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MODIFICATION OF THE LEE-CARTER MORTALITY MODEL AND ITS APPLICATION IN THE PENSION SCHEME

In this paper, a new approach to mortality forecasting is proposed based on an improved model of the Lee-Carter type. The standard Lee-Carter model and its modified version were introduced and compared using mortality data for Poland and some other European countries. Forecasts of log-central age-specific death rates were then derived and used to predict death probabilities and life expectancies for males and females in Poland, which are the main parameters of the so-called dynamic life tables (also known as mortality tables). The application of the proposed methodology in calculations of the present values of future pension annuities is presented in the article. Scenarios of monthly pensions obtained with the use of dynamic life tables were considered and compared with analogous scenarios based on the static (period) life tables published every year by the Central Statistical Office of Poland.

Keywords: mortality forecasting, Lee-Carter model, dynamic life tables, pension annuities

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1. INTRODUCTION

Population ageing is increasing rapidly in many countries. Based on the forecasts until 2060, it appears that countries such as Poland, Greece, Portugal, Slovakia, Slovenia and Spain will be ageing at the fastest pace while Italy will remain one of the countries with the oldest populations (OECD 2019).

Population ageing indicates that pension expenditures tend to increase and the future pensions tend generally to be lower. According to forecasts from 2019 made for OECD countries, the full-career replacement rate would fall by an average of around 6%among people who retired about 15 years ago and employees just entering the labour market. It was also predicted that in the case of Poland the future replacement rate from mandatory pension schemes would be one of the lowest in the

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OECD countries, i.e. 35% or 27% of previous net earnings for young men and women entering the labour market in 2018.

According to the 2019 forecasts, those entering the labour market in 2018 are expected to spend, on average, 34% of their adult life in retirement compared with 32% for those currently retiring, assuming that adult life expectancy is measured from the age of 20 and subject to surviving until retirement age, and taking into account the increase in the normal retirement age in some countries. In Poland the respective proportions were estimated as 28.6% and 32.9%, respectively (OECD 2019). Based on the old-age to working-age ratio, the Polish population is currently younger than the average in the OECD countries, but the ageing process is proceeding rapidly, thus this ratio is expected to exceed the average.

Unfortunately, the COVID-19 pandemic that broke out in 2020 disrupted labour markets around the world. The lockdowns and workplace closures have reduced individuals' wages and incomes, and resulted in their lower capability to contribute to retirement savings plans. Therefore, the OECD forecasts made in 2019 now seem to need a revision.

The current extraordinary situation requires a continuous adjustment of the pension systems to the demographic and economic changes. In recent decades, many developed countries have started reforms of their pension systems in response to the progressing population ageing, e.g. by expanding pension coverage, enhancing work incentives, extending flexible retirement options, encouraging private pension savings, expanding the coverage of mandatory pensions and establishing a higher retirement age, although the latter solution has been often recognised in some countries as controversial.

Currently, due to political and social pressure, pension reforms have lost their momentum in some countries, which is manifested in the failure to implement some of the measures previously set, e.g. in Slovakia, Italy, the Netherlands (limiting the increase in the retirement age or expanding early-retirement options), Spain (suspending the adjustment of pension benefits with demographic changes), Italy, and Portugal (easing early-retirement conditions). In Poland, the decision to increase the retirement age to 67 for both men and women by 2020 and 2040, respectively, was withdrawn in 2017, returning to the age of 65 for men and 60 for women. Thus, the gender gap in the normal retirement age, i.e. the difference between the retirement age for men and women, has been maintained.

The purpose of the paper was to assess the future pension annuities by modelling the assumed evolution of mortality in Poland with the well-known Lee-Carter model. The author created the so-called dynamic mortality tables used for valuing pensions annuities, which the pension funds are expected to provide. Such mortality tables include both the future changes in mortality and mortality experience relevant to the given population.

2. PENSION PLANS IN POLAND

Pension schemes vary among countries and involve a number of different programmes. The funded defined contribution plans (FDC) are mandatory for future retirees in some OECD countries. In these schemes payments affect each individual account. The accumulation of premiums and returns on investment is usually converted into a monthly retirement upon retirement.

Up to 1998, the pension system in Poland was a fully pay-as-you-go (redistributive) system. The main idea was based on the so-called intergenerational agreement to cover current benefits from current contributions. However, this system became unstable as a result – among others – of the limited economic activity of the population. Retirement privileges, early retirements and a growing unemployment rate resulted in an increase in the number of people at the post-working age in relation to the working age population. In order to guarantee the solvency of the system, the amount of obligatory social security contributions was gradually increased from 15.5% in 1981 to 45% in 1998. However, despite this, the contributions were not sufficient to cover current benefits, because at the same time unfavourable demographic changes accelerated (a decrease in fertility and mortality, an increase in life expectancy).

In 1999, the old system was replaced by a three-pillar pension system to meet challenges of the ageing society. The first pillar was a mandatory system based on a defined contribution system (NDC) managed by the Social Insurance Institution (ZUS). This was a public pay-as-you-go system with individual accounts that applied a hypothetical rate of return to contributions made. Every participant had a virtual account, which contained all contributions made over her/his working life. At retirement, the accumulated notional capital was converted into a monthly pension using a formula based on life expectancy. The second pillar took the form of Open Pension Funds (OPF), a type of the funded defined contribution plans (FDC). Portfolio regulations imposed certain restrictions on investments made by OFEs. The third pillar consisted of voluntary occupational retirement provision accounts (PPE). These were FDC plans with limited tax incentives. If an employer established a PPE, it was obliged to pay contributions for its staff. Employees could make additional contributions that supplemented those of the employer, and could not be withdrawn before reaching retirement age. Compared to the OPFs, the PPEs also had more investment possibilities.

In 2004, the institution of individual retirement accounts (IKE) was introduced, followed by individual retirement security accounts, and by employee capital plans. On January 1, 2013, the gradual increase in the retirement age for men and women to the same level of 67 years was implemented. These provisions were withdrawn in 2017 under social and political pressure.

In 2014, 51.5% of participation units accumulated in OFE were redeemed by the Polish parliament. In addition, the so-called safety slider was implemented, according to which the entire pension benefit is paid by ZUS together with benefits from the pay-as-you-go part, calculating them in the same way as before. In order to eliminate the problem related to the redemption of participation units of an OFE member during a possible downturn in the financial markets, the process of transferring funds was extended over time, i.e. ten years before the OFE member reaches retirement age, the process of monthly redemption of participation units on his/her account begins. Upon reaching retirement age, the OFE member has no units in OFE, but only funds in the sub-account in ZUS. During this period, ZUS also does not transfer contributions to the OFE.

Thus, the pay-as-you-go system is currently dominant, supplemented with funds accumulated in open pension funds. It is planned that in the near future funds from OFEs will be transferred to private individual retirement accounts. The law on this matter is pending approval by the Polish parliament, as its enactment was suspended due to the pandemic.

3. MODIFICATION OF THE LEE-CARTER MORTALITY MODEL

Mortality tables are needed for valuing pension annuities. Such tables can account only for the differences in mortality by age (static tables), or can additionally account for the evolution of mortality over time (dynamic tables).

Dynamic mortality tables are usually based on some assumptions on mortality rates and their predictions in order to sufficiently account for future improvements in mortality. Two components must be assessed in order to develop such tables: the current level of mortality and its expected trend, i.e. the mortality improvement since mortality tends in general to decline. The level of mortality is usually assessed based on e.g. 3 recent years of mortality experience for a specified population. To assess mortality improvement, significantly more data is needed and therefore it is more challenging. The general population mortality data are used as inputs to formal mortality projection models. Once mortality forecasts are made, they can be applied to create a mortality table at any future point in time.

In general, two groups of mortality prediction models are considered in the literature. The first group, which is the widest one, comprises static models, i.e. the log-odds function of the death probability or mortality rates are expressed in analytical forms that could be linear or nonlinear functions of age. The second group includes extrapolative models, where the probability of death or mortality rates are expressed as nonlinear functions of age and calendar time. In both types of models there are additionally some scalar parameters that have to be estimated.

One of the popular extrapolative models was proposed by Lee and Carter (1992). It includes two risk factors, i.e. age and time, and uses matrix decomposition to extract a set of age-related parameters, and time-varying indices which are then used for forecasting. In other words, the log-central age-specific mortality rates $y_{xt} = \ln m_x(t)$ for individuals at age x (age x is rounded to an integer) in year t is estimated by three parameters a_x, b_x, k_t, where a_x can be interpreted as the mean mortality at age x, k_t is the time trend parameter, and b_x modulates the influence of the time trend at the given age x. The Lee-Carter model (LC) has the form:

$$\ln m_x(t) = a_x + b_x k_t + \varepsilon_{xt}, \quad x = 0,1,\ldots,X, \quad t = 1,2,\ldots,T, \tag{1}$$

or equivalently

$$m_x(t) = \exp\{a_x + b_x k_t + \varepsilon_{xt}\}, \quad x = 0,1,\ldots,X, \quad t = 1,2,\ldots,T, \tag{2}$$

where X is an upper age limit, $\{a_x\}$ and $\{b_x\}$ are sets of some constants that are different for different age groups x, and $\{k_t\}$ is a set of time components viewed as a discrete-time stochastic process.

Terms ε_{xt} represent random errors reflecting particular age-specific influences not captured by the model. It is assumed that ε_{xt} are independent random variables, normally distributed with the mean equal 0 and common variance σ^2. In practice, this homoscedasticity assumption is often violated, since the variance of the random term is not evenly distributed among age groups. Some of the differences in the variability can be explained by the cohort effect, however, this effect was not included in the LC model.

Model (1) is fitted to the central age-specific death rates

$$m_x(t) = \frac{D_x(t)}{N_x(t)} 1000, \tag{3}$$

where $D_x(t)$ denotes the number of deaths observed at age x and time t, and $N_x(t)$ is the midyear population at age x in year t.

Model (1) or (2) is undetermined without additional constraints. Let us assume, for instance, that we have an empirical data matrix of logarithms of specific mortality rates, i.e. a matrix with elements $y_{xt} = \ln m_x(t)$ in the body, where $x = 0,1,...,X$ denotes the age group (matrix rows), whereas $t = 1,2,...,T$ are calendar years (matrix columns). Let model (1) be valid for a set of parameters { a_x }, { b_x }, $x = 0,1,...,X$ and { k_t }, $t = 1,2,...,T$. It is easy to verify that for any constant c and the set of parameters { $a_x - cb_x$ }, {b_x}, { $k_t + c$ } or { a_x }, { cb_x }, { k_t / c }, model (1) also holds. Hence, parameters k_t are determined to the transformation $k_t + c$ or k_t / c, parameters b_x are determined to the multiplicative constant, whereas parameters a_x – to the additive constant.

To ensure the unique parameters of model (1), it is necessary to define certain additional constraints. To this end, Lee and Carter assumed that the sum of parameters b_x for all age groups (indexed by ages *x*) equals 1, whereas the sum of parameters k_t (indexed by t) equals 0. Thus, in the standard LC model, parameters b_x and k_t are assumed to satisfy the following constraints:

$$\sum_{x=0}^{X} b_x = 1, \tag{4}$$

and

$$\sum_{t=1}^{T} k_t = 0. \tag{5}$$

In the original methodology, the Singular Value Decomposition (SVD) is used to estimate parameters a_x, b_x, k_t (see e.g. Good (1969) for more details). Therefore, the author's proposal was to impose additional constraints allowing to simplify the estimation of the LC model parameters without using the SVD method. The additional restrictions take the form:

$$\sum_{t=1}^{T} y_{xt} = \sum_{t=1}^{T} \left(a_x + b_x k_t \right), \tag{6}$$

$$\sum_{x=0}^{X} y_{xt} = \sum_{x=0}^{X} \left(a_x + b_x k_t \right), \tag{7}$$

$$\sum_{x=0}^{X} k_t y_{xt} = \sum_{x=0}^{X} k_t \left(a_x + b_x k_t \right). \tag{8}$$

The LC model with its attendant constraints (6) to (8) was called the Modified Lee-Carter Model (MLC). In other words, the MLC model is defined as the origin LC model (1) with constraints (4) and (5), but it is supplemented with additional three constraints (6) to (8). It follows from (6) and (5) that

$$a_x = \frac{1}{T} \sum_{t=1}^{T} \ln m_x(t), \tag{9}$$

from (7) and (4) there is

$$k_t = \sum_{x=0}^{X} \left(y_{xt} - a_x \right), \tag{10}$$

while from (8) and (5) we get

$$b_x = \frac{\sum_{t=1}^{T} k_t y_{xt}}{\sum_{t=1}^{T} k_t^2}. \tag{11}$$

Under these constraints, parameters a_x describe the age pattern of mortality averaged over time, parameters k_t describe the effects of the calendar time t on a change in the mortality, whereas b_x are the regression coefficient representing the mean change of mortality rates y_{xt} in response to unit change of component k_t.

The Lee-Carter methodology and the subsequent modifications were broadly discussed in the literature, e.g. Carter (1996), Lee (2000), Alho (2000), Tuljapurkaret al. (2000), Booth et al. (2002), Brouhns et al. (2002a,b), Renshaw, Haberman (2003a,b,c), Li et al. (2004), Lundström, Qvist (2004), Brouhns et al. (2005), Koissi et al. (2006), Denuit, Dhaene (2007), Rossa (2011), Haberman, Renshaw (2012), Danesi et al. (2015).

4. MORTALITY FORECASTING

Let us note that parameters a_x and b_x in model (1) are constant in time, which means that estimates of the parameters, once derived, can be used in the future. The mortality forecasts can be obtained by modelling k_t as a time series. The forecasts concerning the forecasted values of k_t, together with the estimates of parameters a_x and b_x allow for, based on model (1), forecasting mortality, and more specifically, forecasting the log-central deaths rates $\ln m_x(t)$ for $t > T$.

As proposed by Lee and Carter (1992), finding the values of k_t for $t = 1, 2, ..., T$ provides a starting point for modelling a time series { k_t } as a random walk with drift that can be described using the formula

$$k_t = c + k_{t-1} + \eta_t, \tag{12}$$

where c stands for a constant (a drift), and η_t is an error term with normal distribution with the mean 0 and a finite variance.

The estimator of drift c has the form

$$\hat{c} = \frac{k_T - k_1}{T-1}. \tag{13}$$

Its variance estimator is given by the formula

$$\sigma^2 = \frac{1}{T-1}\sum_{t=2}^{T}\left(k_t - k_{t-1} - c\right)^2. \tag{14}$$

Estimation of constant c allows making forecasts of k_t for $t > T$. Inserted in model (1), together with estimates of parameters a_x and b_x, they allow to make forecasts of future log-central mortality rates, and consequently, of future mortality rates using formula (2), i.e.

$$\tilde{m}_x(t) = \exp\left\{a_x + b_x\tilde{k}_t\right\}, \tag{15}$$

where $\tilde{m}_x(t)$, $t = T+1, T+2, \ldots$ denote the forecasted mortality rates based on forecasted components $\tilde{k}_t$ for $t > T$ derived from the random walk model (12).

Next, it was possible to estimate other parameters of the mortality table, e.g. the probability of death during a year for individuals attaining age x in year t or inversely – the probability that an individual aged x at time t will survive the next year (the survival probability). These two parameters are often denoted as $q_x(t)$, $p_x(t)$, respectively, where $p_x(t) = 1 - q_x(t)$.

Assuming the so-called linear interpolation model (e.g. Rossa (2011), pp. 50–55), the approximate relation between $q_x(t)$ and cohort mortality rate $m_x(t)$ is as follows

$$q_x(t) \approx \frac{2m_x(t)}{2 + m_x(t)}. \tag{16}$$

Another important parameter is the remaining life expectancy for individuals aged x in year t, regarded as the additional number of years on average an individual of a given age x can expect to live; life expectancy is usually denoted as $e_x(t)$.

It is worth noting that there is a significant distinction between period and cohort life expectancy. The period approach does not account for changes in mortality beyond the year under study. For instance, the period life expectancy at age x in is calculated on the basis of the survival probabilities $p_x(t)$ of individuals at ages $x, x+1, x+2, \ldots$ in the same year t. In such an approach, life expectancy can be calculated directly from one period life table without accounting for projections. However, because longevity tends to improve with time, period life expectancies systematically underestimate the actual expected lifetime, whereas cohort life expectancy is calculated taking into account improvements in mortality. For instance, the cohort life expectancy for an individual at age x is calculated based on the survival probability at age x in current year t, at age $x+1$ in year $t+1$, at age

$x+2$ in year $t+2$ etc. However, such calculations require making projections of future probabilities $p_{x+k}(t+k)$ for subsequent years $t+1,\ t+2,\ldots$. in the future. This is possible usually by using a prediction model.

Thus life expectancy can be expressed as

$$e_x(t) = 0.5 + \sum_{k=1}^{\infty} {}_k p_x(t), \tag{17}$$

where

$${}_k p_x(t) = \begin{cases} p_x(t)\, p_{x+1}(t) \ldots p_{x+k}(t), & \text{for period approach,} \\ \\ p_x(t)\, p_{x+1}(t+1) \ldots p_{x+k}(t+k), & \text{for cohort approach.} \end{cases} \tag{18}$$

5. ESTIMATION OF THE MLC AND LC MODELS FOR POLAND

One can estimate parameters a_x, b_x, k_t of the LC model (1) with constraints (4) and (5) using the SVD method, as well as the analogous parameters of the MLC. Using the extended set of constraints (4) to (8) for this model, the estimates of parameters a_x, b_x, k_t can be derived directly from formulas (9) to (11).

In this section, the estimation of the MLC and LC models was based on the historical mortality data for the period 1965–2020, taken from the database of the Polish Central Statistical Office (stat.gov.pl).

The input data are annual probabilities of death transformed to mortality rates based on the approximate relation (16) between $q_x(t)$ and $m_x(t)$. Furthermore, it was assumed that mortality rates are additionally multiplied by 1000, i.e.

$$m_x(t) \approx \frac{2q_x(t)}{2-q_x(t)} 1000, \tag{19}$$

where $q_x(t)$ is the probability that an individual aged x will die within one year.

Estimates of parameters a_x for ages $x = 0,1,\ldots,100$ obtained by means of the LC and MLC models were the same, and are presented in Figure 1. Estimates of b_x in both approaches differ very slightly, as illustrated in Figures 2 and 3. More pronounced, but still small differences, can be seen when comparing estimates of parameters k_t in both models (Figures 4 and 5). The forecasts of k_t up to 2040 from the random walk model (12) are also added in Figures 4 and 5.

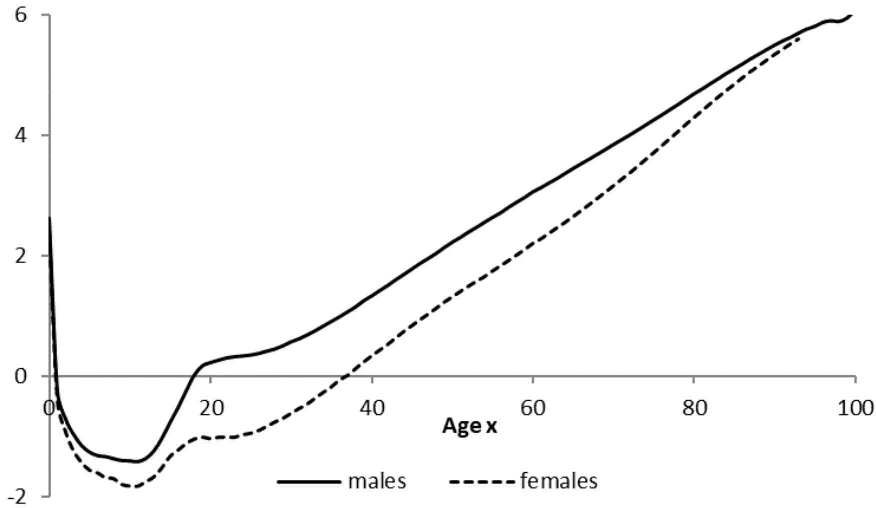

Fig. 1. Estimates of a_x for $x \in [0,100]$ from both the MLC and LC models – fitting based on the period life tables for 1965–2020.

Source: elaborated by the author.

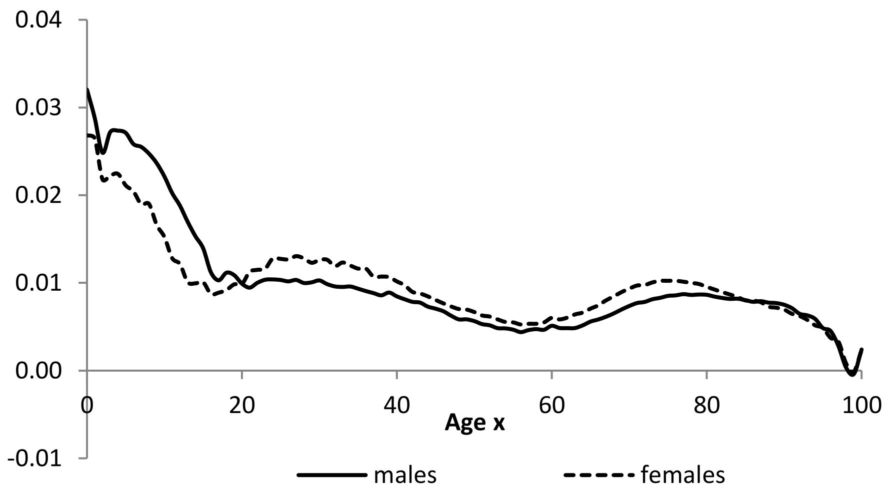

Fig. 2. Estimates of b_x for $x \in [0,100]$ from the MLC model – fitting based on the period life tables for the period 1965–2020.

Source: elaborated by the author.

Fig. 3. Estimates of b_x for $x \in [0,100]$ from the LC model – fitting based on the period life tables for the period 1965–2020.

Source: elaborated by the author.

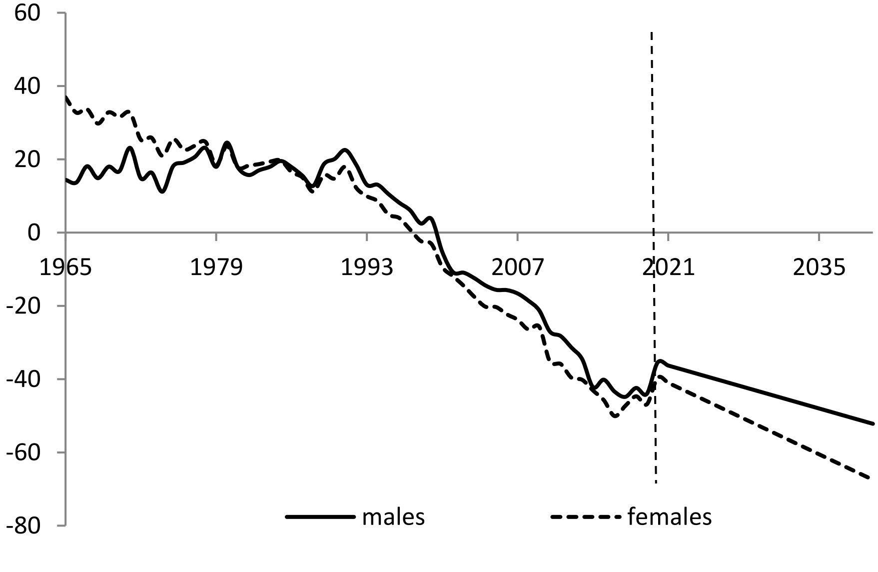

Fig. 4. Estimates of k_t from the MLC model for the period 1965–2020 and their 2021–2040 forecasts – fitting based on the period life tables

Source: elaborated by the author.

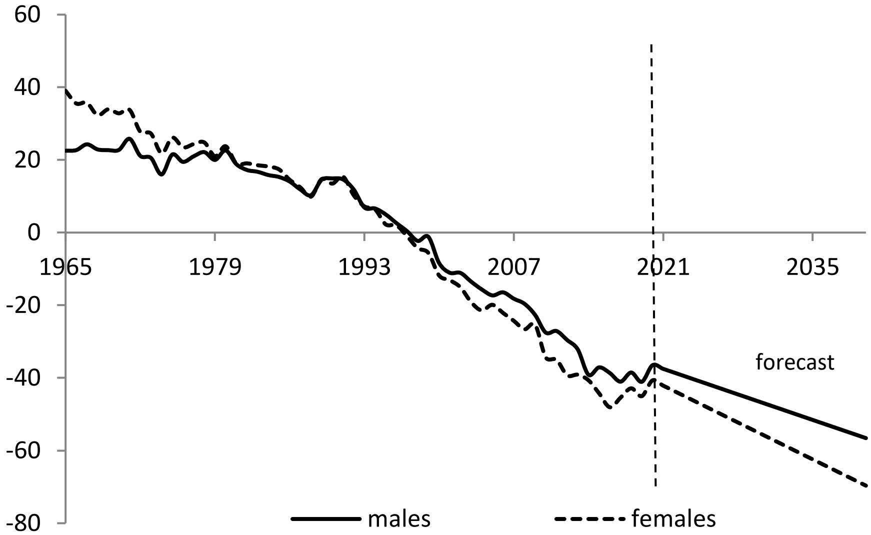

Fig. 5. Estimates of k_t from the LC model for the period 1965–2020 and their 2021–2040 forecasts – fitting based on the period life tables

Source: elaborated by the author.

The results illustrated in Figures 1 to 6 show that the MLC model provides similar estimates of the model parameters as the LC model. The advantage is that the estimation is much simpler, i.e. it does not require employing sophisticated methods such as the singular value decomposition technique. This approach also does not require making assumptions about the distribution of the number of deaths. Such an assumption is needed, for example, in the maximum likelihood estimation.

The curves in Figure 1 show profiles of mortality for males and females, calculated in both models as averages of log-central mortality rates means, therefore they are of the same shapes both in the MLC and LC models. The mortality profiles for males and females in Figure 1 show typical "bath tubs", i.e. with high values around the infant ages, followed by minimal rates at childhood, higher accidental mortality at young adulthood, and increasing mortality at adulthood and old age with an almost constant rate of increase. The "accident hump" at adolescence stands for higher mortality rates due to accidental deaths caused by augmented risk-taking behaviour, as well as increased suicide rates; note that the more demonstrable hump refers to males. As mentioned above, estimates of parameters a_x are identical with both the LC and MLC models. In both cases a_x represent the average values of log-central death rates for ages x.

The arrangements of estimated parameters b_x obtained from the MLC and LC models (Figures 2 and 3) are also very similar. On the basis of both figures, it can be seen that that the rates for young and older males are more sensitive to temporal changes in mortality than analogous rates for females. The reverse relation applies to the remaining age groups.

The curves illustrating time series of k_t in Figures 4 (model MLC) and 5 (model LC), are generally declining both for males and females, with the decline being faster in case of the subpopulation of women, except for the years 1989-1990 (the economic transformation and the health crisis in Poland) and 2020 (the first year of the COVID-19 pandemic), when significant upward shifts of both curves can be observed. Note that estimates of k_t obtained with the use of the MLC model are slightly different from the results obtained with the standard singular value decomposition (SVD) used in the LC method. Estimates of age-related parameters a_x, b_x and the forecasts of time-related parameters k_t were used next to forecast mortality rates for $t > T$. The forecasts are based on formula (15).

6. CALCULATIONS OF OLD-AGE PENSION BENEFITS

According to the current regulations in Poland (Act of 17 December 1998 on pensions and disability pensions from the Social Insurance Fund – Journal of Laws of 2017, item 1383 as amended), the old-age pension benefit is the equivalent of the amount resulting from dividing the pension capital by the remaining lifetime expectancy. Thus, pension benefit K was calculated according to the simple formula

$$K = C / e_x(t), \tag{20}$$

where C is the amount of the indexed pension capital, and $e_x(t)$ is the remaining expected lifetime at age x at which the person intends to retire.

Formula (20) is used by the Social Insurance Institution (ZUS) to determine the amount of the pension benefits in Poland. It shows that benefit B is proportional to the amount of accumulated capital C, and inversely proportional to expected remaining lifetime $e_x(t)$ at retirement age x. The pension benefit grows with the amount of pension contributions, so the longer the person works, the greater his/her benefit. On the other hand, the higher the age of retirement, the shorter the remaining expected lifetime and thus, again the higher his/her benefit.

The amount of pension capital C consists of:

- the pension contributions recorded on the individual’s account indexed by the end of the month preceding the month from which the pension is due,
- the indexed initial capital calculated for the period before 1999,
- in the case of individuals who joined open pension funds, the amount of funds recorded on their sub-accounts.

The remaining expected lifetimes $e_x(t)$ are also called lifetime expectancies. They are published annually by the Central Statistical Office (GUS) in a table which is the basis for calculating pension benefits for applications submitted from April 1 to March 31 of the following calendar year. Lifetime expectancies are determined jointly for men and women on the basis of the mortality data recorded in the previous year. The joint determination of these parameters for both sexes means that when determining the amount of benefits, some funds are shifted from people living on average shorter to those living on average longer. This is called intra-generation solidarity.

Table 1 reveals the life expectancies for both sexes, derived by means of the same method that is used by the Central Statistical Office (see Trwanie życia, 2021). The body of the table demonstrates the lifetime expectancy in months for a person retiring at a certain age. The age at which a person intends to retire is expressed in completed years and months, specified in the first column of the table (in completed years, i.e. 30, 31, ..., 90) and in the head of the table (in completed months: 0, 1, ..., 11).

The values included in the body of Table 1 can be interpreted as expected remaining lifetimes of a hypothetical generation of people which at birth was 100 000 individuals, assuming that during the life of this hypothetical generation, the risk of death in each age group would be the same as in the given calendar year (in this study – 2020).

Table 1

Lifetime expectancies $e_x(t)$ in Poland derived from the 2020 mortality data (both sexes)

Age	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
30	568.1	567.2	566.2	565.2	564.3	563.3	562.4	561.4	560.5	559.5	558.5	557.6
31	556.6	555.7	554.7	553.7	552.8	551.8	550.9	549.9	549.0	548.0	547.0	546.1
32	545.1	544.2	543.2	542.3	541.3	540.3	539.4	538.4	537.5	536.5	535.6	534.6
33	533.7	532.7	531.8	530.8	529.9	528.9	527.9	527.0	526.0	525.1	524.1	523.2
34	522.2	521.3	520.3	519.4	518.4	517.5	516.5	515.6	514.6	513.7	512.7	511.8
35	510.8	509.9	508.9	508.0	507.0	506.1	505.2	504.2	503.3	502.3	501.4	500.4
36	499.5	498.5	497.6	496.6	495.7	494.8	493.8	492.9	491.9	491.0	490.0	489.1
37	488.1	487.2	486.3	485.3	484.4	483.4	482.5	481.6	480.6	479.7	478.7	477.8
38	476.9	475.9	475.0	474.0	473.1	472.2	471.2	470.3	469.4	468.4	467.5	466.5
39	465.6	464.7	463.7	462.8	461.9	460.9	460.0	459.1	458.1	457.2	456.3	455.3
40	454.4	453.5	452.5	451.6	450.7	449.7	448.8	447.9	446.9	446.0	445.1	444.2
41	443.2	442.3	441.4	440.4	439.5	438.6	437.7	436.7	435.8	434.9	434.0	433.0
42	432.1	431.2	430.3	429.3	428.4	427.5	426.6	425.7	424.7	423.8	422.9	422.0
43	421.0	420.1	419.2	418.3	417.4	416.5	415.5	414.6	413.7	412.8	411.9	411.0
44	410.0	409.1	408.2	407.3	406.4	405.5	404.6	403.7	402.7	401.8	400.9	400.0
45	399.1	398.2	397.3	396.4	395.5	394.6	393.7	392.8	391.9	391.0	390.1	389.2
46	388.2	387.4	386.5	385.6	384.7	383.8	382.9	382.0	381.1	380.2	379.3	378.4
47	377.5	376.6	375.7	374.8	373.9	373.0	372.1	371.3	370.4	369.5	368.6	367.7
48	366.8	365.9	365.0	364.2	363.3	362.4	361.5	360.6	359.8	358.9	358.0	357.1
49	356.2	355.3	354.5	353.6	352.7	351.9	351.0	350.1	349.2	348.4	347.5	346.6
50	345.7	344.9	344.0	343.2	342.3	341.4	340.6	339.7	338.8	338.0	337.1	336.2

Age	0	1	2	3	4	5	6	7	8	9	10	11
51	335.4	334.5	333.7	332.8	332.0	331.1	330.3	329.4	328.6	327.7	326.9	326.0
52	325.1	324.3	323.5	322.6	321.8	320.9	320.1	319.2	318.4	317.6	316.7	315.9
53	315.0	314.2	313.3	312.5	311.7	310.8	310.0	309.2	308.3	307.5	306.7	305.8
54	305.0	304.2	303.4	302.5	301.7	300.9	300.1	299.2	298.4	297.6	296.8	295.9
55	295.1	294.3	293.5	292.7	291.9	291.0	290.2	289.4	288.6	287.8	287.0	286.2
56	285.3	284.5	283.7	282.9	282.1	281.3	280.5	279.7	278.9	278.1	277.3	276.5
57	275.7	274.9	274.1	273.3	272.6	271.8	271.0	270.2	269.4	268.6	267.8	267.0
58	266.2	265.4	264.7	263.9	263.1	262.3	261.6	260.8	260.0	259.2	258.4	257.7
59	256.9	256.1	255.4	254.6	253.8	253.1	252.3	251.5	250.8	250.0	249.2	248.5
60	247.7	247.0	246.2	245.5	244.7	244.0	243.2	242.5	241.7	240.9	240.2	239.4
61	238.7	238.0	237.2	236.5	235.7	235.0	234.3	233.5	232.8	232.1	231.3	230.6
62	229.8	229.1	228.4	227.7	227.0	226.2	225.5	224.8	224.1	223.3	222.6	221.9
63	221.2	220.5	219.7	219.0	218.3	217.6	216.9	216.2	215.5	214.8	214.1	213.4
64	212.6	211.9	211.3	210.6	209.9	209.2	208.5	207.8	207.1	206.4	205.7	205.0
65	204.3	203.6	202.9	202.2	201.6	200.9	200.2	199.5	198.8	198.1	197.5	196.8
66	196.1	195.4	194.7	194.1	193.4	192.7	192.1	191.4	190.7	190.1	189.4	188.7
67	188.0	187.4	186.7	186.1	185.4	184.8	184.1	183.4	182.8	182.1	181.5	180.8
68	180.1	179.5	178.9	178.2	177.6	176.9	176.3	175.6	175.0	174.3	173.7	173.0
69	172.4	171.8	171.1	170.5	169.9	169.2	168.6	168.0	167.3	166.7	166.1	165.4
70	164.8	164.2	163.5	162.9	162.3	161.7	161.0	160.4	159.8	159.2	158.5	157.9
71	157.3	156.7	156.1	155.5	154.9	154.2	153.6	153.0	152.4	151.8	151.2	150.6
72	149.9	149.3	148.7	148.1	147.5	146.9	146.3	145.7	145.1	144.5	143.9	143.3
73	142.7	142.1	141.6	141.0	140.4	139.8	139.2	138.6	138.0	137.4	136.8	136.2
74	135.6	135.1	134.5	133.9	133.3	132.8	132.2	131.6	131.0	130.4	129.9	129.3
75	128.7	128.1	127.6	127.0	126.5	125.9	125.3	124.8	124.2	123.6	123.1	122.5
76	121.9	121.4	120.8	120.3	119.7	119.2	118.6	118.1	117.5	116.9	116.4	115.8
77	115.3	114.7	114.2	113.7	113.1	112.6	112.0	111.5	111.0	110.4	109.9	109.3
78	108.8	108.3	107.8	107.2	106.7	106.2	105.7	105.1	104.6	104.1	103.6	103.0
79	102.5	102.0	101.5	101.0	100.5	100.0	99.4	98.9	98.4	97.9	97.4	96.9
80	96.4	95.9	95.4	94.9	94.4	93.9	93.4	92.9	92.5	92.0	91.5	91.0
81	90.5	90.0	89.5	89.1	88.6	88.1	87.6	87.2	86.7	86.2	85.7	85.3
82	84.8	84.4	83.9	83.4	83.0	82.5	82.1	81.6	81.2	80.7	80.3	79.8
83	79.4	79.0	78.5	78.1	77.7	77.2	76.8	76.4	75.9	75.5	75.1	74.7
84	74.2	73.8	73.4	73.0	72.6	72.2	71.8	71.4	71.0	70.6	70.2	69.8
85	69.4	69.0	68.6	68.2	67.8	67.5	67.1	66.7	66.3	66.0	65.6	65.2
86	64.8	64.5	64.1	63.7	63.4	63.0	62.7	62.3	62.0	61.6	61.3	60.9
87	60.6	60.3	59.9	59.6	59.2	58.9	58.6	58.2	57.9	57.6	57.3	57.0
88	56.7	56.3	56.0	55.7	55.4	55.0	54.7	54.4	54.1	53.8	53.5	53.3
89	53.0	52.7	52.4	52.1	51.8	51.5	51.2	50.9	50.6	50.3	50.1	49.8
90	49.6	49.3	49.0	48.7	48.4	48.1	47.9	47.6	47.3	47.1	46.9	46.6

Source: elaborated by the author based on the 2020 life tables published by GUS (stat.gov.pl).

However, in the context of the pandemic that started in 2020 and continued in 2021, the basic assumption (namely that the mortality pattern will remain the same in the future at the level observed in 2020) seems to be unrealistic. In this case, it would be rather reasonable to assume that during the pandemic period, the age-specific mortality rates remain increased, and then decline (at some specific pace) to a level that is consistent with the overall trend observed in the pre-pandemic years. This approach requires employing a mortality model that can forecast the post-pandemic mortality rates. In this study, the MLC model was used.

Table 2

Lifetime expectancies *ex*(*t*) (both sexes) derived via the MLC model – the adjusted dynamic approach

Age	0	1	2	3	4	5	6	7	8	9	10	11
65	231.0	230.4	229.7	229.1	228.4	227.8	227.1	226.5	225.8	225.2	224.5	223.9
66	223.2	222.6	221.9	221.3	220.6	220.0	219.4	218.7	218.1	217.4	216.8	216.1
67	215.5	214.8	214.2	213.6	212.9	212.3	211.6	211.0	210.3	209.7	209.1	208.4
68	207.8	207.1	206.5	205.8	205.2	204.6	203.9	203.3	202.6	202.0	201.3	200.7
69	200.0	199.4	198.8	198.1	197.5	196.8	196.2	195.5	194.9	194.2	193.6	192.9
70	192.3	191.7	191.0	190.4	189.7	189.1	188.4	187.8	187.1	186.5	185.8	185.2
71	184.5	183.9	183.3	182.6	182.0	181.3	180.7	180.0	179.4	178.7	178.1	177.4
72	176.8	176.2	175.5	174.9	174.3	173.6	173.0	172.3	171.7	171.0	170.4	169.8
73	169.1	168.5	167.9	167.2	166.6	166.0	165.3	164.7	164.1	163.4	162.8	162.2
74	161.5	160.9	160.3	159.7	159.0	158.4	157.8	157.2	156.5	155.9	155.3	154.7
75	154.0	153.4	152.8	152.2	151.6	151.0	150.3	149.7	149.1	148.5	147.9	147.3
76	146.6	146.0	145.4	144.8	144.2	143.6	143.0	142.4	141.8	141.2	140.6	140.0
77	139.4	138.8	138.2	137.6	137.0	136.4	135.9	135.3	134.7	134.1	133.5	132.9
78	132.3	131.7	131.2	130.6	130.0	129.4	128.8	128.3	127.7	127.1	126.5	126.0
79	125.4	124.8	124.3	123.7	123.1	122.6	122.0	121.4	120.9	120.3	119.8	119.2
80	118.6	118.1	117.5	117.0	116.5	115.9	115.4	114.8	114.3	113.7	113.2	112.6
81	112.1	111.6	111.0	110.5	110.0	109.5	108.9	108.4	107.9	107.4	106.8	106.3
82	105.8	105.3	104.8	104.2	103.7	103.2	102.7	102.2	101.7	101.2	100.7	100.2
83	99.7	99.2	98.7	98.2	97.7	97.2	96.7	96.2	95.7	95.3	94.8	94.3
84	93.8	93.3	92.8	92.4	91.9	91.4	90.9	90.5	90.0	89.5	89.1	88.6
85	88.1	87.7	87.2	86.7	86.3	85.8	85.4	84.9	84.5	84.0	83.6	83.1
86	82.7	82.2	81.8	81.3	80.9	80.4	80.0	79.6	79.1	78.7	78.3	77.8
87	77.4	76.9	76.5	76.1	75.7	75.2	74.8	74.4	74.0	73.5	73.1	72.7
88	72.3	71.9	71.5	71.0	70.6	70.2	69.8	69.4	69.0	68.6	68.2	67.8
89	67.4	67.0	66.6	66.2	65.8	65.4	65.0	64.6	64.2	63.8	63.4	63.0
90	62.6	62.2	61.8	61.4	61.1	60.7	60.3	59.9	59.5	59.1	58.8	58.4
91	58.0	57.7	57.3	56.9	56.5	56.1	55.8	55.4	55.0	54.7	54.3	54.0
92	53.6	53.3	52.9	52.5	52.2	51.8	51.4	51.1	50.7	50.4	50.1	49.7
93	49.4	49.0	48.7	48.3	48.0	47.6	47.3	47.0	46.6	46.3	46.0	45.7
94	45.4	45.0	44.7	44.3	44.0	43.7	43.3	43.0	42.7	42.4	42.1	41.8
95	41.5	41.2	40.9	40.6	40.2	39.9	39.6	39.3	39.1	38.8	38.5	38.2
96	38.0	37.7	37.4	37.1	36.8	36.5	36.2	36.0	35.7	35.5	35.2	35.0
97	34.8	34.5	34.2	33.9	33.6	33.4	33.1	32.9	32.7	32.5	32.3	32.1
98	31.9	31.6	31.4	31.1	30.9	30.6	30.4	30.2	30.0	29.8	29.7	29.5
99	29.4	29.2	28.9	28.7	28.4	28.2	28.0	27.8	27.7	27.5	27.4	27.3
100	27.3	27.0	26.8	26.5	26.3	26.1	25.9	25.8	25.6	25.5	25.4	25.4

Source: elaborated by the author.

Table 2 shows lifetime expectancies e_x for $x=65,66,67,...,100$ determined under some revised assumptions. In particular, the increased age-specific mortality rates in the 2020–2022 period were set at the level observed in 2020, and for the post-pandemic years forecasted mortality rates were adopted, based on the overall mortality trend extrapolated from the pre-pandemic 1965–2019 period using the MLC model. Such an approach was called the adjusted dynamic approach as opposed to the static approach used by the Central Statistical Office. It can be noted that the life expectancies estimated in this way are significantly higher than the life expectancies for the relevant ages obtained using the static approach.

7. MONTHLY LIFE ANNUITIES

The author considered here the amount of a retirement benefit to be paid out by a private pension provider to a person aged *x* years in the form of a life annuity. To determine the amount of monthly payments, the study used an actuarial formula employed to calculate the present value of the life annuity payable at the beginning of each month (see e.g. Skałba (2002)).

Hence, assuming that life annuity is paid out m times within a year (for the monthly payments $m=12$), at the beginning of each subperiod the length of which is given by $1/m$ of a year, with the instalment amount being $1/m$ PLN, so that the total annual amount due will be 1 PLN. This is the so-called normalised case, also assuming that the last payment is effected at the beginning of the subperiod in which the pensioner dies.

Let $T(x)$ be a non-negative random variable representing the remaining lifetime of a person aged x, while $T^*(x)=\lfloor T(x) \rfloor$ is the integer part of $T(x)$, while $S^{(m)}$ denote the rounded-up part of the recipient's last year of life with an accuracy of $1/m$ subperiod.

Note that $S^{(m)}$ is a discrete random variable taking values from the set

$$\left\{\frac{1}{m},\frac{2}{m},\ldots,\frac{m-1}{m},1\right\}. \tag{21}$$

The present value $Y^{(m)}$ of the payments is a random variable, which can be defined with the following formula:

$$Y^{(m)}=\frac{1}{m}\left(1+\upsilon^{\frac{1}{m}}+\upsilon^{\frac{2}{m}}+\ldots+\upsilon^{T^*(x)+S^{(m)}-\frac{1}{m}}\right), \tag{22}$$

where $\upsilon=1/(1+i)$ is the discount rate and i is an average rate of return calculated for several periods.

The formula for the sum of a geometric series is the following

$$aq^0 + aq^1 + aq^2 + \ldots + aq^{m-1} = a\frac{1-q^m}{1-q}. \tag{23}$$

The variable $Y^{(m)}$ can be expressed as

$$Y^{(m)} = \frac{1}{m}\left[\left(\upsilon^{\frac{1}{m}}\right)^0 + \left(\upsilon^{\frac{1}{m}}\right)^1 + \left(\upsilon^{\frac{1}{m}}\right)^2 + \ldots + \left(\upsilon^{\frac{1}{m}}\right)^{mT^*(x)+mS^{(m)}-1}\right]. \tag{24}$$

Thus, based on (24)

$$Y^{(m)} = \frac{1}{m}\frac{1-\left(\upsilon^{\frac{1}{m}}\right)^{mT^*(x)+mS^{(m)}}}{1-\upsilon^{\frac{1}{m}}} = \frac{1}{m}\frac{1-\upsilon^{T^*(x)+S^{(m)}}}{1-\upsilon^{\frac{1}{m}}}. \tag{25}$$

Then the author calculated the expected value of $Y^{(m)}$ denoted as $\ddot{a}_x^{(m)}$. Assuming that $T^*(x)$ and $S^{(m)}$ are independent,

$$\ddot{a}_x^{(m)} = \mathbf{E}\left[Y^{(m)}\right] = \frac{1}{m}\frac{1}{1-\upsilon^{\frac{1}{m}}}\left(1-\mathbf{E}\left(\upsilon^{T^*(x)+S^{(m)}}\right)\right) = \frac{1}{m}\frac{1}{1-\upsilon^{\frac{1}{m}}}\left(1-\mathbf{E}\left(\upsilon^{T^*(x)+1}\right)\mathbf{E}\left(\upsilon^{S^{(m)}-1}\right)\right). \tag{26}$$

The expected value $\mathbf{E}\left(\upsilon^{T^*(x)+1}\right)$ represents the present value of a benefit (equal 1 PLN), payable at the end of year of death. In other words, it is an actuarial value of infinite life insurance on being 1 PLN, payable at the end of the year of death. In the actuarial notation, it is denoted by A_x. This value is equal to

$$A_x = \mathbf{E}\left(\upsilon^{T^*(x)+1}\right) = \sum_{k=0}^{\infty}\upsilon^{k+1}\,{}_k p_x q_{x+k}, \tag{27}$$

where probabilities

$${}_k p_x = P\left(T(x) > k\right),\ \ q_{x+k} = P\left(T(x+k) \le 1\right). \tag{28}$$

denote, respectively, the probability of surviving k consecutive years giving survival to x, and the probability of dying within a year giving survival to $x+k$.

The second expected value, $\mathbf{E}\left(\upsilon^{S^{(m)}-1}\right)$, can be determined assuming that variable $S^{(m)}$ takes values from the set (21) with identical probabilities, equal 1/*m*. Then

$$\mathbf{E}\left(\upsilon^{S^{(m)}-1}\right)=\sum_{k=1}^{m}\upsilon^{\frac{k}{m}-1}\frac{1}{m}=\frac{1}{m}\frac{1}{\upsilon}\upsilon^{\frac{1}{m}}\sum_{k=0}^{m-1}\left(\upsilon^{\frac{1}{m}}\right)^{k}=\frac{1}{m}\frac{1}{\upsilon}\upsilon^{\frac{1}{m}}\frac{1-\upsilon}{1-\upsilon^{\frac{1}{m}}}, \tag{29}$$

or after straightforward transformation

$$\mathbf{E}\left(\upsilon^{S^{(m)}-1}\right)=\frac{1}{m}\frac{1-\upsilon}{\upsilon}\frac{\upsilon^{\frac{1}{m}}}{1-\upsilon^{\frac{1}{m}}}. \tag{30}$$

Formulae (27) and (30), together with (26) and probabilities (28), allow for calculating the expected value $\ddot{a}_x^{(m)}$ (also $\ddot{a}_x^{(12)}$).

Let K denote the amount of funds accumulated at the OPF by a person retiring at age x, B denote the monthly pension annuity (benefit) that a pensioner receives from the annuity provider. Assume that pension amount B is derived from the following equation, related to the present value of the life annuity paid monthly in advance (Szumlicz (2007))

$$K=12B(1-\gamma)\ddot{a}_x^{(12)}, \tag{31}$$

where γ is a share of charges for the annuity provider.

8. SCENARIOS OF PENSION BENEFIT CALCULATIONS

To compare the amounts of monthly benefits of a retiring person, two different formulas (20) and (31) were considered. In order to derive benefit amount K using life annuity formula (31), it was necessary to find probabilities (28), next the actuarial value of life annuity $\ddot{a}_x^{(12)}$ defined in (26), and finally benefit K given in (31). In this study, the revised assumptions were adopted to compute probabilities (28) (see the adjusted dynamic approach described in the previous section). The results obtained (option I) were then compared with those obtained via formula (20).

Assume the following input data:

- the retirement age in completed years $x = 65.0$ years;
- the calendar year at retirement – 2020;
- the pension capital $C = 500{,}000$ or $600{,}000$ or $700{,}000$ PLN;
- the share of charge $= 3\%$;
- the rate of return $i = 2\%$ or 7%.

The benefits derived from formula (20) were obtained in two ways, i.e. by assuming the static approach based on the period life tables (option II), or the dynamic approach using the MLC model (option III). The results are shown in Table 3.

Table 3

Values of pension benefits (in PLN, both sexes) – options I, II, III

Option no.	$C = 500{,}000$		$C = 600{,}000$		$C = 700{,}000$	
	$i = 2\%$	$i = 7\%$	$i = 2\%$	$i = 7\%$	$i = 2\%$	$i = 7\%$
I	1096	2940	1315	3528	1535	4117
II	2448		2937		3427	
III	2164		2597		3030	

Source: elaborated by the author.

CONCLUSIONS

The obtained results reveal substantial variations in the pension benefits calculated using different approaches, and indicate that lower values of monthly payments are provided through using the dynamic approach in the calculations (option III), instead of the static one (option II).When using the life annuity approach (option I), the results clearly differ depending mainly on the value of the rate of return In the case of a low rate of return, the resulting annuities are very low.

The illustrative results presented in this study show that calculating the benefits based on the period life tables (option II) may expose the pension provider to a risk of the considerable overestimation of payments, and thus may cause difficulties with covering future liabilities. From this point of view, option III is recommended.

It is also worth noting that even though gender is a distinct determinant of different benefits, in practice the pension benefits are calculated using the common life tables for both sexes. This means that the pension benefits are overestimated for women and underestimated for males.

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