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PARETO DISTRIBUTION IN NON-PROPORTIONAL REINSURANCE

Summary: In this paper we use the Pareto model to estimate risk premium for excess of loss treaties with high deductibles, where loss experience is insufficient and could therefore be misleading. Pareto distribution is possible use to estimate the unknown frequency of losses exceeding any given high deductible if we know the frequency at a low deductible. Having learnt how to extrapolate frequencies and to determine expected excess losses, we can calculate the expected excess loss burden, or the risk premium. If the risk premium for one layer of an excess of loss programme is known, it becomes possible to extrapolate risk premiums directly for additional layers.

1. Introduction

The worldwide property-liability insurance industry has been rocked by the increasing catastrophes in recent years and increased demand for catastrophe cover (e.g., per occurrence excess of loss reinsurance), leading to a capacity shortage in property catastrophe reinsurance. Catastrophe events in last years are associated with increases in premiums for some lines of business. These market developments are particularly important for non-proportional reinsurance because this coverage is designed to cover the tail of the loss distribution and is triggered only when losses are unexpectedly high.

Modelling of the tail of the loss distributions in non-life insurance is one of the problem areas, where obtaining a good fit to the extreme tails is of major importance. Thus it is of particular relevance in non-proportional reinsurance if we are required to choose or price a high-excess layer. Pareto distribution plays a central role in this matter and an important role in quotation in non-proportional reinsurance.

2. Non-proportional reinsurance treaties

A reinsurance treaty is a contractual agreement between a direct (primary) insurance company and a reinsurance company stipulating which share of future losses will be borne by the reinsurance company and the premium, which the direct insurance company is required to pay to the reinsurance company for this. There are two types of excess of loss treaty:

1. WXL/R (working excess of loss). In this form of non-proportional reinsurance the reinsurer takes on a share X_Z of each loss X in excess of a previously agreed limit a (priority or deductible), albeit only up to a limit L. For this cover the reinsurer requires a premium which is calculated independently from the origin premium. The area between a and a + L is often referred to a layer. This type of treaty protects the direct insurer from individual major losses. This can be written as:

$$X_Z = \begin{cases} 0 & \text{if} \quad X \le a, \\ X - a & \text{if} \quad a < X \le L, \\ L & \text{if} \quad X > L. \end{cases} \qquad (1)$$

Fig. 1 presents a graphic form of WXL/R reinsurance.

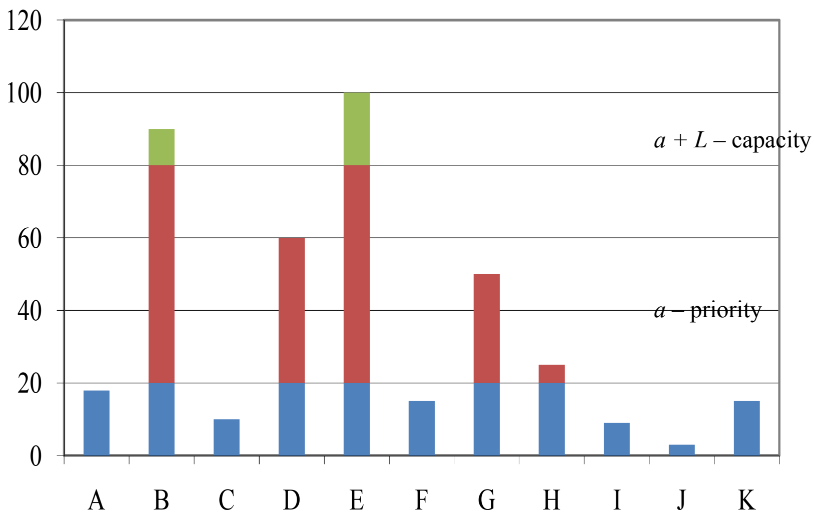

Fig. 1. Non-proportional reinsurance of the type WXL/R

Source: own working-out.

2. CatXL (catastrophe excess of loss). This is a per event cover common in property insurance, whereby the direct insurer retains a deductible a per event (e.g. earthquake, storm, hail). This type of treaty is used if many risks can be affected by a loss event at the same time. This type of reinsurance can be written as:

$$X_z = \begin{cases} 0 & \text{pro} \quad \sum_{i=1}^{n} X_i \le a, \\ \sum_{i=1}^{n} X_i - a & \text{pro} \quad \sum_{i=1}^{n} X_i \rangle a, \end{cases} \qquad (2)$$

3. Pareto model for excess of loss

The Pareto model is often used to estimate risk premiums for excess of loss treaties with high deductibles, where loss experience is insufficient and could therefore be misleading. This model is likely to remain the most important mathematical model for calculating property excess of loss premiums for some years to come [Schmitter 1997].

The Pareto distribution function of the losses X_a that exceed known deductible a is:

$$F_a(x) = 1 - \left(\frac{a}{x}\right)^b, \qquad x \geq a. \tag{3}$$

The density function can be written:

$$f_a(x) = \frac{b \cdot a^b}{x^{b+1}}, \qquad x \geq a. \tag{4}$$

Through this paper we will assume that the lower limit a is known as very often will be the case in practice when the reinsurer receives information about all losses exceeding a certain limit which could for instance be the priority of the excess of loss treaty.

The parameter b is the Pareto parameter and we need it estimate. Let us consider the single losses in a given portfolio during a given period, usually one year. As we want to calculate premiums for XL treaties, we may limit our attention to the losses above a certain amount, the "observation point" *OP*. Of course, the *OP* must be lower than the deductible of the layer for which we wish to calculate the premium [Schmutz, Doerr 1998].

Let losses above this *OP*:

$$X_{OP,1}, X_{OP,2}, \ldots, X_{OP,n} \tag{5}$$

be independent identically Pareto distributed random variables with distribution function:

$$F_{OP}(x) = 1 - \left(\frac{OP}{x}\right)^b, \qquad x \geq OP. \tag{6}$$

The maximum likelihood estimation of Pareto parameter b is given by formula:

$$\frac{n}{\sum_{i=1}^{n} \ln\left(\frac{X_{OP,i}}{OP}\right)}. \tag{7}$$

There are particularly advantageous as it has been in practice shown that a typical value b can be associated with a certain loss potential. The following rules of thumb have been established: earthquake/storm $b \approx 1$, fire $b \approx 2$, fire in industry $b \approx 1{,}5$, motor liability $b \approx 2{,}5$, general liability $b \approx 1{,}8$ and occupational injury

$b \approx 2$ [Antal 2003]. We can thus describe this relation: the smaller the Pareto parameter *b*, the larger the expected loss.

4. Modelling premiums using Pareto distribution

We use the Pareto distribution to model risk premiums. However, the technical premium also contains loadings for administration costs. Risk premiums are usually calculated using the following equation:

risk premium = expected frequency $\times$ expected loss

The frequency is the average number of losses per year. For a given portfolio we should set *OP* low enough to have a sufficient number of losses to give a reasonable estimation of the frequency *LF* (*OP*).

If the frequency at the observation point *OP* is known than it is possible to estimate the unknown frequency of losses exceeding any given high deductible *a* as:

$$LF(a) = LF(OP) \cdot P(X_{OP} \rangle\, a) = LF(OP) \cdot (1 - F_{OP}(a)) = LF(OP) \cdot \left(\frac{OP}{a}\right)^b = \tag{8}$$

$$= \begin{cases} LF(OP) \cdot OP^b \cdot \dfrac{a^{1-b}}{1-b} \cdot (RL^{1-b} - 1) & \text{if} \quad b \neq 1, \\ LF(OP) \cdot OP \cdot \ln RL & \text{if} \quad b = 1. \end{cases}$$

We can see that the lower the Pareto parameter, the higher the frequency of large losses in relation to the frequency of small losses.

Given the Pareto parameter, the deductible and the cover, the expected excess of loss can be calculated. We set as *L* the cover of reinsurer over the deductible *a*. For losses between *a* and *a* + *L* the excess loss is equal to *x* – *a*. Thus, we may calculate the expected excess loss as:

$$EXL = \int_a^{a+L} (x-a) \cdot f_a(x)dx + \int_{a+L}^{+\infty} L \cdot f_a(x)dx = \tag{9}$$

$$= \begin{cases} \dfrac{a}{1-b}(RL^{1-b} - 1) & \text{if} \quad b \neq 1, \\ a \cdot \ln RL & \text{if} \quad b = 1. \end{cases}$$

where $RL = \dfrac{a+L}{a}$ is the relative length of the layer. The risk premium *RP* for our layer can now be calculated as follows:

$$RP = LF\left(a\right) \cdot EXL = \begin{cases} LF(OP) \cdot OP^b \cdot \dfrac{a^{1-b}}{1-b} \cdot (RL^{1-b} - 1) & \text{if} \quad b \neq 1, \\ LF(OP) \cdot OP \cdot \ln RL & \text{if} \quad b = 1. \end{cases} \tag{10}$$

5. Graphical solution of risk premium calculation

The Pareto distribution is very serviceable in many situations. Its main advantages are the following: it can be used in many situations (WXL/R, CatXL). The distribution is characterized with only one parameter. Mathematically it is simple to handle. It is widely used and there is a good amount of knowledge on typical parameter values for certain perils. Another advantage of this is possibility of graphical solution of risk premium calculation.

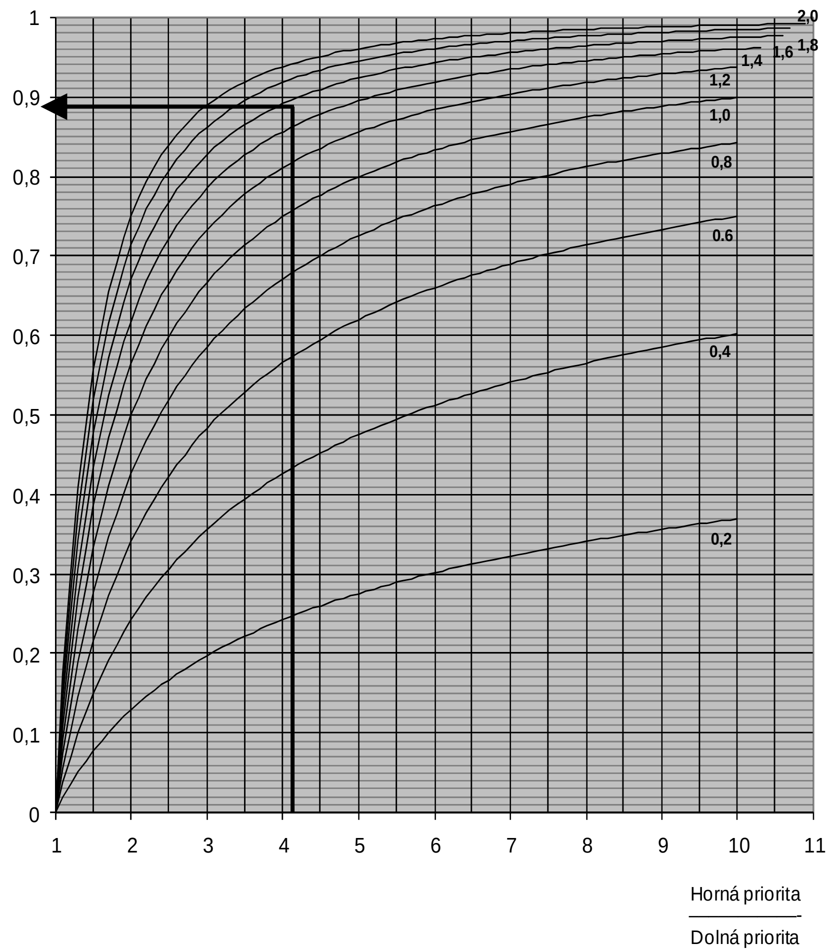

Fig. 2. Pareto distribution functions for various values of parameter *b*

Source: own working-out.

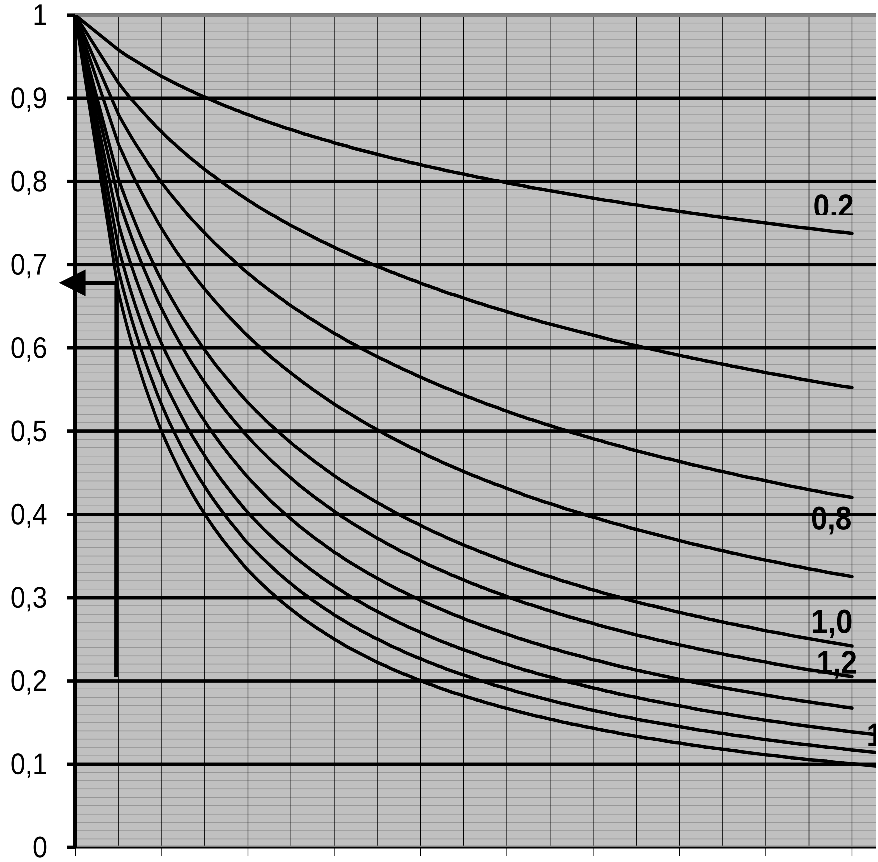

Fig. 3. Pareto distributions: expected excess loss for various values of parameter *b*

Source: own working-out

In practice we can determine risk premium with the help of the curves in Fig. 2 and Fig. 3. To do so, we need to know the Pareto parameter *b*, the deductible *a* and the cover *L*. We can then establish expected frequency and excess loss and hence risk premium using following steps.

Let us assume a Pareto parameter $b = 1{,}6$ and a frequency at the deductible 100 000 $LF(100\,000) = 4{,}5$. Our goal is to calculate premium when $a = 500\,000$ and $L = 500\,000$. First we determine the expected frequency at the deductible 500 000 for the value:

$$\frac{a}{OP} = \frac{500\,000}{100\,000} = 5$$

as $LF(500\,000) = P(X_a > 500\,000) \cdot 4{,}5 = (1 - 0{,}92) \cdot 4{,}5 = 0{,}36$, when with help of the fig. 2 we identified $P(X_a \le 500000) = 0{,}92$.

Secondly, we determine the expected excess loss. With help of the fig. 3 for the value:

$$\frac{a+L}{a} = \frac{500\,000 + 500\,000}{500\,000} = 2$$

we identified curve value 0,57. Then $EXL = 0{,}57 \cdot L = 285\,000$. Finally the risk premium is:

$$RP = LF(500\,000) \cdot EXL = 0.36 \cdot 285\,000 = 102\,600.$$

Literature

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ZASTOSOWANIE MODELU PARETO W REASEKURACJI

Streszczenie: W artykule zastosowano model Pareto do wyceny świadczeń za ryzyko szkód dodatkowych z wysokim wkładem własnym, kiedy doświadczenie w zakresie strat jest niewystarczające i przez to może być mylące. Model Pareto może być użyty do wyceny nieznanej częstotliwości szkód przewyższającej wysoki wkład własny, kiedy znamy częstotliwość z niskim udziałem własnym. Wiedząc, jak oszacować częstotliwość i ustalić przewidywane straty dodatkowe, możemy wycenić obciążenia za ryzyko szkód. Jeśli świadczenie za ryzyko znane jest dla jednej warstwy programu, możliwe jest oszacowanie świadczeń za ryzyko dla warstw dodatkowych.