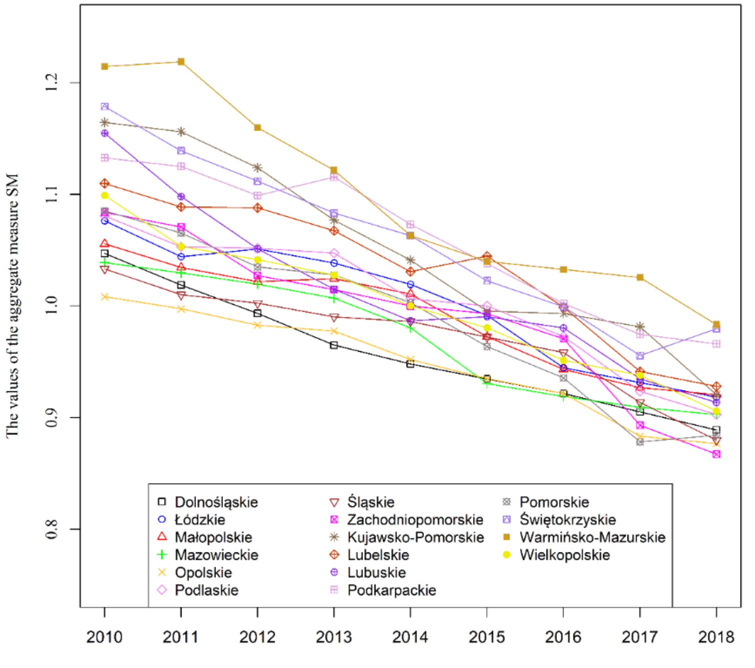

Fig. 1. Changes in the level of social cohesion for 16 provinces of Poland in the period 2010-2018

Source: own calculations and plots made using R.

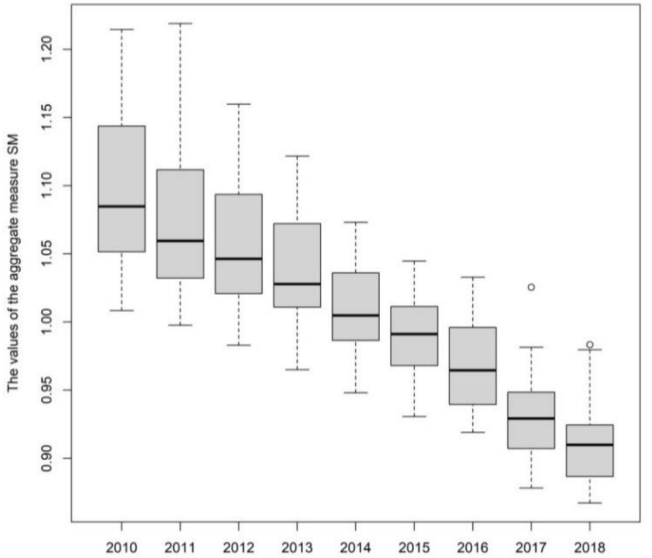

Fig. 2. Box plots visualising distribution of the level of social cohesion (values of the aggregate measure (19)) in the period 2010-2018

Source: own calculations and plots made using R.

It can be noted that over the entire reference the values of the aggregate measure and its variability systematically declined (Table 5, Figures 1 and 2). This represents, first of all, a systematic improvement in the level of social cohesion in Poland. Secondly, there is less variation between the provinces in terms of social cohesion in consecutive years. The rate of this improvement was faster in those province where the level of social cohesion was lower at the start of the reference period (including Kujawsko-Pomorskie, Lubuskie, Warmińsko-Mazurskie and Zachodniopomorskie). In 2018, the range of the aggregate measure $SM_{i2018} \in [0.8672; 0.9833]$ was clearly smaller ($R_{i2018} = 0.1161$) than in 2010 ($R_{i2010} = 0.2061$).

Changes in the mean level (arithmetic mean and median) and variability (standard deviation, absolute median deviation) of social cohesion in the provinces of Poland in 2010-2018 are shown in Figure 3.

Over the course of nine years it was possible to observe some changes in the ranking of provinces in terms of social cohesion. Kendall's rank correlation coefficients represent the degree of similarity between rankings in two consecutive years of the reference period (see the last row of Table 5). Their values ranged from 0.644 to 0.900. The biggest changes in the rankings occurred in 2017 relative to 2016 (Kendall's rank correlation coefficient was equal to 0.644), while the smallest changes were observed for 2011 relative to 2010 (Kendall's rank correlation coefficient equalled 0.900).

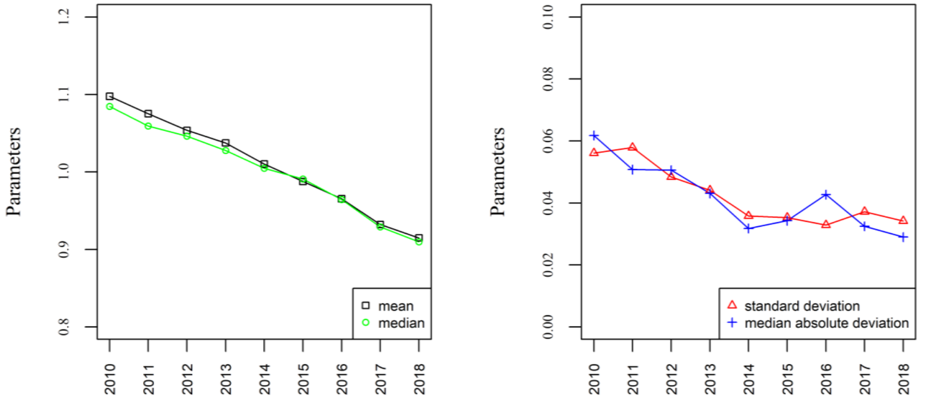

Fig. 3. Changes in the mean level and variability of social cohesion in the provinces of Poland in 2010-2018

Source: own calculations and plots made using R.

The changing assessments of social cohesion in the provinces in subsequent years of the reference period are shown in choropleth maps in Figure 4. The sequential presentation of changes in social cohesion helps to capture the relations between the objects (provinces) in each year and highlights the trends identified earlier. The gradually decreasing colour saturation represents the systematic improvement in social cohesion and the decreasing disparities between the provinces.

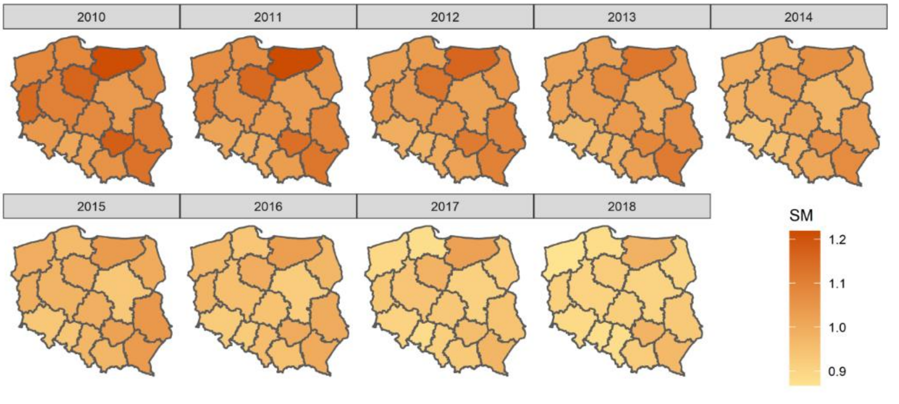

Fig. 4. Changes in the level of social cohesion in the province of Poland in the period 2010-2018

Source: own calculations and plots made using R.

CONCLUDING REMARKS

The assessment of the level of social cohesion over time and space is a complex task which calls for a multivariate and dynamic approach. The novelty of this study consists in formulating and applying a dynamic approach to relative taxonomy to assess changes in social cohesion of the provinces of Poland in the period 2010--2018. In contrast to the static approach, the dynamic variant not only shows the relations between objects in particular periods, but also the changes in the level of the phenomenon over the entire reference period. In other words, the approach is suitable for tracking changes from a cross-sectional and longitudinal perspective, offering the possibility of assessing relations between the objects in each study period. Another important benefit of the proposed method is that it can be applied to datasets containing NA values.

The article also provides a review of the literature on relative taxonomy, including a wide range of concepts for developing methods of linear ordering. This theoretical background serves as the point of departure for the presentation of a fairly new method of relative taxonomy developed in 2013. Finally, the article provides a comparison between the static and dynamic approach to relative taxonomy, illustrated with a simple empirical example.

The results of the study of social cohesion in Poland clearly indicate that between 2010 and 2018 the level of social cohesion in all the provinces systematically improved, although the rate of these changes varied: it was faster in those provinces where the level of social cohesion was lower at the start of the reference period. The changes that occurred over the course of nine years brought about a decrease in the amount of variation between the provinces as regards social cohesion.

The calculations were made using R scripts written by one of the authors (R Core Team 2020).

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