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## USING OPTIMIZATION MODELS TO DETERMINE THE INVENTORY LEVEL IN THE PRODUCTION PROCESS

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**Abstract:** Growing requirements from customers concerning not only product quality but also time handling combined with pressure, cause managers to increasingly seek solutions that will allow them to optimize drawn up production plans. Logistic support to the production process ceases therefore to be only connected with coordinating various activities and to guarantee proper movement, but is extended by the elements optimizing the production level of inventories in the course of production. This article presents an example of a company that has faced such a challenge. A comparative study has been carried out of the various types of planning, which has led to attempting to use an optimization model to minimize the inventory and the production costs for one of the departments that has been studied.

**Key words:** production planning, bottom-up planning, push production system.

### 1. Introduction

The progressing increase of the consumer role in production processes causes the change of the rule used so far: “manufacture the product and find its consumer” into: “find the consumer and manufacture on their wish” [Skowronek, Saryusz-Wolski 2008]. You can observe then that some companies change their production strategies switching from the *push* production system (production to warehouse) to *pull* (i.e. production on order). The *pull* strategy is based on “sucking” system whose main idea is the delaying process. This involves delaying “changes in the product form or identity until the last minute” [Hopp, Spearman 2001; Schary, Skjott-Larsen 2002]. This way the manufacturer avoids too much inventory and adjusts the product to the customer’s actual requirements. Such procedures allow you to limit the weaknesses of the traditional production process in the *push* system.

The *pull* strategy is not however a universal solution for all difficulties related to production planning and is not dedicated to all cases. It is true that the pull system

strives to eliminate waste in order to achieve the same objectives while push systems attempt to determine an optimal number of buffers in order to prevent flow line breakdowns [Razim, Rahnejat, Khan 1998]. Due to that fact in recent years many authors have paid particular attention to the application of hybrid *push/pull* systems. G.J. Bose, A. Rao [1988], G.J. Miltenburg [1990] and C.A. Ptak [1991] agree that each approach has advantages and disadvantages. Their combination then will allow a strategy of reciprocal completion and adjusted to conditions in which companies operate. J. Ming-wei and L. Shi-lian [1992] underline that in practical situations there is always a need of hybrid solutions. Because of that, production delaying in companies can have various forms (based on Schary, Skjott-Larsen, 2002).

- Delaying can apply to designing and manufacturing products. The project preparation and the production process are initiated when the order is taken by the client.
- In the case of the ordered product development based on the catalogue offer the moment of the start of full production is delayed. In this case the client takes an order for the product in the catalogue that is not made from standardized components, subassemblies, semi-finished products.
- Delaying can only apply to the final assembly time. This happens when they manufacture products on order-product customization. The finished product is made from standardized components, subassemblies, semi-finished products but its assembly follows the client's individual order. In this case products are made in a transition form (e.g. modules), and delaying applies only to the final assembly of the product.

Companies, looking for optimal solutions, should consider meeting the material as well as economic goals that production management faces [Nowosielski 2001]. When making a decision on which moment to choose for delaying, managers should be aware that production delaying allows higher flexibility when adjusting to market requirements and greater individualization of the finished product. It requires, however, a longer period of product preparation. The pressure from competition and growing clients' expectations require product delivery quickly and on time. At the same time the company tries to maximize its profit. Product individualization can increase demand for it, having a positive effect on the profit. But the "on order" production applies to individual production which is connected with high manufacturing costs.

Regarding the company being the subject of the research, the authors paid attention to the production-related hybrid system in particular. The system is related to "on order" production but follows the catalogue offer. If the company manufactures products based on the catalogue offer, and made from standardized components – it is essential to only delay the final moment of the finished product assembly. Thanks to that it is possible to shorten the client's order procedures, maintaining product customization at the same time. Simultaneous production of subassemblies and necessary components in the *push* system allows to estimate the volume of the production batch, and that enables reaching an optimal level of combined production

and inventory costs. However, in order to make it possible it is necessary to prepare an effective planning system that will be integrated with an efficiently operating forecasting system for the future demand for components needed in assembly.

## 2. Identification of problems appearing in an analyzed company

The subject of the research and analysis is a manufacturer of furniture for hair salons and beauty parlours. As in most companies from this sector, production is usually carried out in the *pull* system, i.e., on order by a customer. Only some products (e.g. cheap standard chairs) are manufactured for the warehouse. The demand orientation is dominant in the company. This means that – in most cases – a customer’s contract is initiated just after the order is received and accepted. The order of procedures in the customer service process is presented in Figure 1.

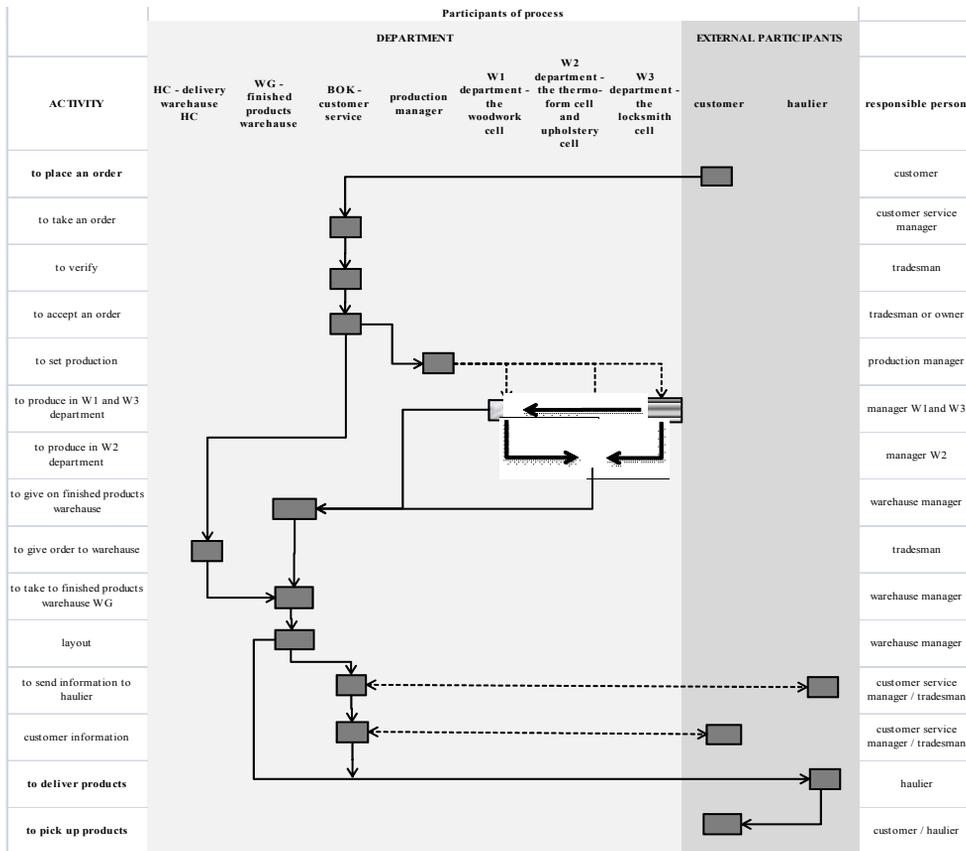
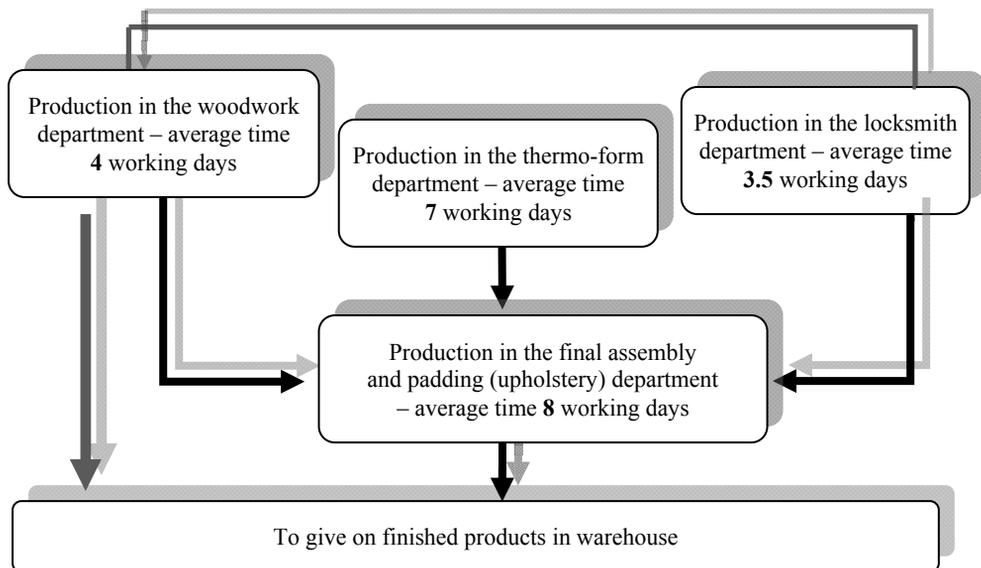


Figure 1. Production process on client’s demand

Source: own study.

The fact that production is carried out in the *pull* system does not mean that the goods are produced in accordance with the project developed with the client (the number of such orders is quite limited). Usually the client orders goods from the current catalogue offer. According to the customer service policy the order time is two weeks (10 working days). This time has been established based on the standards set by competition, and after taking into consideration the production capacity of all departments. The admitted order is processed to become a production order and that initiates production in particular departments – woodwork, locksmith, thermo-form and upholstery. Because each department makes individual elements of the finished product, part of the work can be performed in the parallel system. Average operation times in each department are shown in Figure 2.



**Figure 2.** Production process with manufacturing times in each department

Source: own study.

As you can see, the operations performed in the woodwork department, thermo-form, and locksmith are carried out in parallel. Despite that, the average time from the order to the finished product in the warehouse is 15 days. This means that the declared delivery time for customer service is usually delayed by about one week (5 working days). This causes a significant decrease of the delivery timing rate and that has a negative impact on the quality of services rendered. The resulting situation causes dissatisfaction of current clients and their negative opinion of the company in the market.

It was necessary then to conduct a client's order analysis. Firstly, it was verified whether the actual capacity in each department allows us to carry out the standard

order on time. It appeared that it would be possible if operations in all departments were carried out simultaneously. In fact, such operations are only possible in the woodwork, thermo-form, and locksmith departments. This implies that in the production time we have to be concerned about not only different starting times in different departments but also waiting times at the end of the preceding operations. The delay in the order preparation by even one week went unattended by the managers. This kind of action would weaken the competitiveness of the company's product range. That is why they started to consider other options.

### 3. Proposal of changes

The essential purpose of the project was to meet the order deadlines. Performance times of individual production stages became the centre of the authors' attention. The longest operation time was recorded in the thermo-form department (7 working days) and upholstery (8 working days). These activities were declared the critical path of the process. The operations in the upholstery department could not start before the activities in the remaining three departments finished. They just delivered semi-finished products used later in processing. The thermo-form department then became an area for potential changes.

Due to the fact that clients make orders following the catalogue offer, the idea of whole production process in the *pull* system was analyzed. It appeared that almost all semi-finished products made in the thermo-form department are standardized. It was suggested – in order to shorten the order time – that the production of those elements was carried out in the *for warehouse* system.

Applying the suggested solution required a forecasting system for semi-finished products made in the thermo-form department. At first a production historical data analysis was performed concerning a possibility of demand (requirements) forecasting for the researched semi-finished products. The variable demand analysis set for the thermo-form department allowed us to establish variable data sequences showing the requirement of the components. It appeared that it was possible to set a forecast burdened by a minor error. The oscillation in the demand forecast results as a relation to the actual use reached  $\pm 20\%$  for all semi-finished products made in this department in 2008. A very significant fact concerning the production planning was that the total forecast for the whole year was different from actual requirement volumes by only  $\pm 5\%$ . Because of that it was possible to establish the number of required semi-finished products, and – as a result of that – it allowed us to prepare the necessary inventory in advance.

The change in the production from *pull* to *push* allowed us to attempt to optimize the plan following the set criteria. In order to do so some S&OP operational elements were used. The decision was a result of a literature source analysis following that subject. Several authors see S&OP as a building process for the operation planning based on a consensus – others suggest that it is a technique of fast adjustments in real

time to market changes and operational situations [Smith 2004; Olhager, Rudberg, Wikner 2001; Dwyer 2000; Jain 2005; Grimson, Pyke 2007]. However, the most significant fact is that the system leads to the synchronization of supply and demand through collaboration between managers in the sales, production, and logistics areas. That is why the authors decided to use S&OP model rules in the mechanism of production planning. The simplified procedure consists of five steps.

Production preparation process should begin from setting forecast demand for individual components, subassemblies. The forecast is then translated into production plans which according to the S&OP methodology can be prepared as [Bozarth, Handfield 2007]:

- Top-down planning – applied when supply is similar for different products or services and also when the combination of offered products or services is the same in various periods; it involves creating one aggregate supply forecast;
- Bottom-up planning – applied when supply is different for different products or services and also when the combination of products or services is different in various periods; it involves creating a forecast for each product or group of products individually, and then summing up produced volumes in order to achieve a general picture of future needs.

Created plans can be adaptable, equalized and mixed [Cox, Blackstone (Eds.) 2002; Bozarth, Handfield 2007; Pochet, Wolsey 2006]. Financial factors and the company's capability are decisive regarding which of them is accepted. The production plan for required semi-finished products is carried out before the client's order.

Four kinds of production plans were generated within the developed project:

- 1) Adaptable production plan – when the production volume is changeable, adjusted to the sales forecast;
- 2) Equalized production plan – when the production volume is constant;
- 3) Equalized production plan – when the workload for a given position is constant;
- 4) Mixed plan – no other additional constraints apply.

These plans were compared concerning selected criteria in order to choose the best solution. Due to a differentiated volume supply and kind of individual components it was impossible to apply the top-down planning. The implementation of the bottom-up approach was suggested according to the S&OP model.

#### **4. The analysis of selected production plans**

15 semi-finished products are manufactured in the thermo-form department. However – for the purpose of this article – only three of them were used in the analysis:

- 1) semi-finished product PP 01,
- 2) semi-finished product PP 02,
- 3) semi-finished product PP 03.

In order to establish a production plan, a forecast requirements for individual components needs to be set. The forecast was set for selected semi-finished products for one year in monthly periods. In Table 1 there are results of the established forecast for three analyzed semi-finished products. The forecast is for the year 2008 with historical data from 2005–2007 taken under consideration.

**Table 1.** Demand forecast for three analyzed semi-finished products

Months of 2008	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Demand forecast PP 01 [units]	30	26	30	23	23	20	32	21	20	31	22	20
Demand forecast PP 02 [units]	32	32	30	28	27	17	15	18	10	11	7	2
Demand forecast PP 03 [units]	11	15	13	10	9	5	6	11	7	11	14	15

Source: own study.

While establishing a production plan in the *push* system, managers are usually driven by the production and inventory costs minimization. Considering these two criteria an integer programming problem was defined – one that would let establish production plans for the thermo-form department simultaneously for the few semi-finished products manufactured there. The purpose of the designated problem is minimizing costs not only for one manufactured semi-finished product but for other semi-finished products made in this department as well. In this math problem we were looking for the production volume. The total costs are minimized and these are: inventory costs, production costs, production starting costs (rearm the production machine) and labour costs.

The following assumptions were adopted:

- inventory at the period's end cannot be negative,
- production value should at least equal forecast volume,
- 1 employee works 7.5 hours a day.

At the same time the necessary entry parameters shown in Table 2 were estimated for the tested semi-finished products.

**Table 2.** Data related to production process of three analyzed semi-finished products

	$t^n$ – work time [min/unit]	$k^n$ – production cost per unit [zł/unit]	rearm the production machine cost [zł/rearm]	labour cost [zł/h]
$n = 1$ semi-finished product PP 01	70	46.31	37.50	8.00
$n = 2$ semi-finished product PP 02	10	4.18	37.50	8.00
$n = 3$ semi-finished product PP 03	10	49.92	37.50	8.00

Source: own study.

The following symbols were adopted:

$p_t^n$  – sales forecast for semi-finished product  $n$  in period  $t$  [unit],  $n \in \{1, 2, 3\}$ ,  
 $t \in \{0, \dots, 12\}$ ,

$y_t^n$  – production volume for semi-finished product  $n$  in period  $t$  [unit],  $n \in \{1, 2, 3\}$ ,  
 $t \in \{0, \dots, 12\}$ ,

$up^n$  – number of production starts for semi-finished product  $n$ ,  $n \in \{1, 2, 3\}$ .

For each production plan total costs are calculated ( $Kc$ ):

$$Kc = Km + K_{prod} + K_{up} + K_{lab}. \quad (1)$$

Individual symbols are:

$Km$  – inventory costs [zł]

$$Km = \sum_{n=1}^3 \sum_{t=0}^{12} (0,0125 \cdot k^n \cdot mk_t^n), \quad (2)$$

where: 1,25% – monthly ratio of inventory cost for 1 unit of a semi-finished product,

$k^n$  – semi-finished product  $n$  production cost per unit [zł/unit],

$mk_t^n$  – amount of inventory of semi-finished product  $n$  at the period's end  $t$  [unit],

$mk_0^n = y_0^n$ .

$$mk_t^n = mk_{t-1}^n + y_t^n - p_t^n \text{ with } t \in \{1, \dots, 12\}, \quad (3)$$

where:  $y_t^n, p_t^n$  – above;

$K_{prod}$  – production cost [zł]

$$K_{prod} = \sum_{n=1}^3 \sum_{t=0}^{12} (k^n \cdot y_t^n), \quad (4)$$

where:  $k^n, y_t^n$  – above;

$K_{up}$  – production starting cost [zł]

$$K_{up} = \sum_{n=1}^3 (up^n \cdot 37.50), \quad (5)$$

where: 37.50 – rearm the production machine cost [zł/start],

$up^n$  – number of production starts for semi-finished product  $n$ ,

$$up^n = \sum_{t=0}^{12} up_t^n, \text{ where:} \quad (6)$$

$$up_t^n = \begin{cases} 1 & \text{for } y_t^n > 0 \\ 0 & \text{for } y_t^n = 0; \end{cases}$$

$Klab$  – labour cost [zł]

$$Klab = \sum_{n=1}^3 \sum_{t=0}^{12} \left( y_t^n \cdot \frac{t^n}{60} \cdot 8 \right), \tag{7}$$

where:  $\frac{t^n}{60}$  – working time on semi-finished product  $n$  in hours [h/unit],

8 – employee cost [zł/h],

$y_t^n$  – above.

Four production plans for the thermo-form department were established and compared with the cost function defined in this way.

### 4.1. Adaptable plan

The adaptable plan is presented first. The adopted assumption is that production is the same as the volume of established demand forecast:

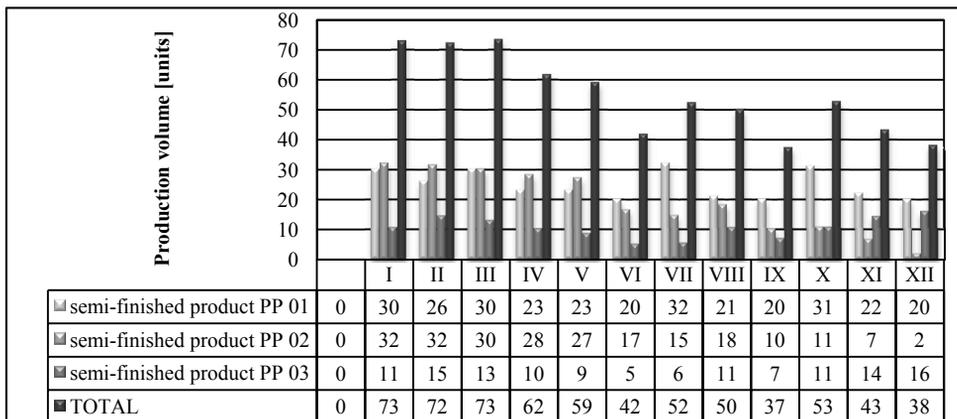
$$y_t^n = p_t^n. \tag{8}$$

Capacity constraint:

$$70y_1^1 + 10y_1^2 + 10y_1^3 \leq 9600, \tag{9}$$

where: 9600 – average working time in one month [min].

A math problem defined this way does not require forming an optimization problem. The adaptable production plan of the three presented semi-finished products is shown in Figure 3.



**Figure 3.** Production plan for semi-finished products PP 01, PP 02 and PP 03

Source: own study.

The total production costs for the established adaptable plan in the thermo-form department was 25,703.62 zł.

### 4.2. Equalized production plan

In the second option the equalized production plan is analyzed. We assume that the production is carried out in a balanced way. That means fluctuations of  $\pm 10\%$  of the average production forecast for a given period according to the demand forecast. Due to that, the production volume will range [45; 55] – average forecast volume for periods from 1 to 12 is 50. For these assumptions we can construct the following problem.

Function is defined by:  $Kc \rightarrow \min.$  (10)

Variable:

$y_t^n$  – production volume for semi-finished product  $n$  in period  $t$  [unit].

Constraints:

$$\sum_{t=1}^{12} p_t^1 = \sum_{t=0}^{12} y_t^1; \sum_{t=1}^{12} p_t^2 = \sum_{t=0}^{12} y_t^2; \sum_{t=1}^{12} p_t^3 = \sum_{t=0}^{12} y_t^3; \tag{11}$$

$$70y_t^1 + 10y_t^2 + 10y_t^3 \leq 9600; \tag{12}$$

$$45 \leq \sum_{n=1}^3 y_0^n \leq 55; 45 \leq \sum_{n=1}^3 y_1^n \leq 55; \dots; 45 \leq \sum_{n=1}^3 y_{12}^n \leq 55; \tag{13}$$

$$y_t^n \geq 0, y_t^n \in C, mk_t^n \geq 0. \tag{14}$$

Solving the above problem we receive a production plan shown in Figure 4.

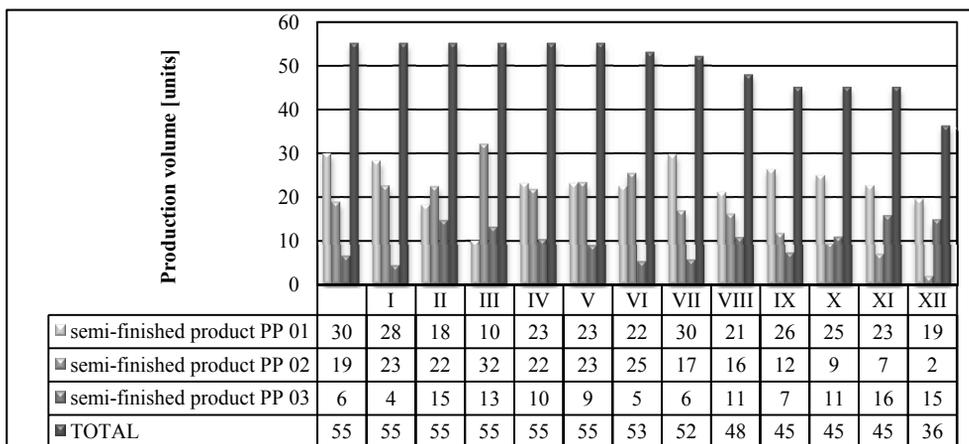


Figure 4. Equalized production plan for semi-finished products PP 01, PP 02 and PP 03

Source: own study.

Total production and inventory costs in 2008 were 25,872.82 zł.

### 4.3. Equalized plan of workload for a given position

Another proposal is an equalized plan of workload. This time we assume that the number of employees will be balanced. We adopt fluctuations of  $\pm 10\%$  from the average number of employees required to produce semi-finished products according to the established supply forecast. It is the number of 4 or 5 workers. The problem has this form:

Function is defined by:  $Kc \rightarrow \min.$  (15)

Variable:

$y_t^n$  – production volume for semi-finished product  $n$  in period  $t$  [unit].

Constraints:

$$\sum_{t=1}^{12} p_t^1 = \sum_{t=0}^{12} y_t^1; \sum_{t=1}^{12} p_t^2 = \sum_{t=0}^{12} y_t^2; \sum_{t=1}^{12} p_t^3 = \sum_{t=0}^{12} y_t^3; \tag{16}$$

$$70y_t^1 + 10y_t^2 + 10y_t^3 \leq 9600; \tag{17}$$

$$4 \leq \sum_{n=1}^3 pp_0^n \leq 5; \dots; 4 \leq \sum_{n=1}^3 pp_{12}^n \leq 5, \tag{18}$$

where:  $pp_t^n$  – number of required workers in period  $t$  for semi-finished product  $n$

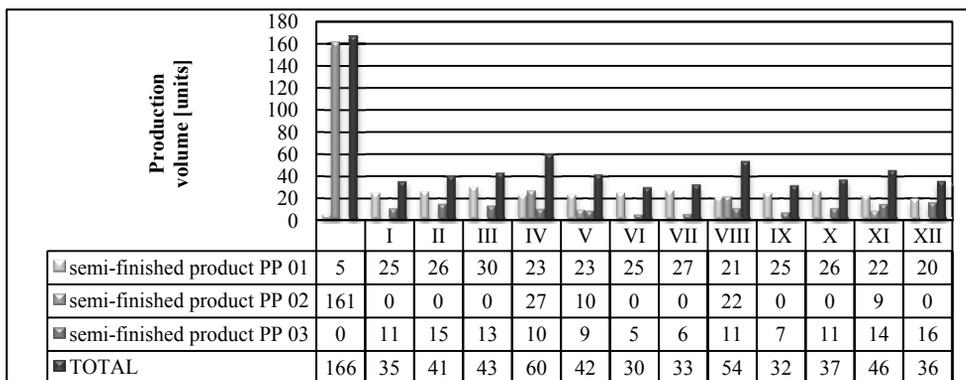
$$pp_t^n = y_t^n \cdot \frac{t^n}{60} / 7.5, \tag{19}$$

where:  $y_t^n$  – above,

$\frac{t^n}{60}$  – working time on semi-finished product  $n$  calculated in hours [h/unit],

7.5 – employee’s working hours – so called day pay [h/1 worker].

$$y_t^n \geq 0, y_t^n \in C, mk_t^n \geq 0, pp_t^n \geq 0, pp_t^n \in C. \tag{20}$$



**Figure 5.** Equalized plan of workload for production positions for semi-finished products PP 01, PP 02, PP 03

Source: own study.

We receive a production structure presented in Figure 5 as the solution of this problem.

Total costs in 2008 were 25,859.32 zł. Typically the semi-finished products whose production time is the shortest are made at the beginning. This allows us to decrease the rearm of the production machine costs.

#### 4.4. Establishing a production plan – mixed plan

The last case is a plan to which we do not introduce any constraints balancing jobs performed at this position (Figure 6). The problem defined like this requires the following model:

Function is defined by:  $Kc \rightarrow \min$ . (21)

Variable:

$y_t^n$  – production volume of semi-finished products  $n$  in period  $t$  [unit].

Constraints:

$$\sum_{t=1}^{12} p_t^1 = \sum_{t=0}^{12} y_t^1; \quad \sum_{t=1}^{12} p_t^2 = \sum_{t=0}^{12} y_t^2; \quad \sum_{t=1}^{12} p_t^3 = \sum_{t=0}^{12} y_t^3; \tag{22}$$

$$70y_1^1 + 10y_1^2 + 10y_1^3 \leq 9600; \tag{23}$$

$$y_t^n \geq 0, \quad y_t^n \in C, \quad mk_t^n \geq 0. \tag{24}$$

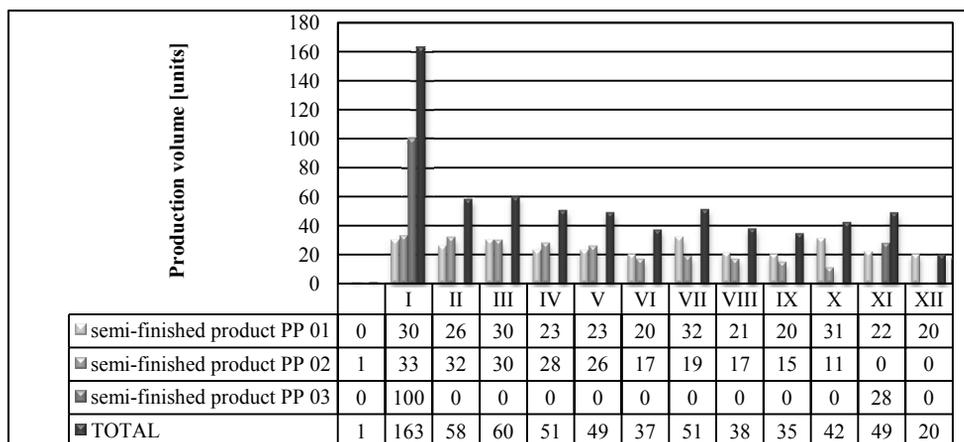


Figure 6. Mixed production plan for semi-finished products PP 01, PP 02 and PP 03

Source: own study.

We receive the total costs of 25,570.95 zł as the solution of the problem.

### 4.5. Comparable analysis of received results

Comparing the received results we can notice that we achieve the highest fluctuations in the production scale with plans III and IV. The situation is shown in Figure 7. We can allow ourselves in this case to sum up production in order to compare volumes.

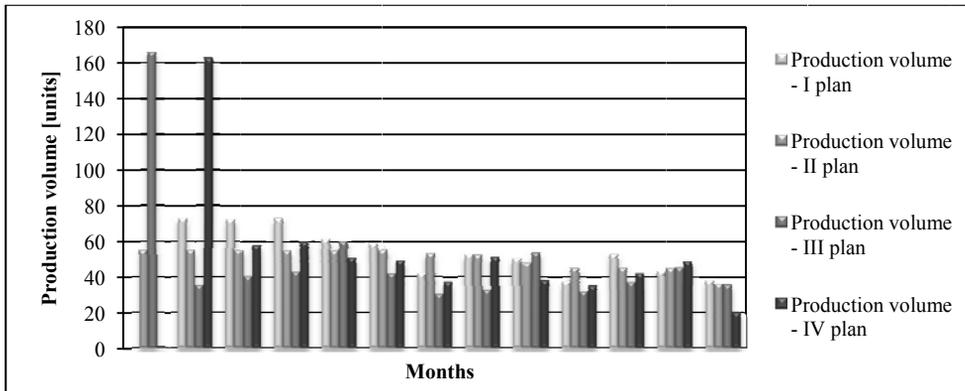


Figure 7. Specification of three production plans for analyzed semi-finished products

Source: own study.

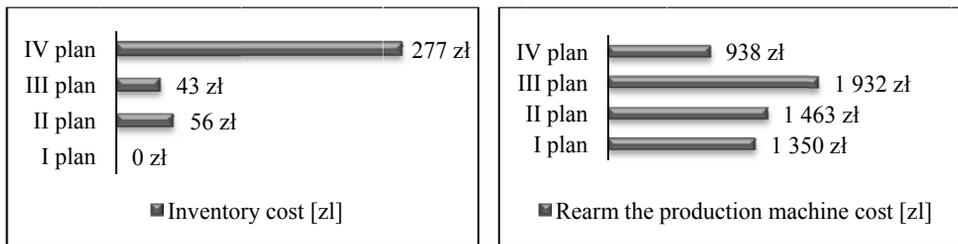


Figure 8. Specification of inventory and production start costs for four analyzed options

Source: own study.

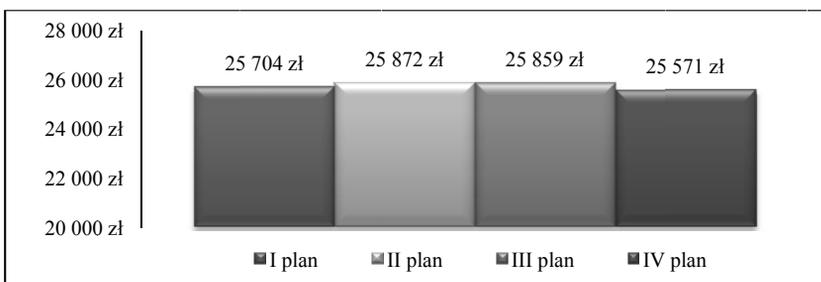


Figure 9. Table of total production and inventory costs for analyzed options

Source: own study.

Comparing the cost specifications (Figure 8–9), we can notice that they do not vary much. This drives you to choose not only the most beneficial for the company cost-wise but other criteria that you cannot overlook. One of them is workload.

We need the following:

option I – from 4 to 6 workers,

option II – from 3 to 6 workers,

option III – from 4 to 5 workers,

option IV – from 4 to 8 workers.

Definitely the option III is the most beneficial for the company when you need 4 to 5 workers to cover that position. This situation secures the workload stability as well as employment in a given production cell.

## 5. Conclusions

The performed analysis of the researched process proved that it makes sense to move the delay of production to the assembly stage. Production of goods from standardized components does not require delaying the whole production process until the client's order is made. Demand for the researched semi-finished products was so characteristic that it became possible to forecast it with a comparably slight margin of error. The implementation of the suggested solution in the thermo-form department would allow us to shorten the whole ordering process by three working days. Apparently an application of the approach model based on the S&OP rules will decide about the success of the operation. This would be the first stage of implementation of the company's goal and that is an improvement of customer service through on-time deliveries. If this step were successful, attempts to forecast production orders in the two remaining departments would have to be taken. This would help to significantly minimize the two-week order management time and achieve a competitive edge over the competitors who still offer the two-week order time.

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## WYKORZYSTANIE MODELI OPTYMALIZACYJNYCH DO WYZNACZANIA POZIOMU ZAPASÓW W PROCESIE PRODUKCYJNYM

**Streszczenie:** Rosnące wymagania klientów dotyczące nie tylko jakości produktów, ale także czasu obsługi w połączeniu z presją kosztową powodują, że menedżerowie coraz częściej poszukują rozwiązań pozwalających im zoptymalizować tworzone plany produkcyjne. Logistyczne wsparcie procesu produkcyjnego przestaje się w związku z tym ograniczać wyłącznie do koordynacji poszczególnych działań i gwarantowania sprawnego przepływu, ale rozszerzone jest o elementy optymalizacji poziomu zapasów produkcji w toku. W artykule przedstawiono przykład przedsiębiorstwa, które przed takim wyzwaniem stało. Przeprowadzono analizę porównawczą różnych typów planowania i podjęto próbę użycia modelu optymalizacyjnego do minimalizacji kosztów magazynowania i produkcji dla jednego z wydziałów badanego przedsiębiorstwa.

**Słowa kluczowe:** planowanie produkcji, planowanie wstępujące, pchający system produkcji.