RESEARCH PAPERS OF WROCŁAW UNIVERSITY OF ECONOMICS
No. 66 — 2009
Towards Information-based Welfare Society

Jiří Trešl
University of Economics, Prague, Czech Republic

MODELLING OF VOLATILITY AT CZECH FINANCIAL MARKETS

Abstract

At present, two principal approaches to volatility modelling exist: the GARCH and stochastic volatility models. In this paper, the GARCH models have been applied both to selected Czech capital market and exchange rates time series. The first aim of investigation consists in possible differences in behaviour with respect to time scale (days, weeks, months). The second level is given by differences between stock prices and exchange rates. Finally, besides univariate models, multivariate GARCH were also used to discover possible relations among different time series.

1. Input data and descriptive methods

The main aim of this study is to compare the behaviour of stock returns and exchange rate returns at Czech financial markets. Input data are given as daily values of corresponding time series values xt during the period 2001-2007, i.e.1757 daily values of stock prices. We have selected four most liquid stocks: **CEZ** (energetics), **KB** (finance), **TEL** (telecommunication) and **UP** (petrochemicals). As for exchange rates, there are 1302 daily values observed in the years 2003-2007, and, namely, **CHF**, **EUR**, **GBP** and **USD**. The subject of our analysis were one-day logarithmic returns expressed as percentage and computed as

$$y_t = 100\,(\ln x_t - \ln x_{t-1})\ . \tag{1}$$

Similarly, 5-day (weekly) and 21-day (monthly) returns were computed too using non-overlapping intervals. Thus, lengths of corresponding weekly time series are 364 (stocks) and 273 (exchange rates). Monthly time series have lengths 86 (stocks) and 65 (exchange rates).

First, some elementary summary statistics related to daily returns were computed. The results obtained are compiled in the following tables.

Table 1. Summary statistics for stock returns

	Average	Median	StDev	LowerQ	UpperQ	InterQ	Skewness	Kurtosis
0.15	0.17	1.95	–0.81	1.20	2.01	–1.11	13.33	
0.75	1.03	3.53	–1.06	2.83	3.89	–0.45	1.89	
2.95	3.87	8.53	–3.19	8.19	11.38	–0.21	0.56	
0.09	0.14	1.83	–0.94	1.08	2.02	–0.08	2.38	
0.43	0.71	3.52	–1.63	2.65	4.28	–0.33	0.72	
1.58	3.00	8.72	–3.57	7.44	11.01	–0.45	–0.01	
0.00	0.02	2.07	–0.82	0.87	1.69	–0.45	5.78	
0.02	0.32	3.90	–1.46	1.90	3.36	–0.62	3.09	
0.08	0.98	9.74	–4.02	4.58	8.60	–0.33	2.96	
0.09	0.12	2.29	–0.75	1.03	1.78	–0.61	10.54	
0.48	0.55	4.63	–1.47	2.68	4.15	–1.39	12.28	
1.79	1.95	11.54	–3.24	7.48	10.72	–0.07	1.93	

Source: own calculations.

Table 2. Summary statistics for exchange rates

	Average	Median	StDev	LowerQ	UpperQ	InterQ	Skewness	Kurtosis
CHF_01	–0.02	–0.02	0.44	–0.26	0.20	0.46	0.15	13.12
CHF_05	–0.11	–0.10	0.64	–0.62	0.37	0.99	–0.07	–0.36
CHF_21	–0.47	–0.52	1.04	–1.34	0.31	1.65	–0.19	–0.43
EUR_01	–0.01	–0.01	0.33	–0.20	0.18	0.38	0.10	2.69
EUR_05	–0.07	–0.06	0.55	–0.42	0.29	0.71	–0.24	0.28
EUR_21	–0.30	–0.44	0.99	–1.00	0.52	1.52	0.03	–0.86
GBP_01	–0.02	0.00	0.50	–0.33	0.29	0.62	0.04	1.18
GBP_05	–0.12	–0.08	0.84	–0.64	0.44	1.08	–0.15	0.22
GBP_21	–0.51	–0.56	1.68	–1.68	0.63	2.31	0.08	0.02
USD_01	–0.04	–0.02	0.66	–0.41	0.35	0.76	0.04	1.33
USD_05	–0.20	–0.24	1.21	–0.98	0.52	1.50	0.26	0.19
USD_21	–0.83	–0.66	2.46	–2.36	1.07	3.43	–0.35	–0.22

Source: own calculations.

2. Univariate modelling

The next step is the possibility of modelling of the return time series. First, the behaviour of daily returns was investigated. In general, ACF values statistically significant at 5% level occurred up to relatively high orders. Therefore, combined

AR-GARCH models suitable for the modelling in the presence of heteroscedasticity were employed. The governing equations are [4; 8]:

$$y_t = \varphi_1 y_{t-1} + \ldots + \varphi_m y_{t-m} + \varepsilon_t \qquad \varepsilon_t = \sigma_t e_t$$
$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \qquad e_t \approx N(0,1) \tag{2}$$

where σ_t denotes conditional standard deviation and et is normal white noise. First, we employed the simplest GARCH (1,1) model, which was quite efficient in most cases. Second, EGARCH (1,1) model was employed to model possible asymmetric reaction with respect to positive and negative shocks et. General form of conditional variance in EGARCH models can be written as [7]

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{q} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right). \tag{3}$$

Clearly, if $\gamma_1 = 0$, then both positive and negative shocks exert the same influence on volatility. On the other hand, for -1< γ_1 0, a positive shocks influence volatility less than negative ones. Indeed, this second case actually occurs, as can be seen from Table 3. In all cases, EGARCH (1,1) model led to slightly better results in comparison with GARCH (1,1).

Table 3. Estimated parameters of AR-EGARCH models for daily stock returns (*SL*=0.05)

	φ (1)	φ (2)	φ (4)	φ (05)	φ (7)	φ (8)	ω	α	β	γ
CEZ_01	0.054	-	-	-	-	0.054	-	0.241	0.892	−0.085
KB_01	0.072	-	0.058	-	−0.047	-	−0.075	0.216	0.924	−0.079
TEL_01	0.048	-	-	-	-	-	−0.174	0.267	0.996	
UP_01	0.052	0.037	-	-	-	-	−0.170	0.420	0.940	−0.090

Source: own calculations.

Clearly, there is strong direct dependence of conditional standard deviation on its previous values, manifested by large values of β parameter. Secondly, β values vary only slightly among individual stocks returns. On the other hand, GARCH (1,1) models proved to be sufficient for exchange rate daily returns and the results are presented in Table 4.

Table 4. Estimated parameters of AR-GARCH models for daily exchange rate returns (*SL*=0.05)

	φ (1)	φ (3)	φ (6)	ω	α	β
CHF_01	−0.056	-	-	-	0.036	0.946
EUR_01	−0.072	-	−0.055	0.002	0.053	0.929
GBP_01	-	−0.052	-	-	0.023	0.971
USD_01	-	-	-	-	0.029	0.967

Source: own calculations.

As for weekly stock returns, the matter is more stratified. There is no need for GARCH model in the case of *KB* and EGARCH is more efficient only for *TEL*. Again, this situation is summarized in Table 5.

Table 5. Estimated parameters of AR-EGARCH models for weekly stock returns (*SL*=0.05)

	φ (1)	φ (2)	φ (3)	φ (4)	φ (5)	ω	α	β	γ
GARCH(1,1)									
CEZ_05	0.307	-	-	-	-	-	-	0.864	-
KB_05	0.190	-	-	-	-	-	-	-	-
UP_05	0.235	-	0.097	-0.113	0.067	-	0.159	0.742	-
EGARCH(1,1)									
TEL_05	0.261	-0.148	-	-	-	0.085	-	0.985	-0.139

Source: own calculations.

On the contrary, the modelling of weekly exchange rate returns does not demand GARCH at all and simple AR models are quite sufficient (Table 6).

Table 6. Estimated parameters of AR models for weekly exchange rate returns (*SL*=0.05)

	CHF_05	EUR_05	GBP_05	USD_05
AR (1)	0.140	0.209	0.201	0.291
AR (2)	−0.121	−0.161	-	-

Source: own calculations.

As for monthly stock returns, classical ARMA models are fully satisfactory (Table 7).

Table 7. Estimated parameters of ARMA models for monthly stock returns (*SL*=0.05)

	CEZ_21	KB_21	TEL_21	UP_21
AR (7)	-	−0.633	-	-
AR(10)	−0.403	-	-	-
MA(7)	-	0.891	0.268	0.271
MA(10)	0.878	-	-	-

Source: own calculations.

The same is true for 21-day exchange rate returns. Moreover, only MA models are needed.

Table 8. Estimated parameters of MA models for monthly exchange rate returns (*SL*=0.05)

	CHF_05	EUR_05	GBP_05	USD_05
MA (1)	-	0.290	0.244	0.283
MA (3)	-	-	-	−0.345
MA (5)	0.427	-	-	-
MA (7)	-	-	-	−0.763
MA (9)	−0.507	-	−0.512	-
MA (10)	-	-	−0.531	-

Source: own calculations.

3. Multivariate modelling

First of all, the values of cross-correlation function for zero time lag were computed. Clearly, it is the case of synchronous correlation between all time series of returns under consideration.

Table 9. Estimated values of sample cross-correlation function for stock returns (*SL*=0.05)

	CEZ	CEZ	CEZ	KB	KB	TEL
	KB	TEL	UP	TEL	UP	UP
Daily	0.465	0.362	0.417	0.408	0.349	0.276
Weekly	0.397	0.294	0.438	0.368	0.256	0.236
Monthly	0.465	0.302	0.558	0.288	-	0.236

Source: own calculations.

Table 10. Estimated values of sample cross-correlation function for exchange rate returns (*SL*=0.05)

	CHF	CHF	CHF	EUR	EUR	GBP
	EUR	GBP	USD	GBP	USD	USD
Daily	0.689	0.456	0.324	0.638	0.531	0.634
Weekly	0.748	0.442	0.425	0.572	0.593	0.615
Monthly	0.739	0.492	0.516	0.660	0.629	0.633

Source: own calculations.

To investigate dynamical dependence among individual returns, Granger causality test was applied in two-dimensional system of jointly stationary time series [1]. We say, variable x Granger cause variable y, if delayed values of x variable improve prediction of y, despite the fact that delayed values of y are introduced as explanatory variables. The model assumed is bivariate VAR(p) in the form

$$\begin{aligned} x_t &= c_1 + \sum_{i=1}^{p} \alpha_{1i} x_{t-i} + \sum_{i=1}^{p} \beta_{1i} y_{t-i} + u_{1t} \\ y_t &= c_2 + \sum_{i=1}^{p} \alpha_{2i} x_{t-i} + \sum_{i=1}^{p} \beta_{2i} y_{t-i} + u_{2t} \end{aligned} \tag{4}$$

Then the test of Granger causality in direction $x \rightarrow y$ can be understood as F-test of parameters $\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2p}$, in regression model (4), whereas the test of Granger causality in direction $y \rightarrow x$ is related to parameters $\beta_{11}, \beta_{12}, \ldots, \beta_{1p}$. The results obtained are summarized in Table 11.

Table 11. Results of testing Granger causality

1-DAY	CEZ → UP	TEL → KB	-	-	-
P-value	0.021	0.047	-	-	-
5-DAY	CEZ → UP	KB → CEZ	UP → KB	-	-
P-value	0.045	0.048	0.008	-	-
21-DAY	-	-	-	-	-
P-value	-	-	-	-	-
1-DAY	USD → EUR	USD → CHF	EUR → CHF		
P-value	0.016	0.014	0.001		
5-DAY	USD → EUR				
P-value	0.029				
21-DAY	USD → EUR	USD → GBP	GBP → USD	GBP → CHF	GBP → EUR
P-value	0.049	0.006	0.005	0.004	0.002

Source: own calculations.

Further, it is of interest to generalize univariate GARCH (1,1) employed formerly to multivariate case. This approach allows us to study time-varying behaviour conditional covariances, which is an important problem in the portfolio theory. A general multivariate GARCH model related to k-dimensional random process et can be written in the form [6]:

$$\varepsilon_t = \mathbf{e}_{\mathrm{t}} \sqrt{\mathbf{H}_t} \qquad \varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{kt})^T, \tag{5}$$

where $\mathbf{e}_{\mathrm{t}}$ is a *k*-dimensional iid process with zero mean and covariance matrix equal to identity matrix. As a generalization of univariate case, $\mathbf{H}_t$ denotes time-varying conditional covariance matrix that needs to be specified. A general representation for the multivariate analogue of the GARCH (1,1) is so-called VEC model [5]:

$$vech\,(\mathbf{H}_{\mathrm{t}}) = \mathbf{\Omega}^* + \mathbf{A}^*\, vech\,(\varepsilon_{t-1}\,\varepsilon_{t-1}{}^T) + \mathbf{B}\, vech\,(\mathbf{H}_{\,t-1})\,, \tag{6}$$

where *vech* operator stacks the lower portion of a matrix in a vector. For example, in the simplest bivariate case, this expression takes the form

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} \omega^*_{11t} \\ \omega^*_{12t} \\ \omega^*_{22t} \end{bmatrix} + \begin{bmatrix} \alpha^*_{11} & \alpha^*_{12} & \alpha^*_{13} \\ \alpha^*_{21} & \alpha^*_{22} & \alpha^*_{23} \\ \alpha^*_{31} & \alpha^*_{32} & \alpha^*_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,\,t-1} \\ \varepsilon_{1,\,t-1}\,\varepsilon_{2,\,t-1} \\ \varepsilon^2_{2,\,t-1} \end{bmatrix} + \begin{bmatrix} \beta^*_{11} & \beta^*_{12} & \beta^*_{13} \\ \beta^*_{21} & \beta^*_{22} & \beta^*_{23} \\ \beta^*_{31} & \beta^*_{32} & \beta^*_{33} \end{bmatrix} \begin{bmatrix} h_{11,\,t-1} \\ h_{12,\,t-1} \\ h_{22,\,t-1} \end{bmatrix} \tag{7}$$

containing 21 parameters to be estimated, and, generally, $(k(k+1)/2)*(1+2(k(k+1)/2))$. Thus, to overcome this shortcoming, diagonal VEC model was constructed with elements [3]:

$$h_{ijt} = \omega_{ij} + \alpha_{ij}\ \varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_{ij}\ h_{ij,t-1} \qquad i,j = 1,2,\dots,k \tag{8}$$

containing generally $3(k(k+1)/2)$ parameters. Again, written explicite for bivariate case $k = 2$

$$\begin{aligned} h_{11t} &= \omega_{11} + \alpha_{11}\ \varepsilon_{1,t-1}^2 + \beta_{11}\ h_{11,t-1} \\ h_{22t} &= \omega_{22} + \alpha_{22}\ \varepsilon_{2,t-1}^2 + \beta_{22}\ h_{22,t-1} \\ h_{12t} &= \omega_{12} + \alpha_{12}\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} + \beta_{12}\ h_{12,t-1} \end{aligned} \tag{9}$$

and there are 9 parameters to be estimated.

Bollerslev developed an alternative approach by assuming time-invariant conditional correlations ρ_{ij} between the elements of ε_t (CCC model). This model can be written as [2]:

$$\begin{aligned} h_{iit} &= \omega_{ii} + \alpha_{ii}\ \varepsilon_{i,t-1}^2 + \beta_{ii}\ h_{ii,t-1} \qquad && i = 1,2,\dots,k \\ h_{ijt} &= \rho_{ij}\sqrt{h_{iit}}\sqrt{h_{jjt}} && i \neq j \end{aligned}$$

The results of computation are summarized in the tables below.

Table 12. Estimated parameters of diagonal VEC-GARCH model for daily stock returns. Parameters ρ_{ij} were computed using CCC model

	φ (1)	ω	α	β	ρ
CEZ	0.055	0.241	0.094	0.851	
KB	0.078	0.145	0.076	0.893	
TEL	0.052	0.019	0.077	0.919	
UP	0.051	0.194	0.131	0.830	
CEZ_KB		0.057	0.044	0.914	0.432
CEZ_TEL		0.032	0.053	0.912	0.401
CEZ_UP		0.071	0.067	0.876	0.389
KB_TEL		0.016	0.041	0.937	0.372
KB_UP		0.051	0.046	0.907	0.352
TEL_UP		0.018	0.052	0.915	0.300

Source: own calculations.

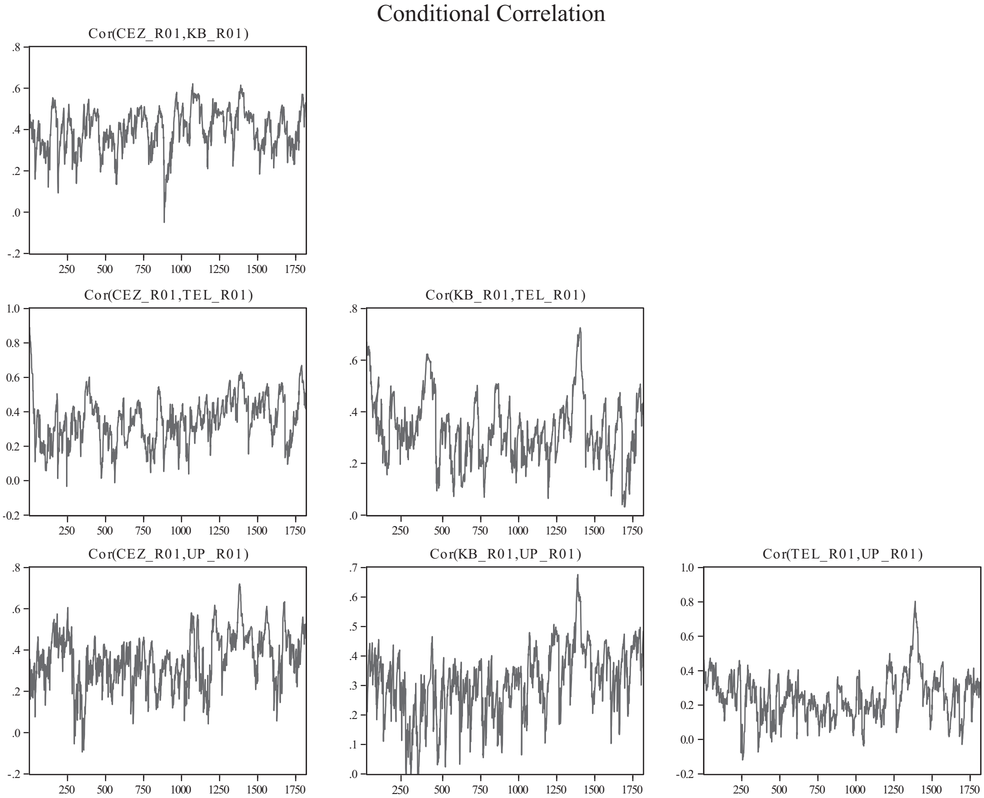

Fig. 1. Time-varying conditional correlation between daily stock returns

Source: own elaboration.

Table 13. Estimated parameters of diagonal VEC-GARCH model for daily exchange rate returns. Parameters ρ_{ij} were computed using CCC model

	φ (1)	ω	α	β	ρ
CHF	−0.078	0.005	0.060	0.904	
EUR	−0.053	0.002	0.044	0.932	
GBP	-	0.002	0.025	0.966	
USD	-	0.003	0.021	0.971	
CHF_EUR		0.002	0.054	0.919	0.758
CHF_GBP		0.002	0.030	0.935	0.475
CHF_USD		0.002	0.038	0.912	0.323
EUR_GBP		0.002	0.029	0.944	0.615
EUR_USD		0.002	0.030	0.939	0.519
GBP_USD		0.001	0.020	0.970	0.616

Source: own calculations.

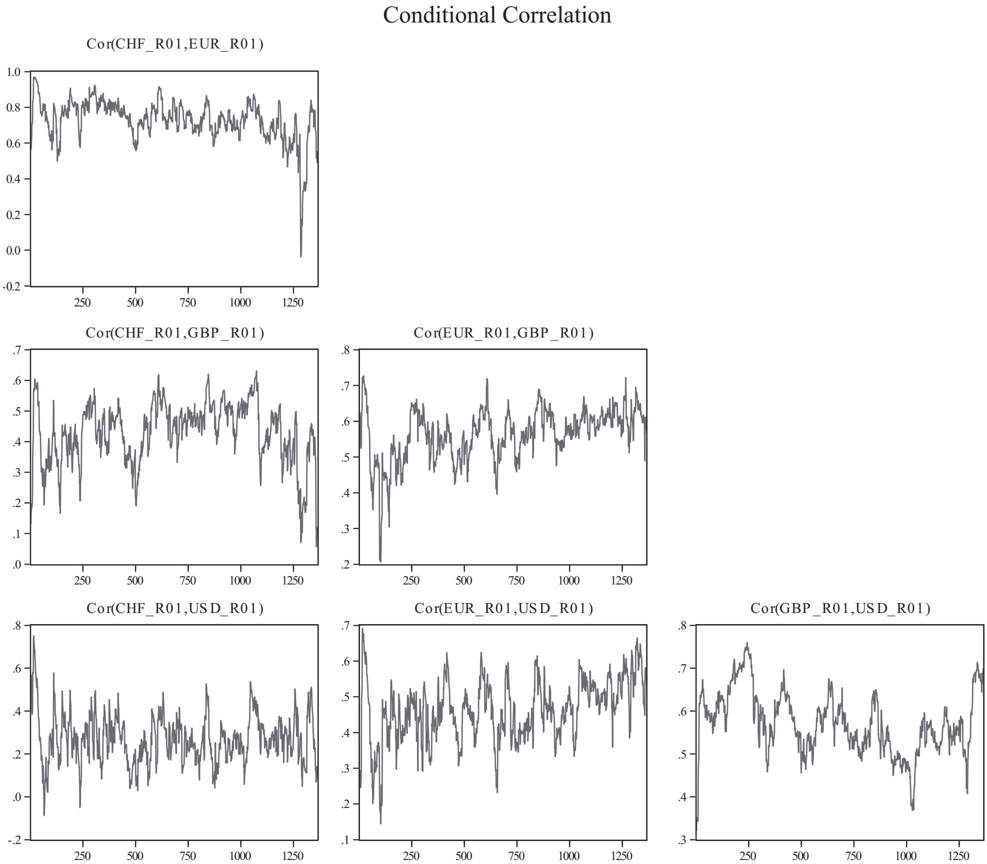

Fig. 2. Time-varying conditional correlation between daily exchange rate returns

Source: own elaboration.

Table 14. Estimated parameters of diagonal VEC-GARCH model for weekly stock returns. Parameters ρ_{ij} were computed using CCC model

	φ (1)	ω	α	β	ρ
CEZ	0.229	-	0.033	0.919	
TEL	0.227	-	0.047	0.941	
UP	0.210	0.772	0.064	0.889	
CEZ_TEL		-	0.073	0.625	0.366
CEZ_UP		-	0.015	0.969	0.393
TEL_UP		-	0.068	0.625	0.309

Source: own calculations.

Again, like in univariate case, there was no need for multivariate GARCH model in the case of weekly exchange rate returns.

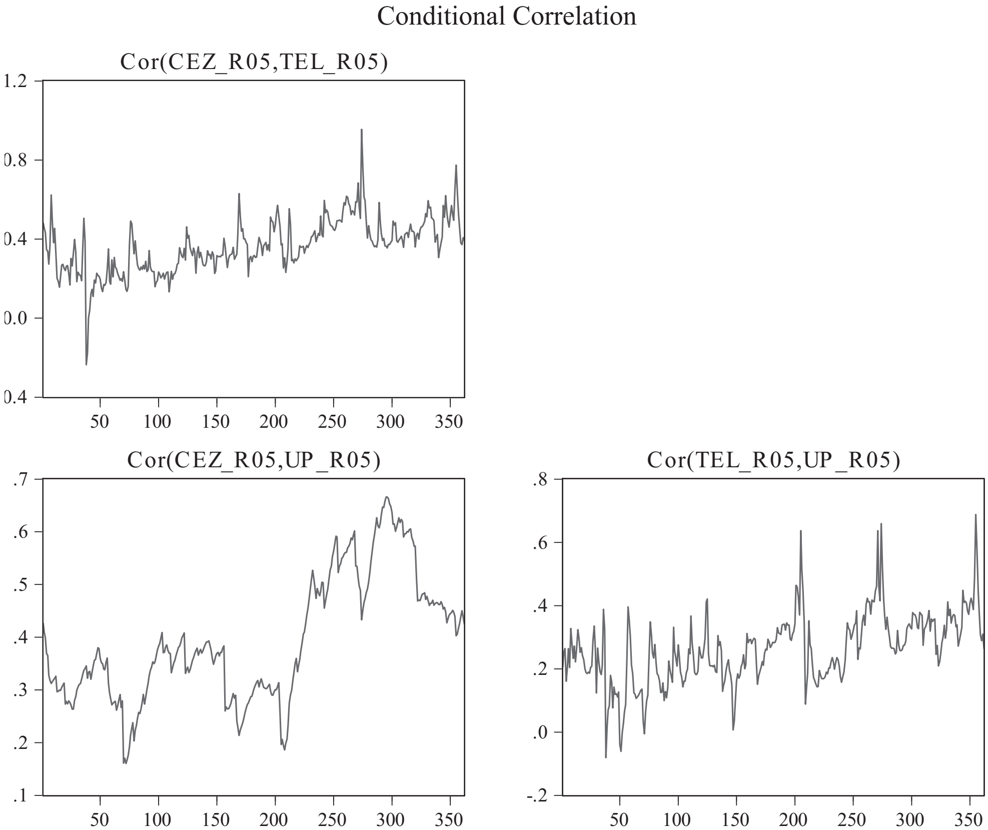

Fig. 3. Time-varying conditional correlation between weekly stock returns

Source: own elaboration.

4. Conclusion

As for descriptive statistics, there is clear tendency to positive kurtosis and negative skewness in the case of daily and weekly stock returns. One the other hand, these findings are not repeated in exchange rate returns. Thus, GARCH models proved to be unavoidable for the modelling of daily and mostly also weekly stock returns and daily exchange rate returns. Secondly, asymmetric EGARCH (1,1) model capable to capture non-symmetry in reaction to positive and negative shocks was needed in the case of daily stock returns. It was manifested that positive shocks influenced volatility less than negative ones. On the contrary, the modelling of weekly and monthly exchange rate returns demands only ARMA models. The same is true also for monthly stock returns.

Further, the values of cross-correlation function were always positive, signalizing some measure of coherent movement among time series. The measure of cross-correlation is higher in the case of exchange rate returns. Granger causality test revealed some directions of possible influence, exerted by lagged values of explanatory time series (for example the influence of lagged CEZ returns on UP ones and lagged USD returns on EUR ones). Finally, the use of multivariate GARCH (1,1) model led to the possibility of modelling dynamical time-varying correlations among individual daily returns. In practice, this can help in the problems connected with portfolio theory.

Bibliography

[1] Alexander C., *Market Models*, Wiley, New York 2001.
[2] Bollerslev T., "Modelling the coherence in short-run nominal exchange rates: Multivariate generalized ARCH approach", *Review of Economics and Statistics* 1990, vol. 72, pp.498-505.
[3] Bollerslev T. et al., "Capital asset pricing model with time-varying covariances", *Journal of Political Economy* 1988, vol. 96, pp.116-131.
[4] Engle R., *ARCH (Selected Readings)*, Oxford University Press, Oxford 1995.
[5] Engle R., Kroner K., "Multivariate simultaneous generalized ARCH", *Econometric Theory* 1995, vol. 11, pp.122-150.
[6] Franses P., van Dijk D., *Non-Linear Time Series Models in Empirical Finance*, Cambridge University Press, Cambridge 2006.
[7] Nelson D., "Conditional heteroscedasticity in asset returns: A new approach", *Econometrica* 1991, vol. 59, pp. 347-370.
[8] Tsay R., *Analysis of Financial Time Series*, Wiley, New York 2002.

MODELOWANIE ZMIENNOŚCI NA CZESKICH RYNKACH FINANSOWYCH

Streszczenie

Istnieją dwa zasadnicze podejścia do modelowania zmienności (volatility): GARCH i modele stochastyczne. W artykule modele GARCH są zastosowane do wybranych szeregów czasowych dotyczących rynku kapitałowego, a także stopy wymiany.

Pierwszy cel badawczy polegał na analizie zróżnicowania zachowań ze względu na zmianę skali czasu (dni, tygodnie, miesiące), drugi zaś na analizie różnicy między cenami giełdowymi a stopami wymiany. Ponadto wielowymiarowa wersja GARCH posłużyła do wykrycia możliwych związków między szeregami czasowymi.